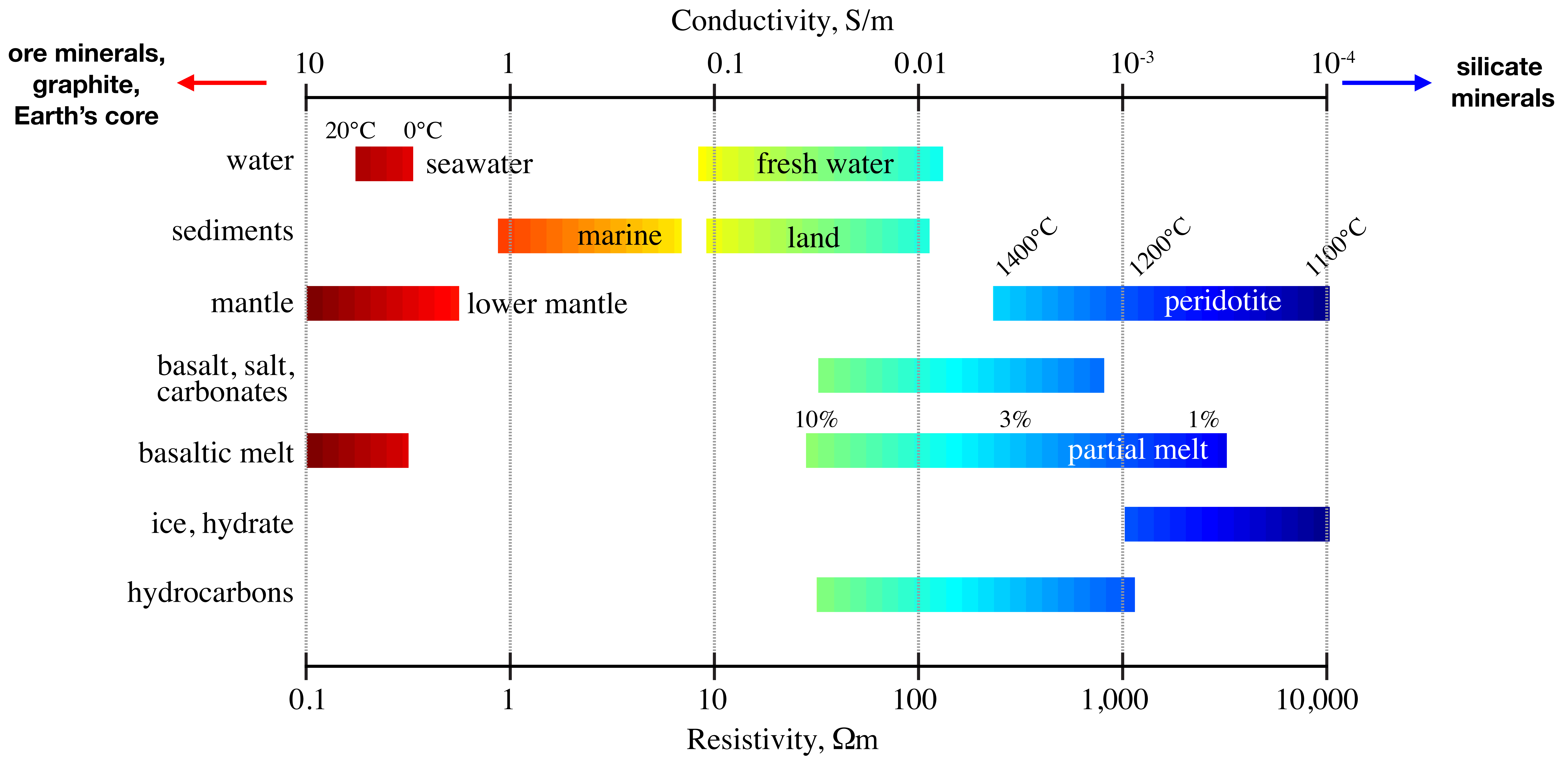
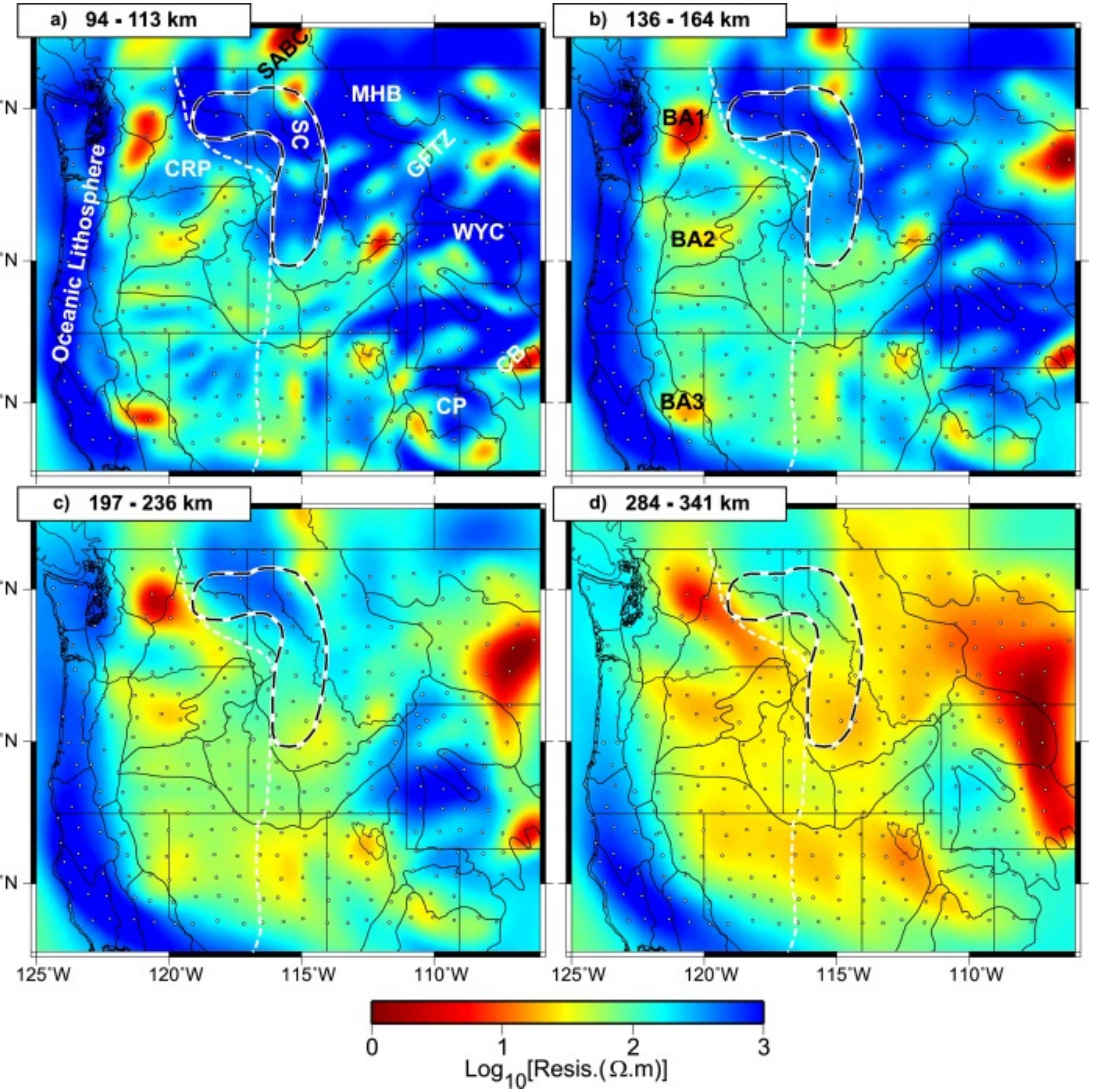
SIO 182

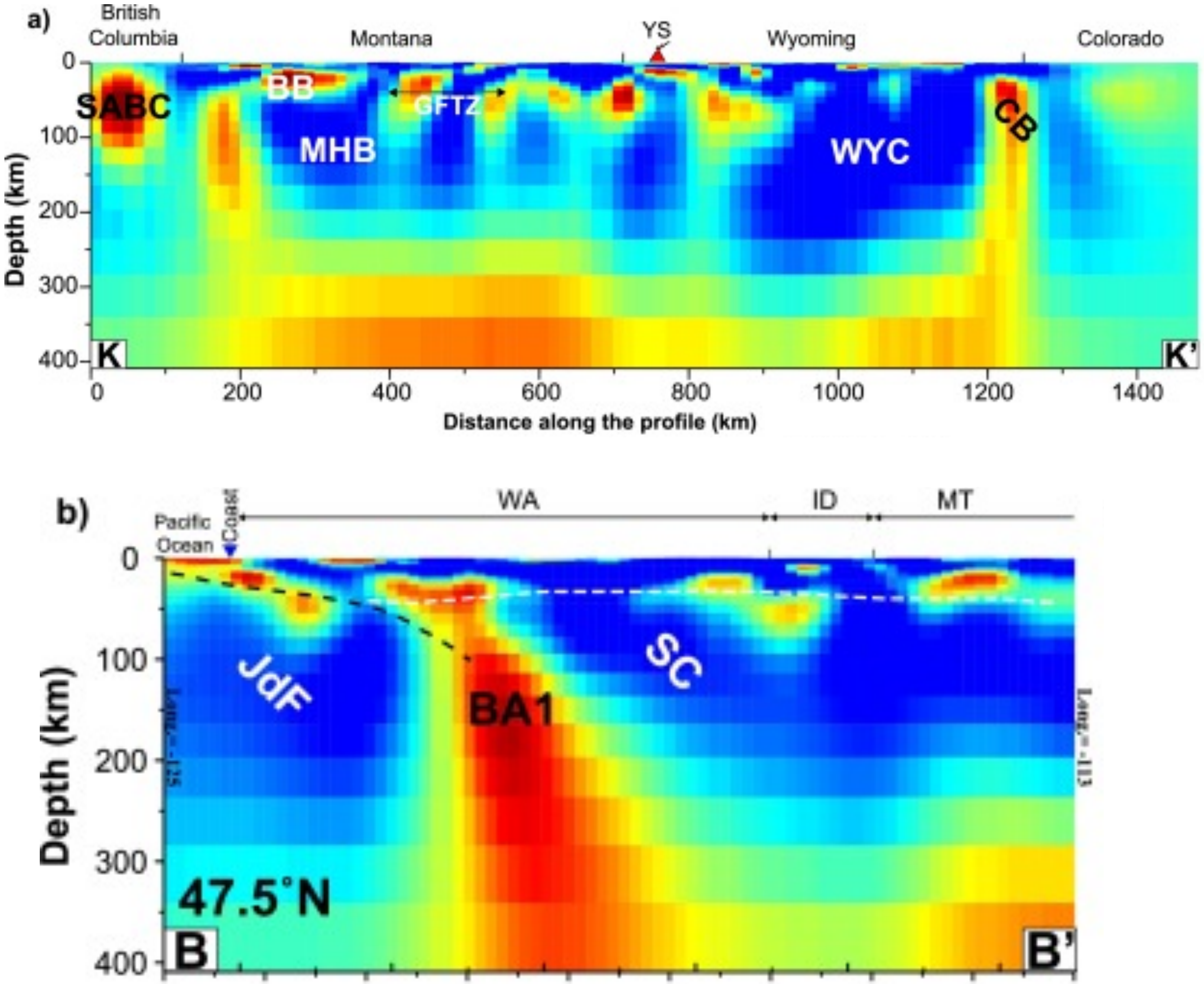
Electromagnetic Induction and the Magnetotelluric Method

Steven Constable

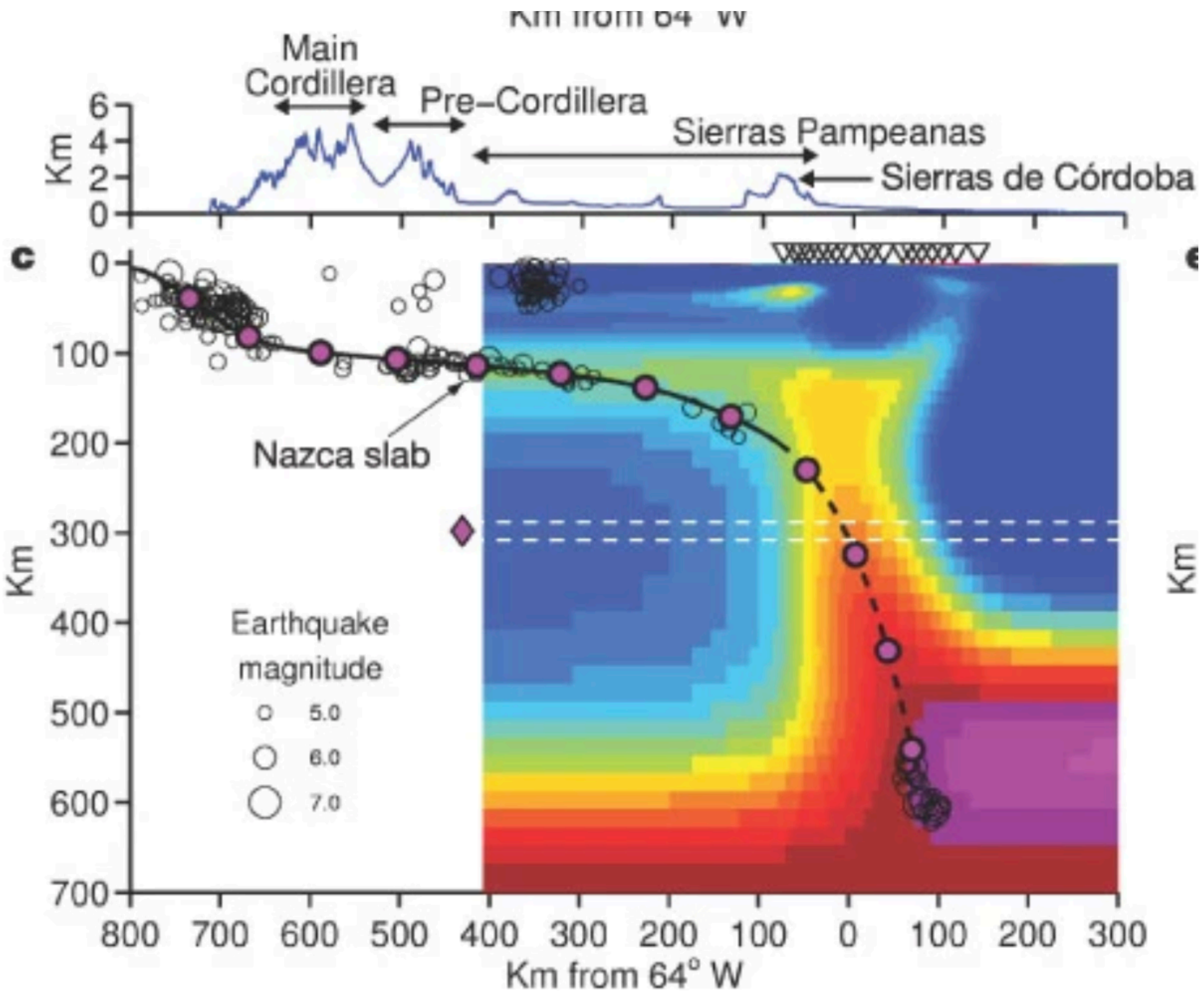
Why electrical methods? Electrical conductivity varies over 5 orders of magnitude in common Earth materials. Conductivity can also be expressed as resistivity:





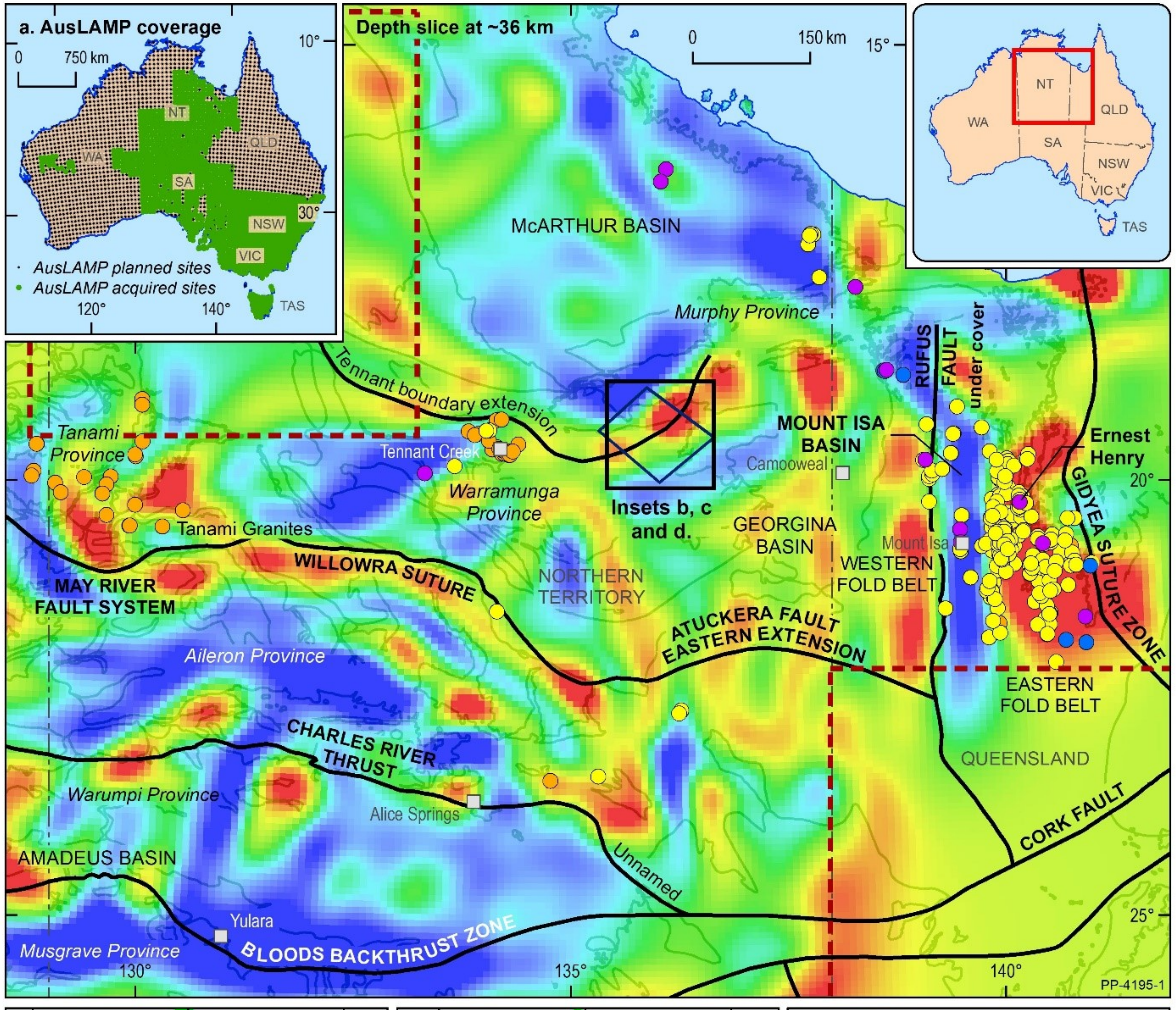


Northwestern USA, from US Array: *Meqbel et al., EPSL, 2014*

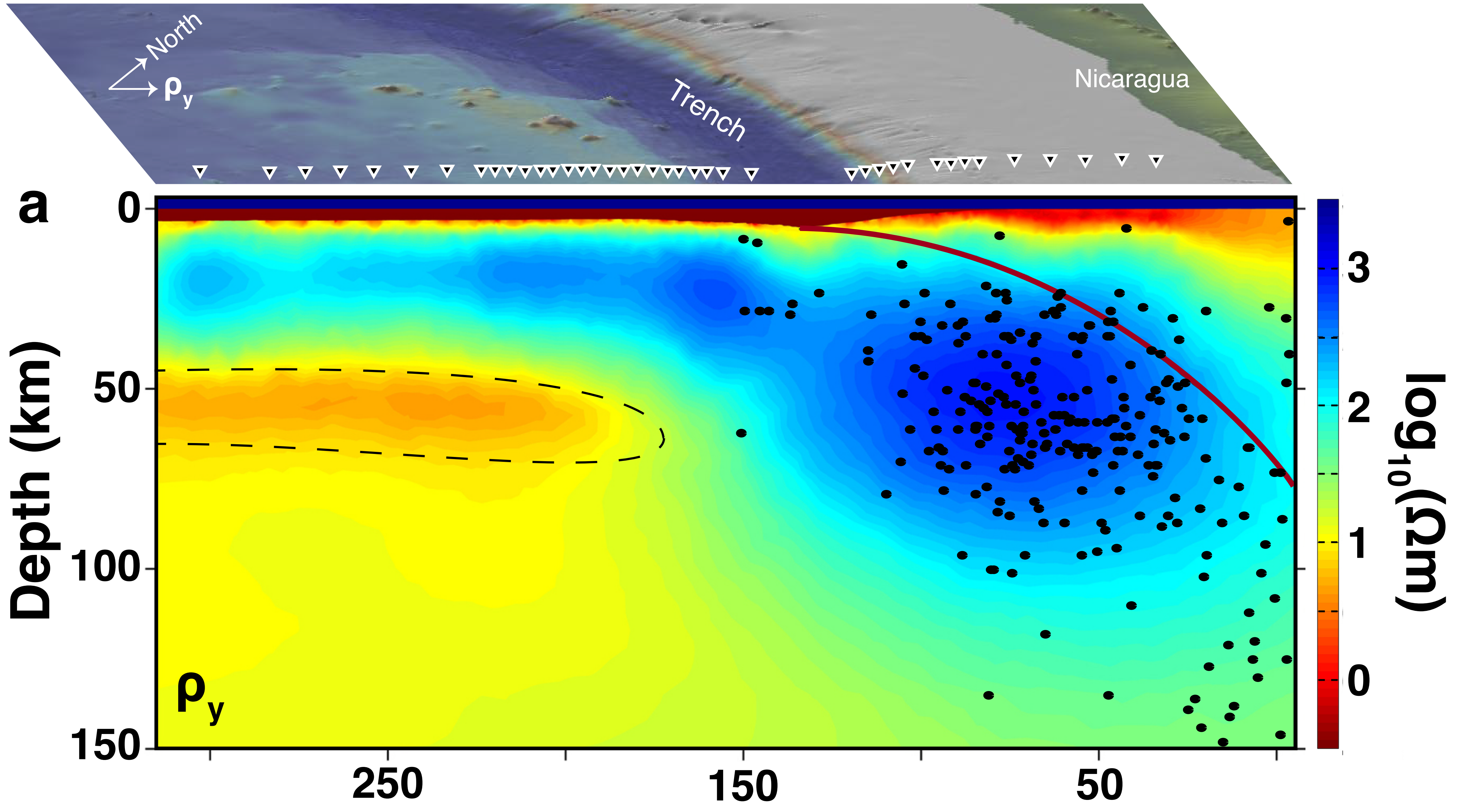


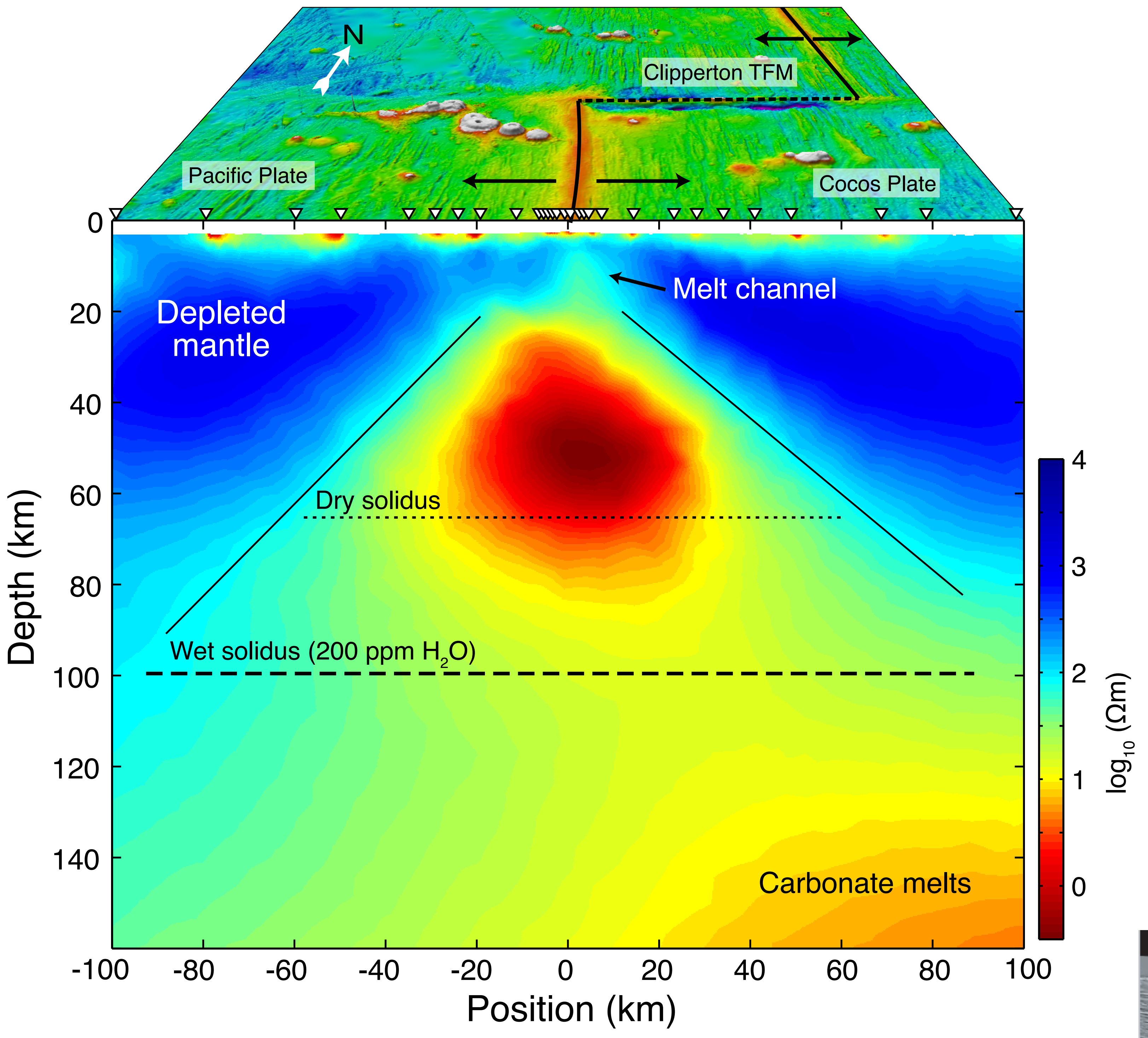
Subduction beneath Argentina: *Booker, Favetto, & Pomposiello, Nature, 2004*

AUSLAMP: *Duan et al., 2021 Geoscience Australia report*

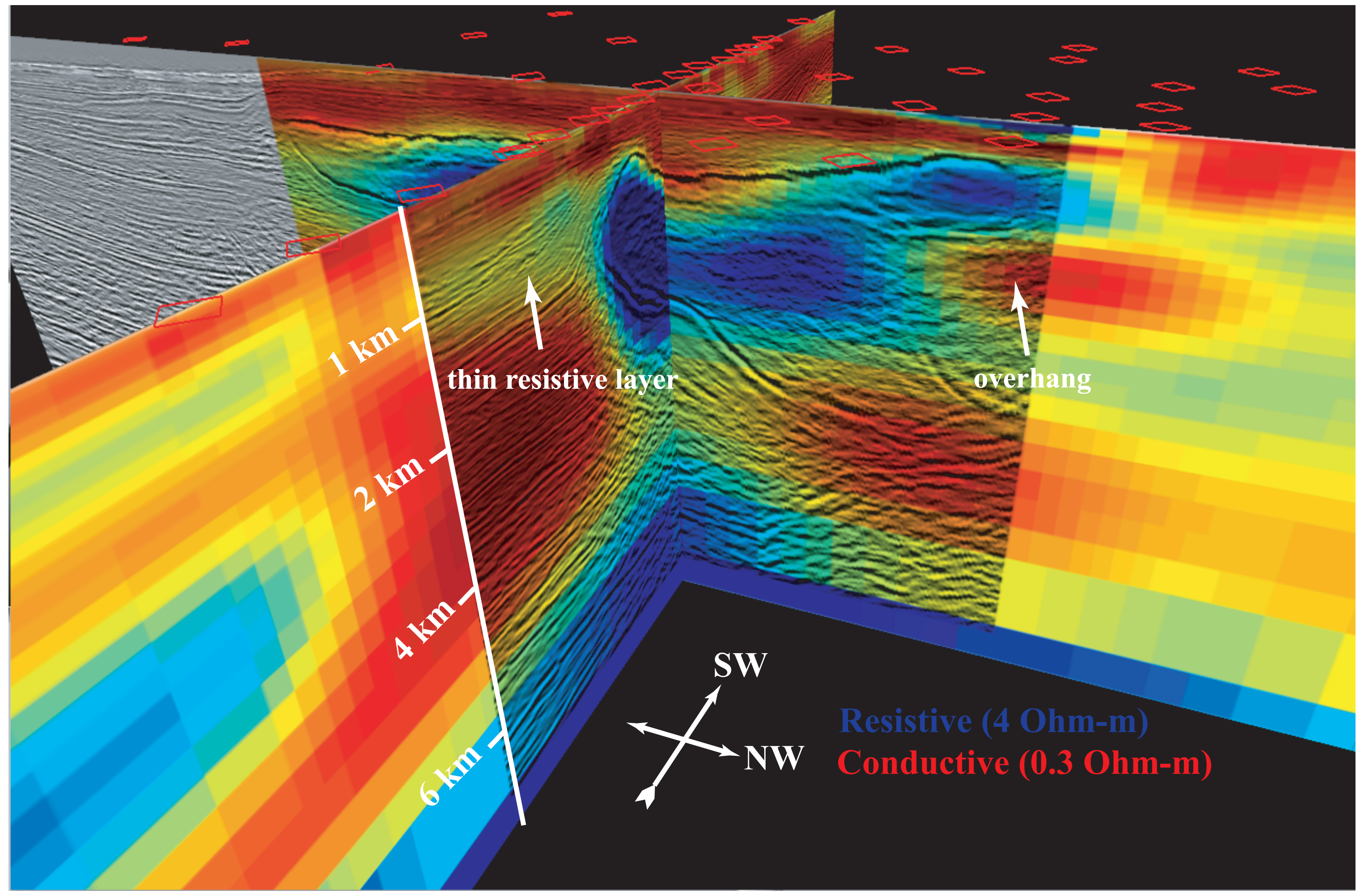


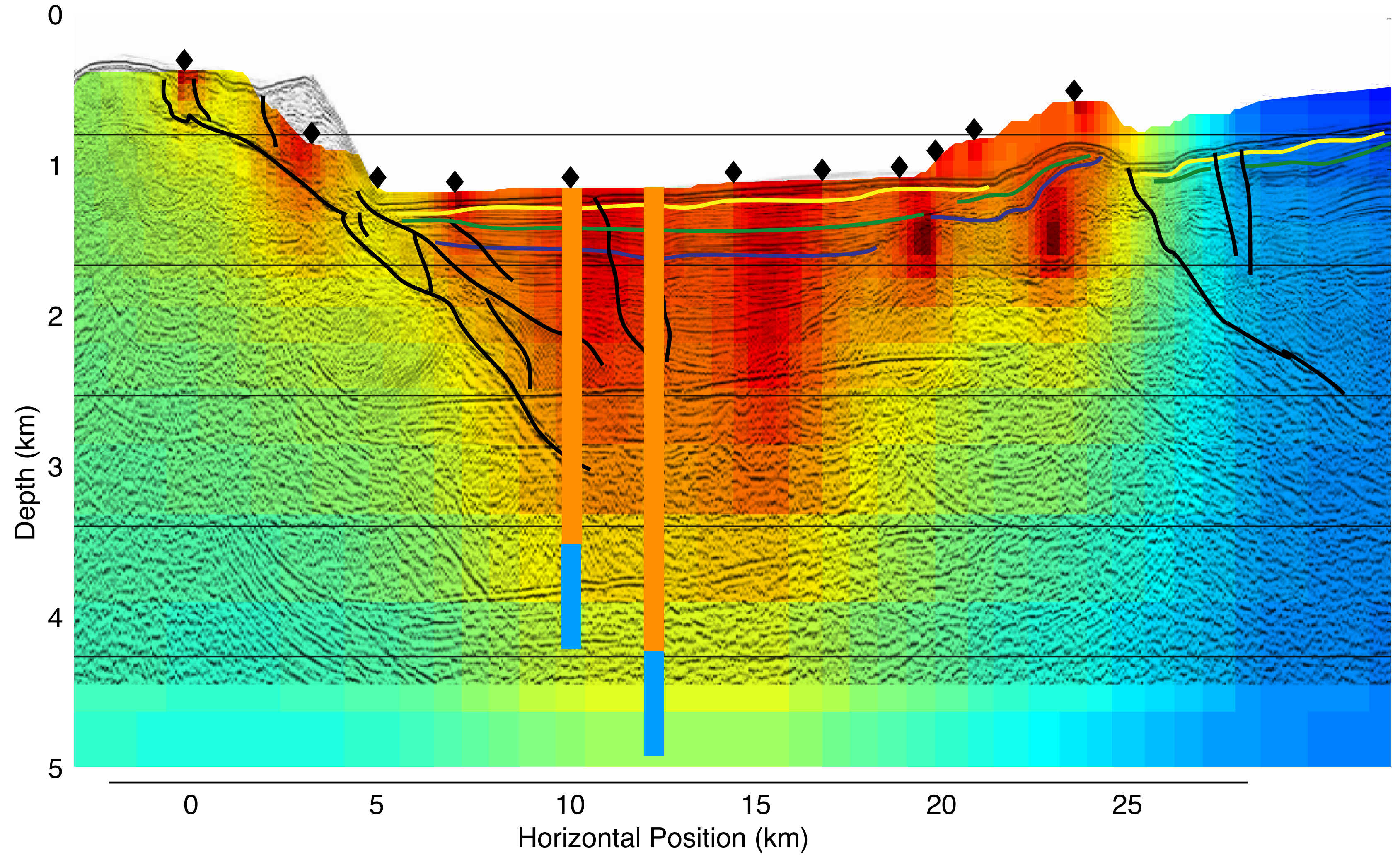
Subduction off Nicaragua





East Pacific Rise





Salt in Gulf of Mexico

Basin structure off California

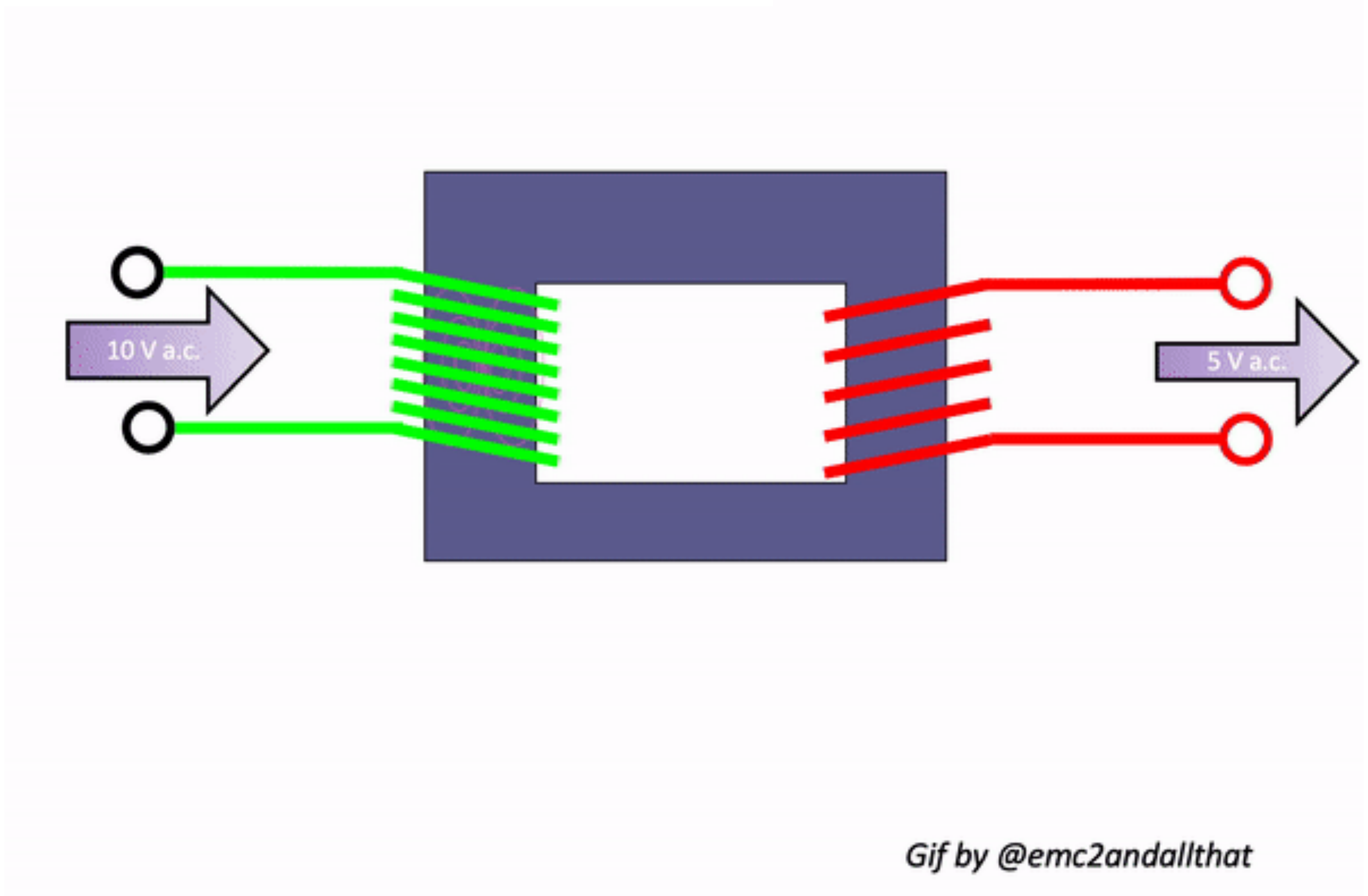
Electromagnetic Induction:

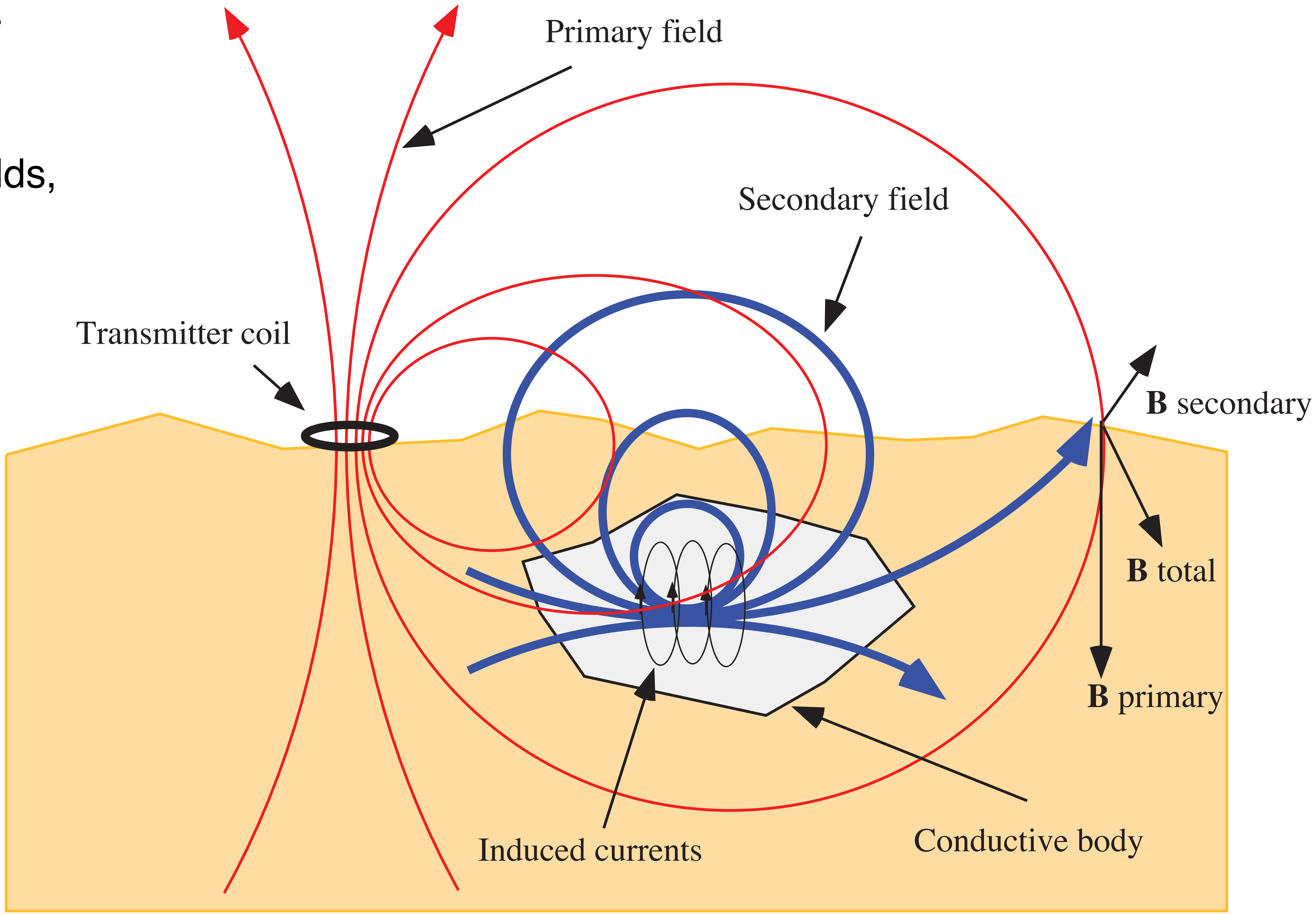
A magnetic field that varies in time or space induces electric currents in conductors, just as in a power transformer.

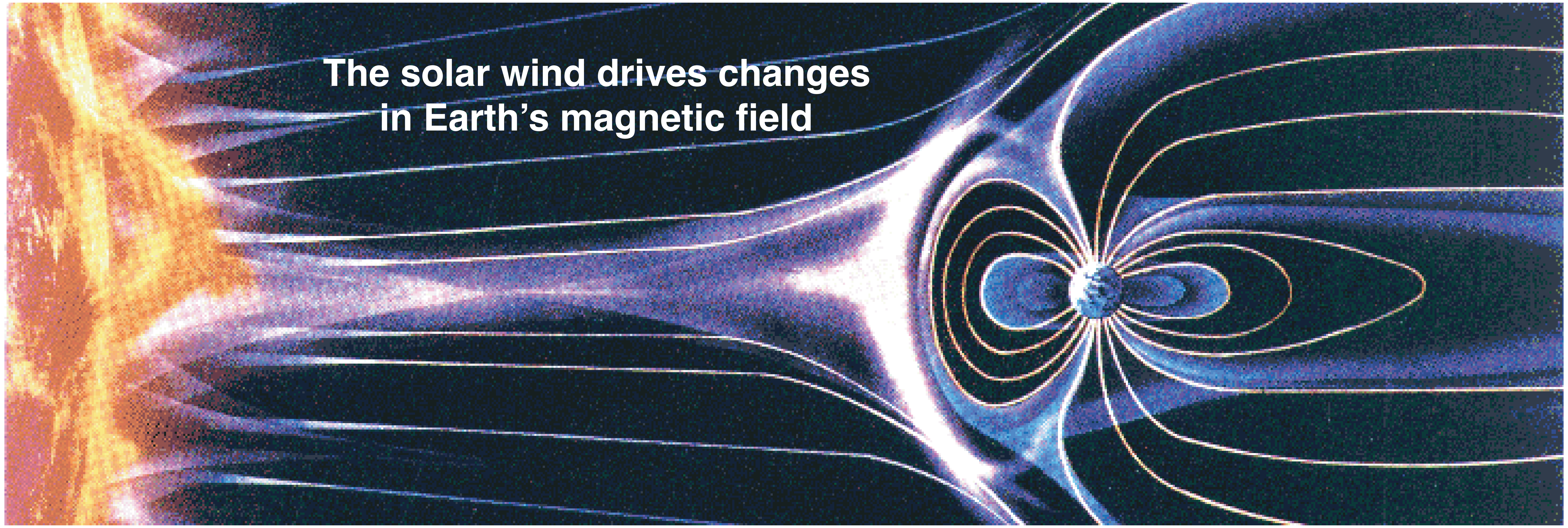
These currents generate secondary magnetic fields, that

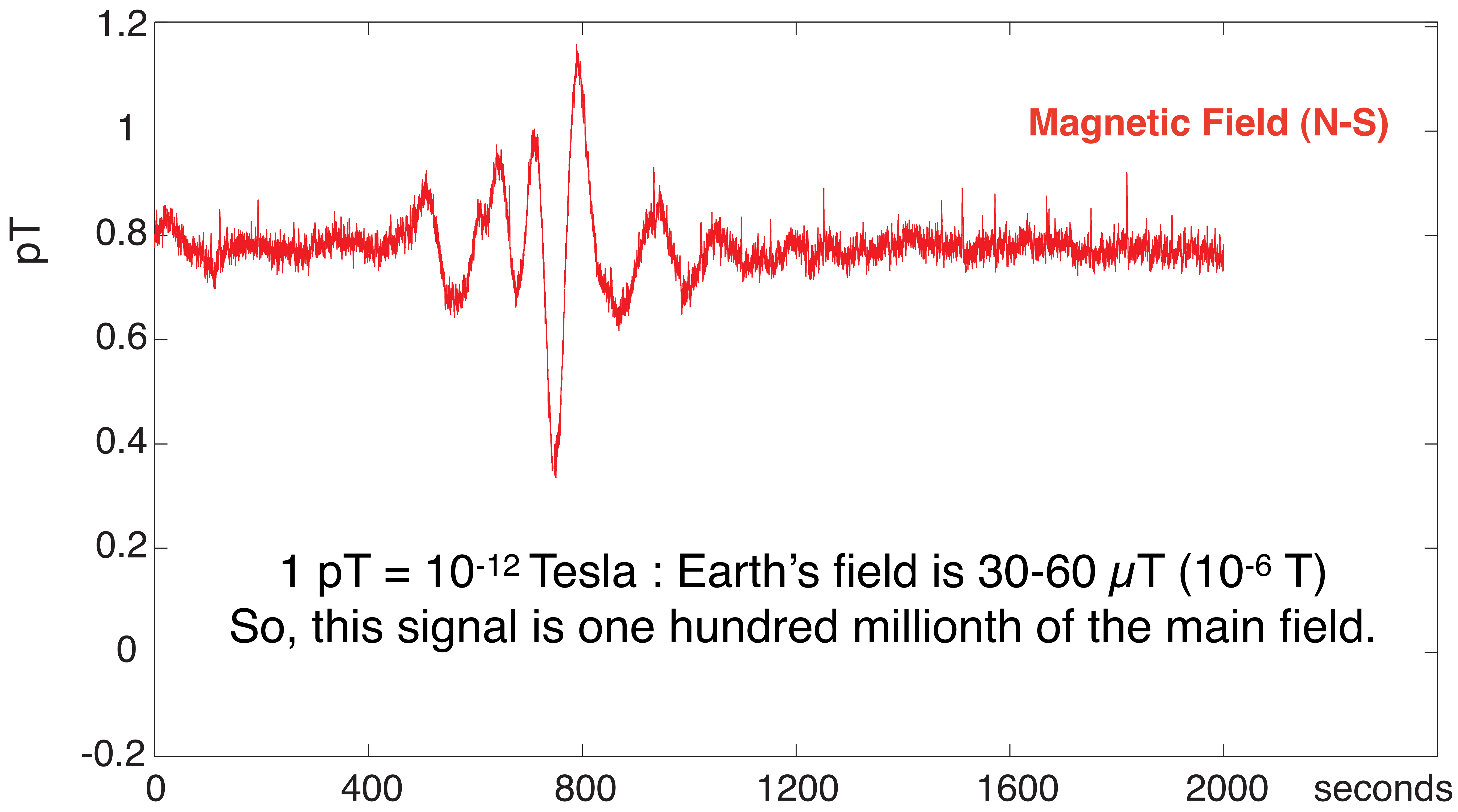
a) are in a different direction
b) are a different magnitude
c) are a different phase

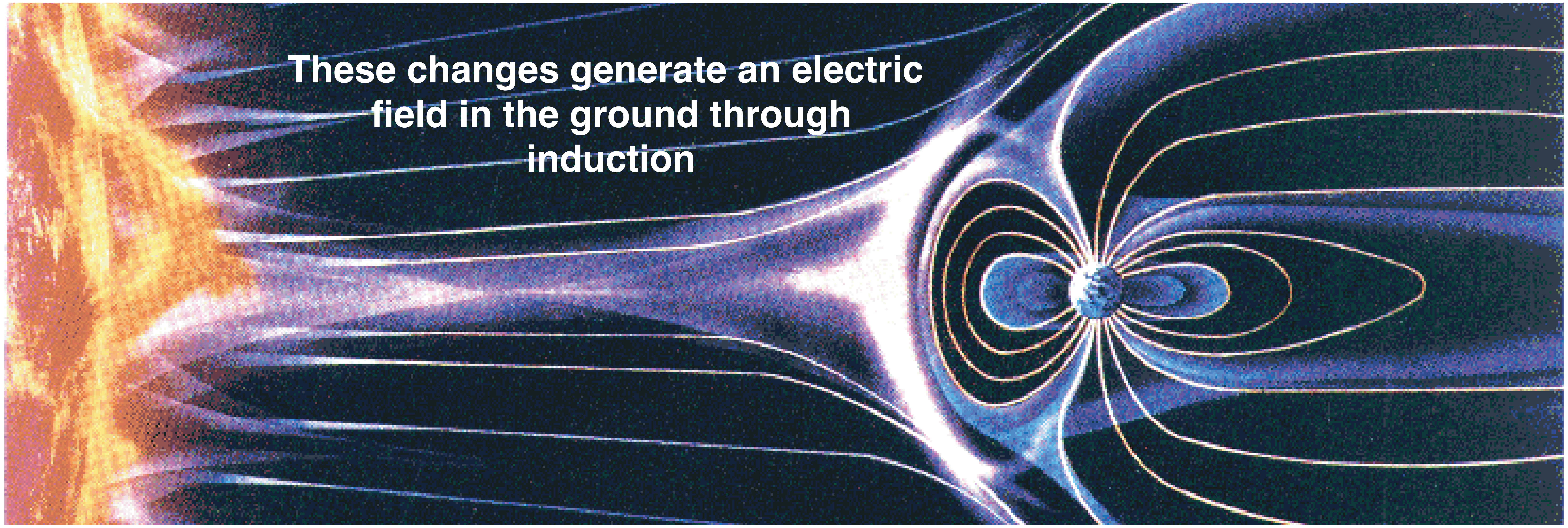
than the primary fields.

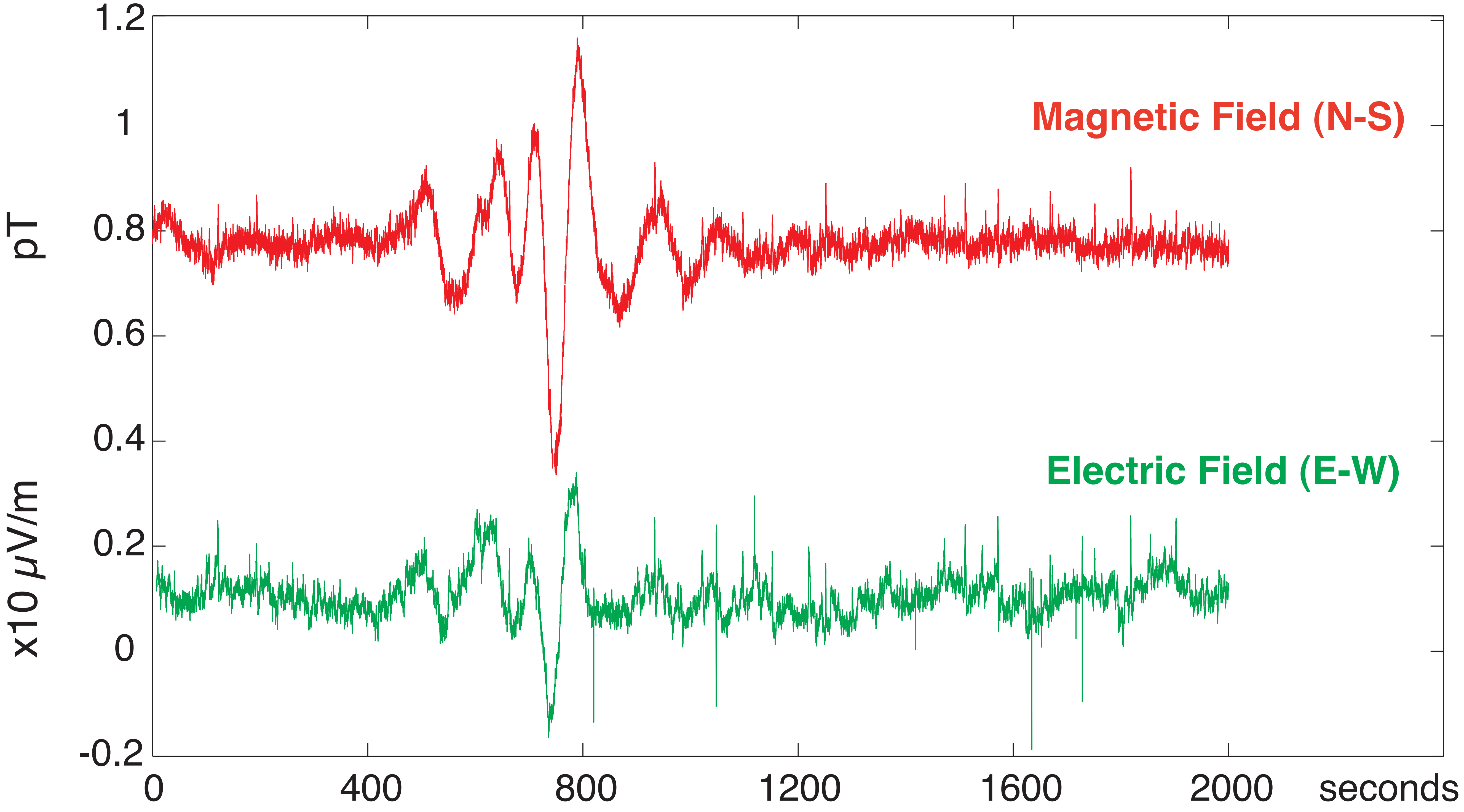





The solar wind drives changes
in Earth's magnetic field


Magnetic Field (N-S)
pT
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
400
800
1200
1600
2000
seconds
1 pT = 10-12 Tesla : Earth's field is 30-60 μT (10-6 T)
So, this signal is one hundred millionth of the main field.

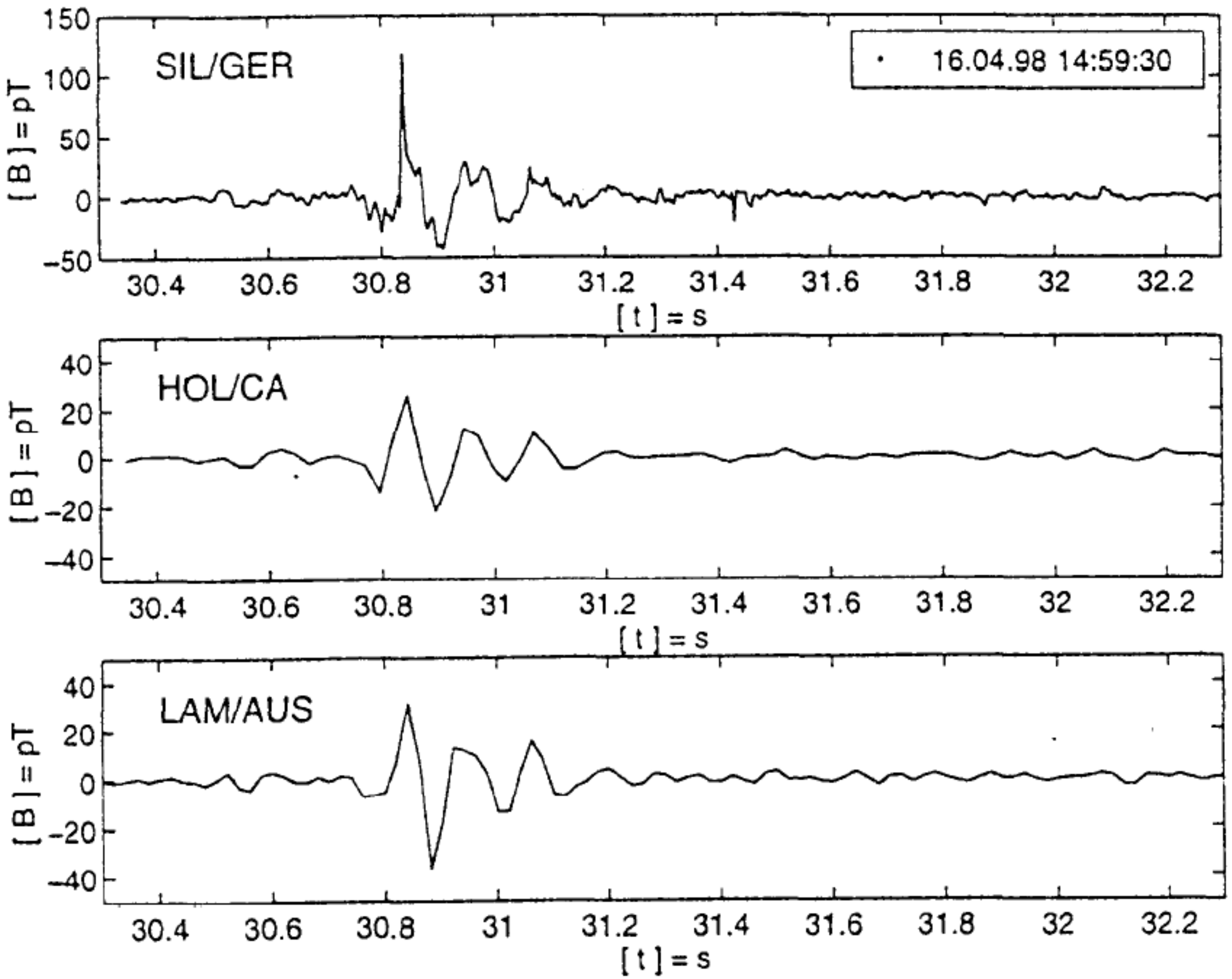

These changes generate an electric field in the ground through induction


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
pT
x10 μV/m
Magnetic Field (N-S)
Electric Field (E-W)
0
400
800
1200
1600
2000
seconds

On land one also sees signals from global lightning strikes.

Global lightning for April 1998 by triangulation:

Füllekrug and Constable, 2000



A single lightning strike in April 1998 measured on three magnetometers.

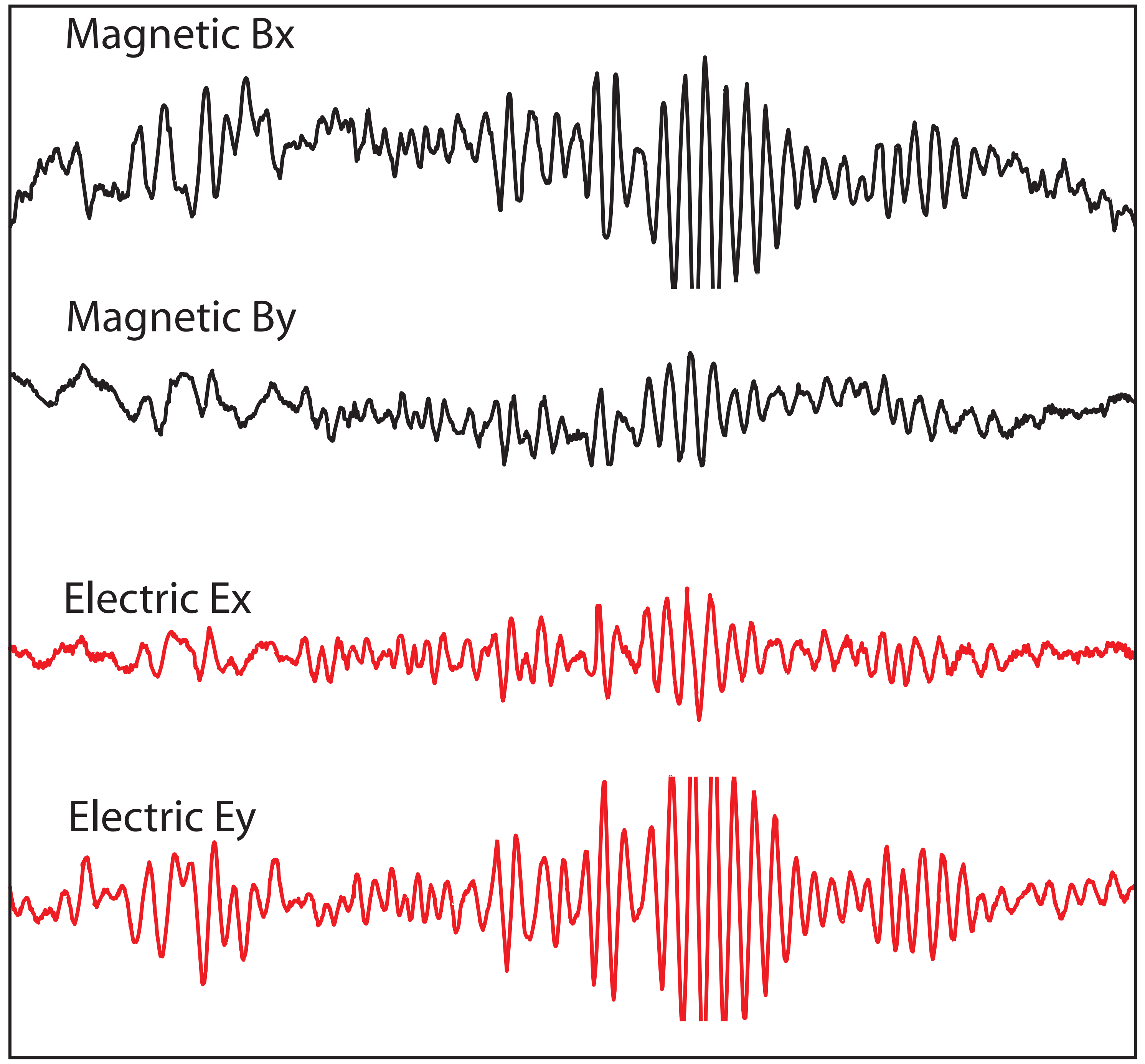
9

Work Flow

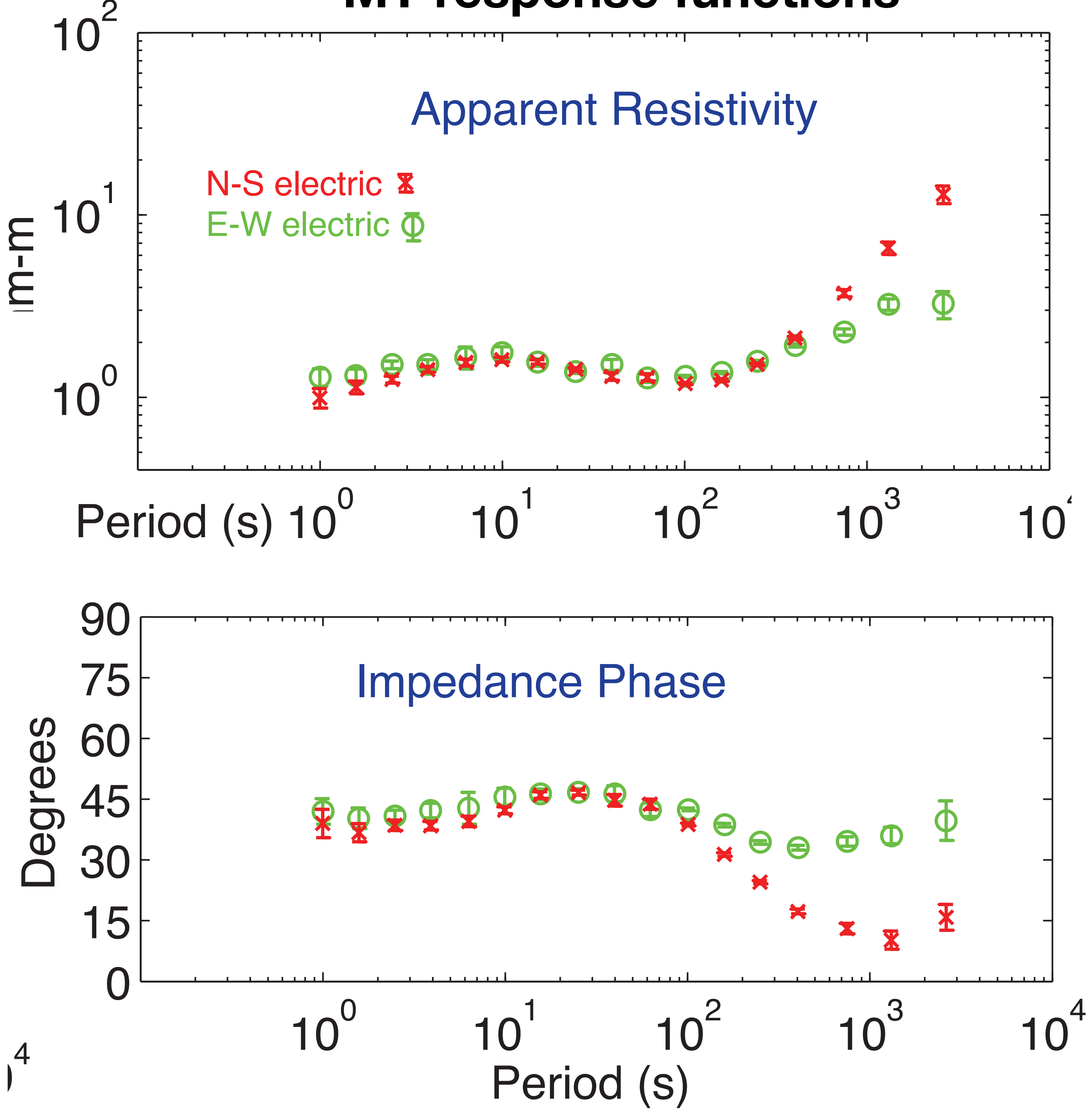
Inversion

Processing → **Inversion** →

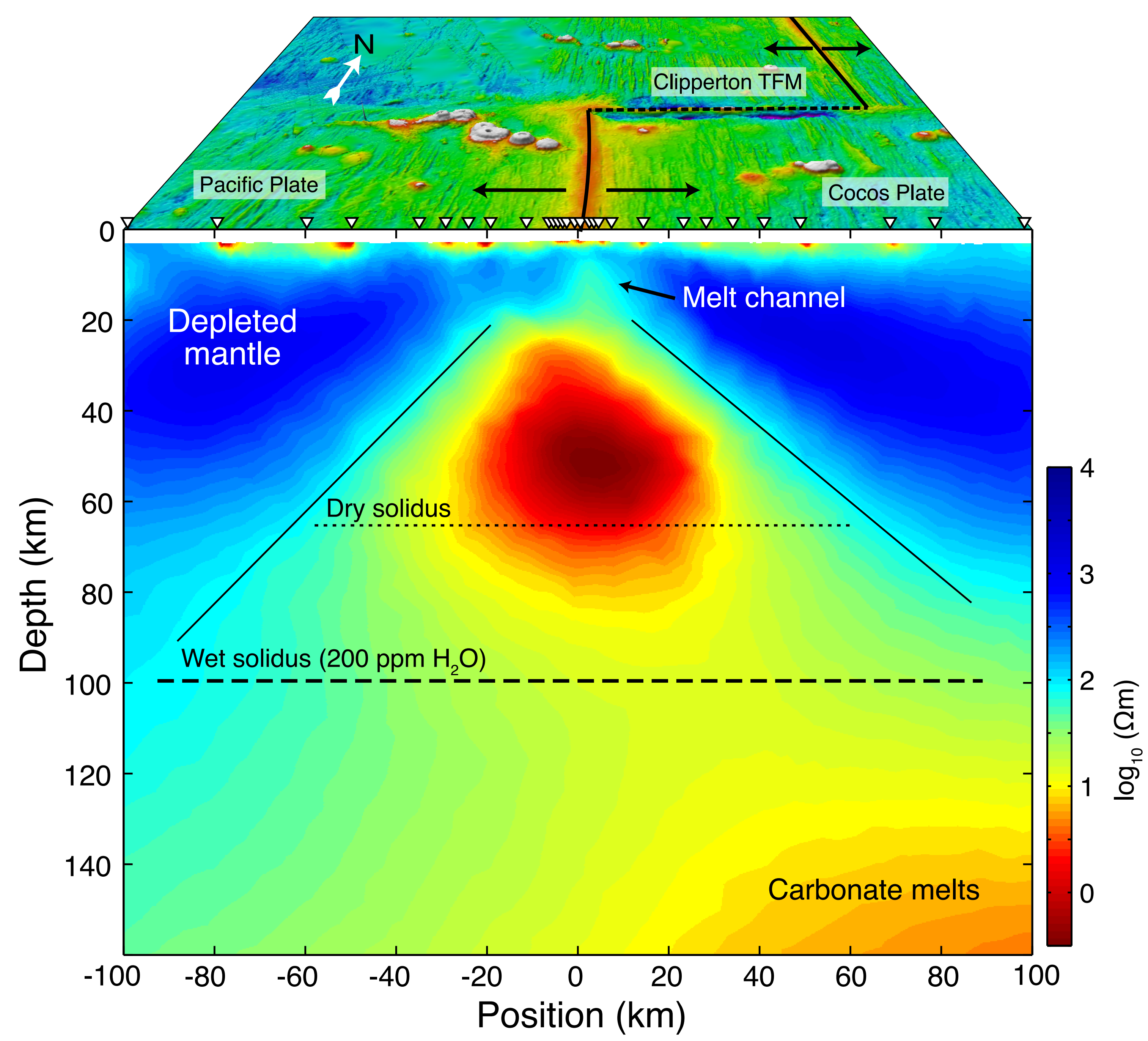
MT timeseries



MT response functions



Resistivity model



Lab data/models

↓

Melt fraction

Water salinity/porosity

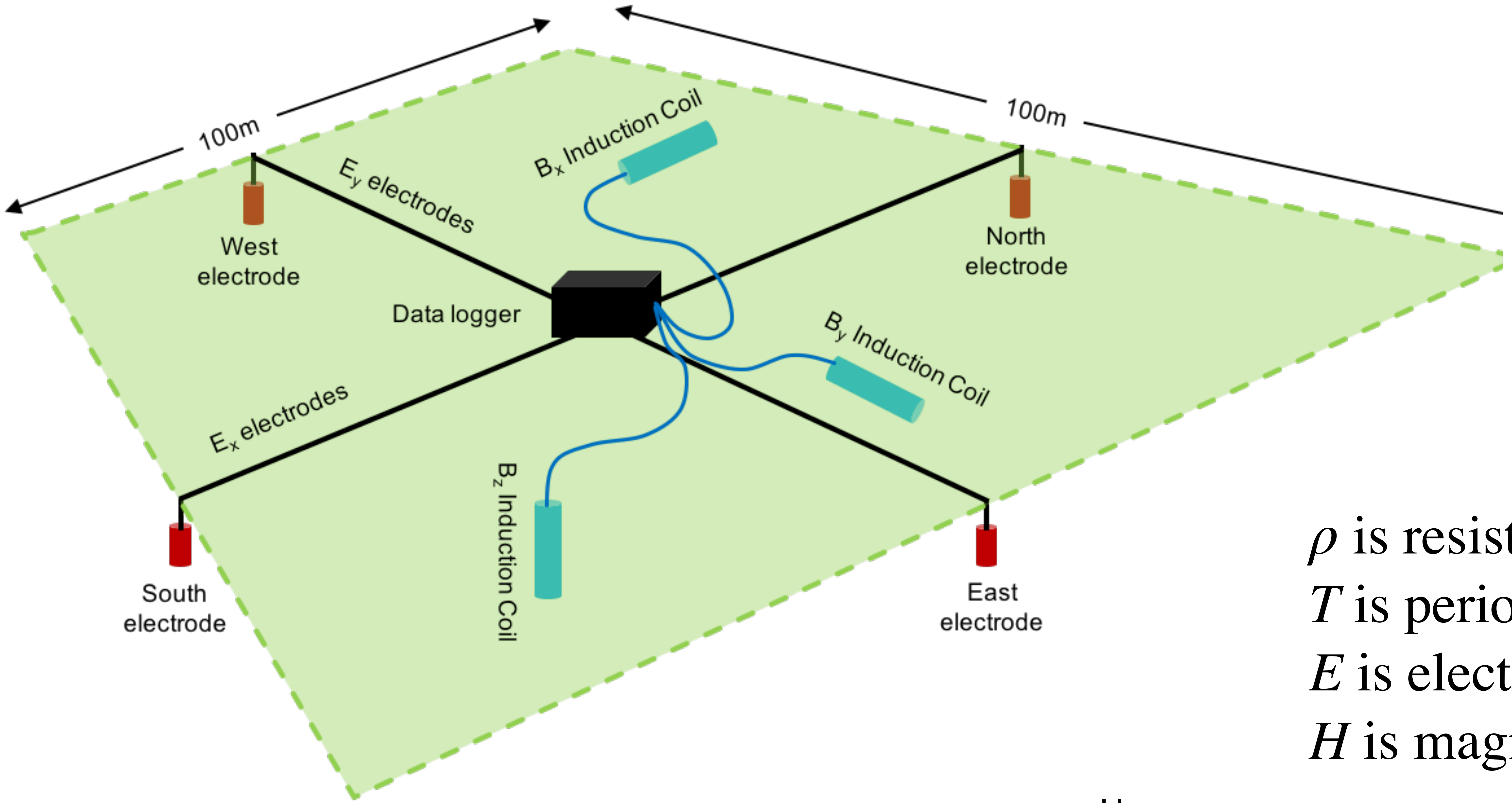
Temperature

Gas/hydrate fraction

Enter the electric field and the magnetotelluric (MT) method.

1950: A.N. Tikhonov, T. Rikitake, Y. Kato, and T. Kikuchi developed mathematical descriptions for the relationship between induced electric and magnetic fields.

1953: Louis Cagniard described a practical method to use measurements of magnetic and electric fields to estimate Earth conductivity, and called it the magnetotelluric method.



$$\rho = 0.2T\left(\frac{E}{H}\right)^2.$$

ρ is resistivity (one over conductivity) (Ωm)
T is period (seconds)
E is electric field as period T (V/m)
H is magnetizing field at period T (A/m)


digging in a magnetometer
programming logger
electrode wires

Some terminology:

σ is electrical conductivity, units S/m (S = 1/Ω). Relates $\mathbf{J} = \sigma\mathbf{E}$.

ρ is electrical resistivity, units Ωm. Just the reciprocal of conductivity.

$\mathbf{B}$ is magnetic field, units of Tesla, although nT is more useful in geophysics. Also called flux density.

$\mathbf{H}$ is magnetizing field, units of A/m. A mathematical construct. Also called magnetic field.

$\mathbf{E}$ is the electric field, units V/m. The field created by a charge.

$\mathbf{J}$ is electric current density, units A/m^2. Flow of charge through a material.

μ is magnetic permeability. A measure of how well a material magnetizes. Relates $\mathbf{B} = \mu\mathbf{H}$.

μ_o is permeability of free space. Almost exactly $4\pi \times 10^{-7}$ Tm/A (H/m).

ϵ is electric permittivity. A measure of how charges polarize in a material. Relates $\mathbf{D} = \epsilon\mathbf{E}$.

ϵ_o is permittivity of free space. It is $1/c^2\mu_o \approx$ 8.85 $\times 10^{-12}$ C/(Vm) (F/m).

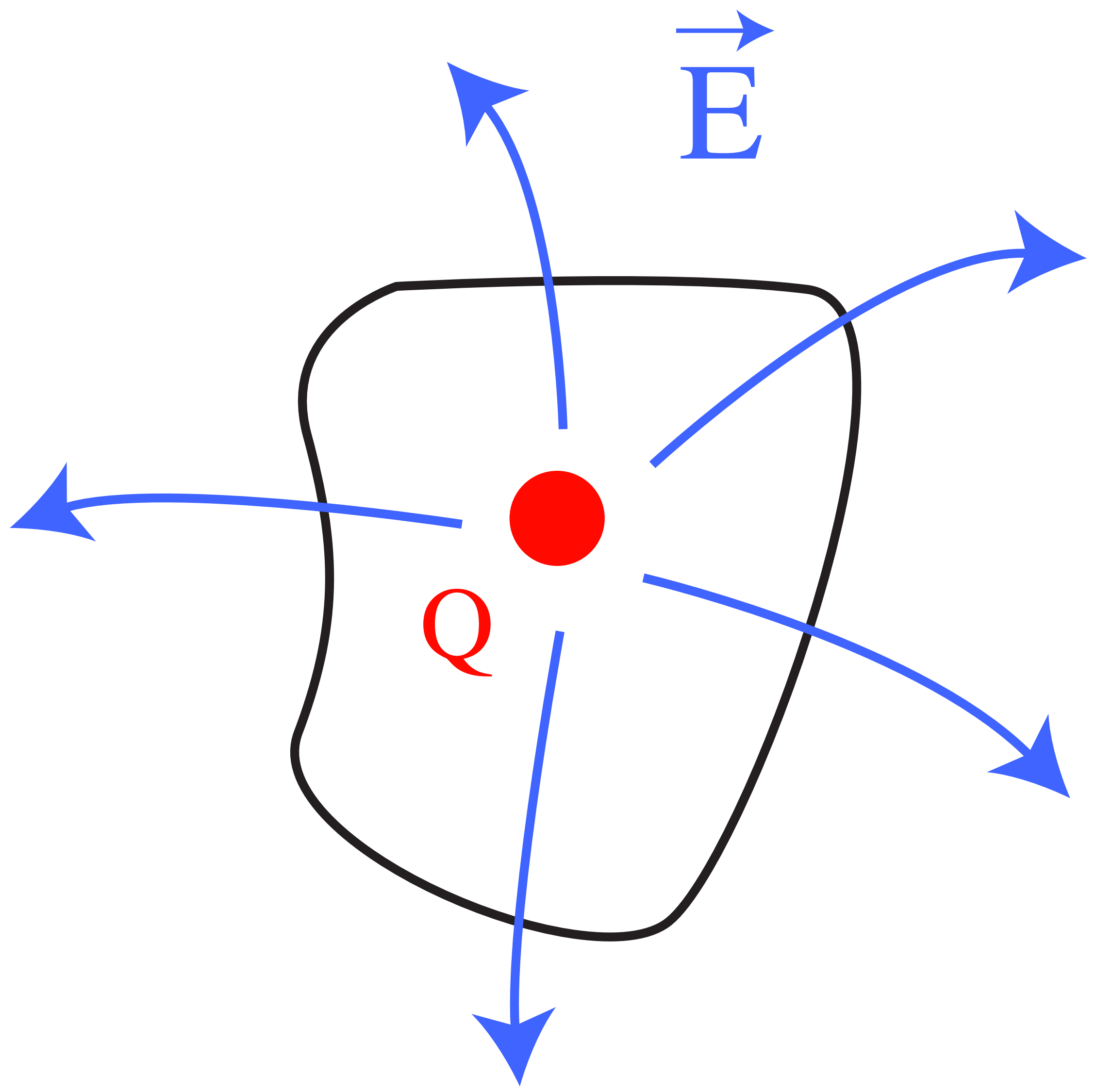
Gauss' Law:

$$\int_{\Omega} \mathbf{E} \cdot \mathrm{ds} = \frac{Q}{\epsilon_o}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

Q is charge, C
ρ is charge density, C/m^3
ϵ_o is permittivity of free space, 8.895 x 10^{-12} F/m

Gauss' Law says that the electric field leaving a volume is proportional to the enclosed charge.

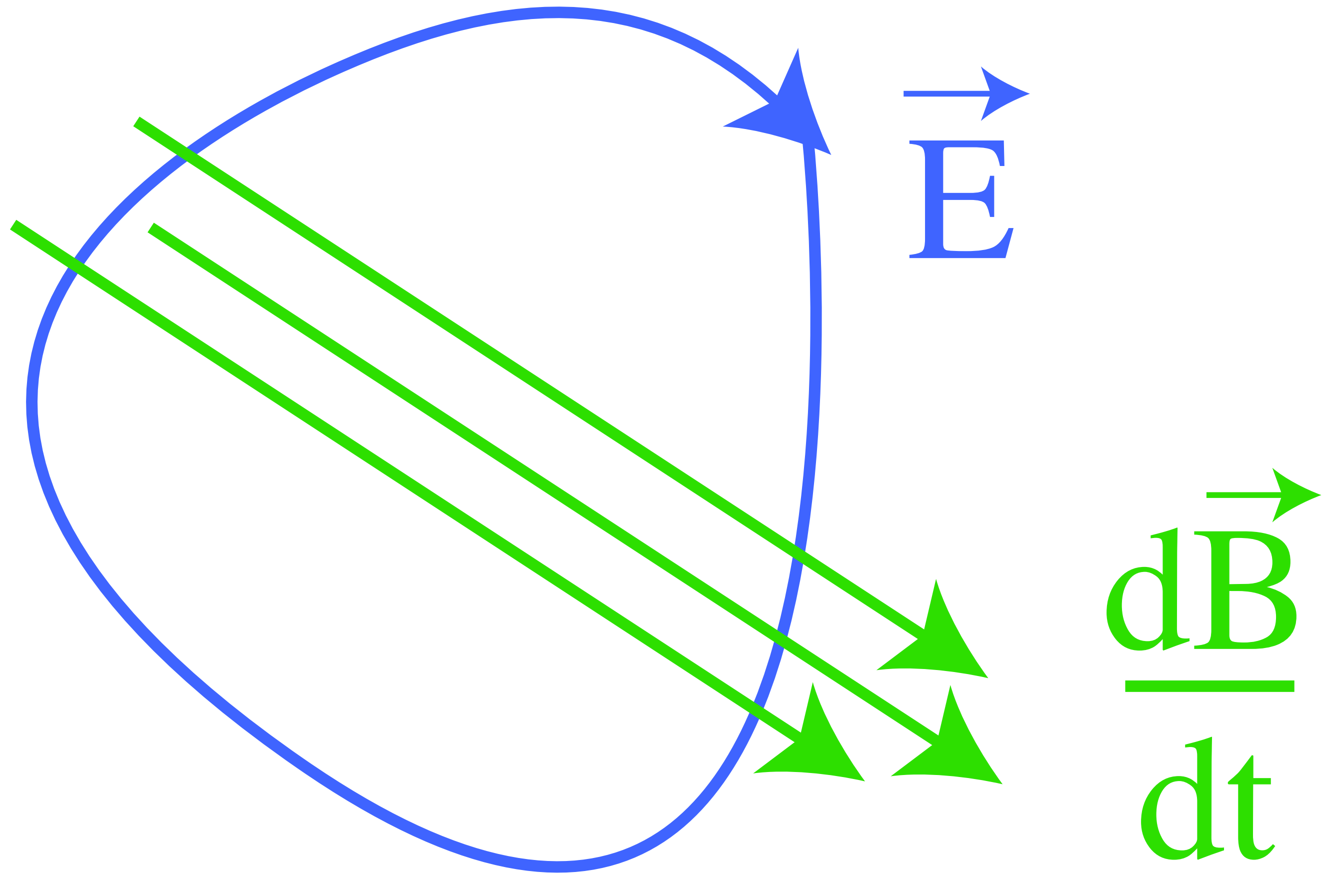


Faraday's Law:

$$\oint_c \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Φ_B is magnetic flux

Faraday's Law says that the electric field integrated around a loop (i.e. the voltage) is given by the time rate of change of the enclosed magnetic flux.

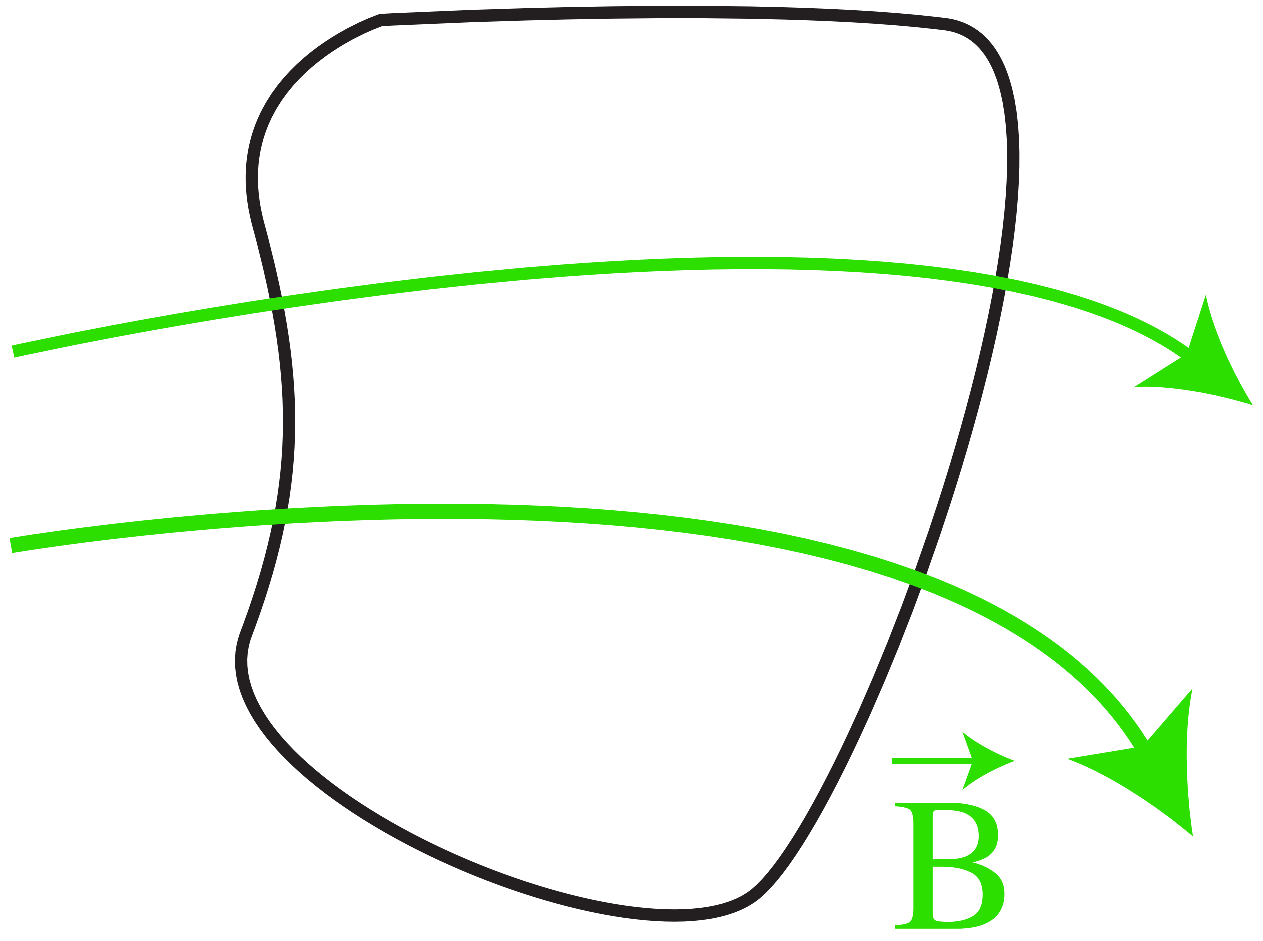


Gauss' Law (magnetism):

$$\int_\Omega \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

Gauss' Law for magnetism says that there are no magnetic monopoles. Any magnetic flux entering a volume has to leave it.

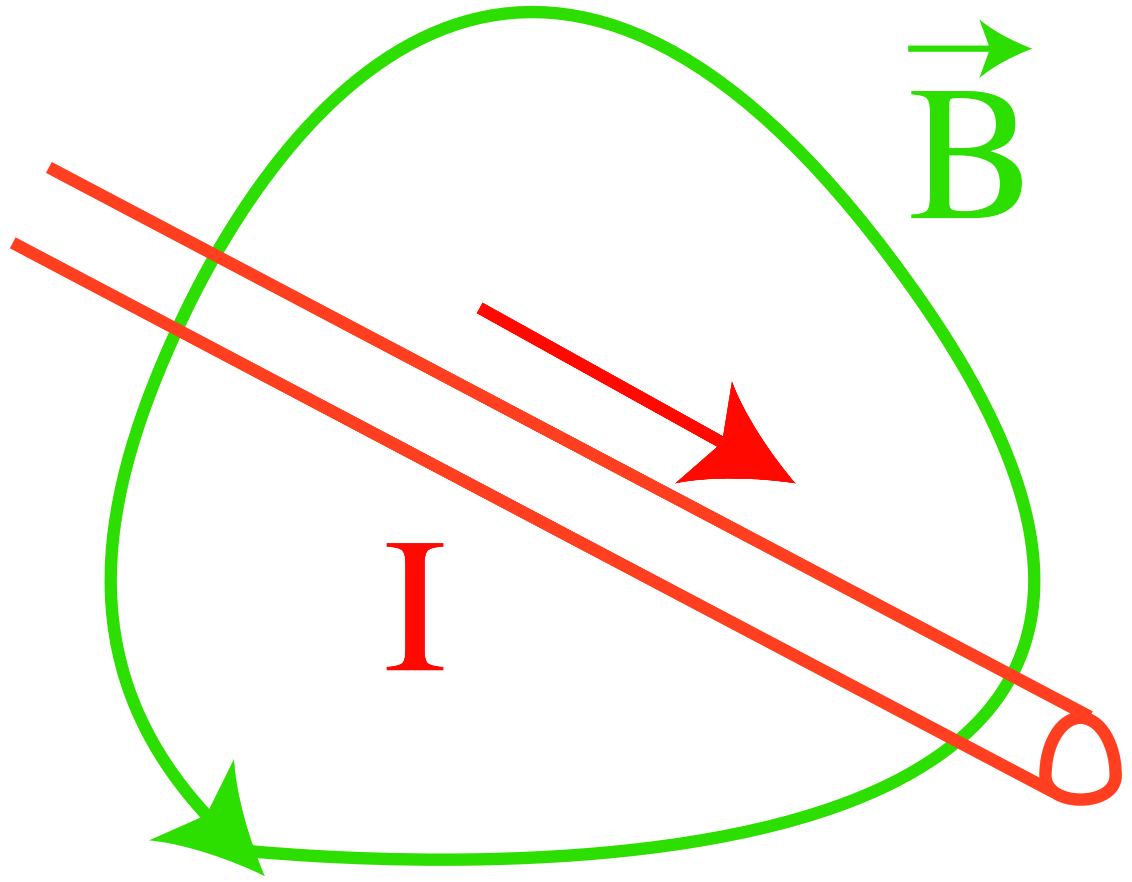


Ampere's Law:

$$\oint_c \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_o I \qquad \nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

I is electric current, A
$\boldsymbol{J}$ is electric current density, A/m²
μ_o is the permeability of free space, about 4π x 10^{-8} H/m

Ampere's Law says that an electric current will generate a circulating magnetic field.



Maxwell's Equations (in a vacuum):

Faraday's Law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Gauss' Law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

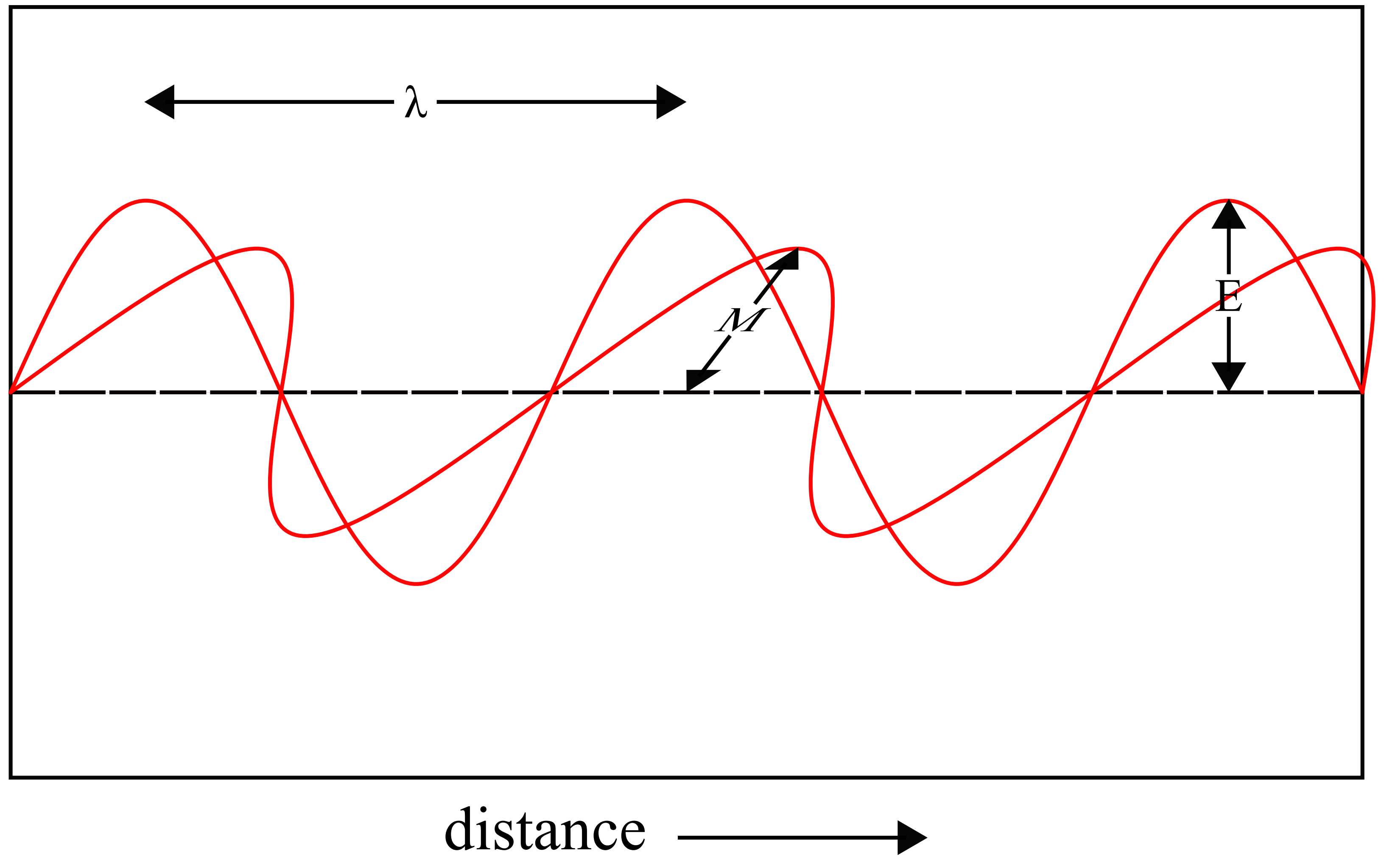
Ampere's Law:

$$\nabla \times \mathbf{B} = \mu_o \left(\mathbf{J} + \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \right)$$

Gauss' Law (magnetism):

$$\nabla \cdot \mathbf{B} = 0$$

The extra term in Ampere's Law was added by Maxwell to allow fields to exist without charges or currents. This allows electromagnetic radiation to propagate in a vacuum with speed c, where $c^2 = 1/(\mu_o \epsilon_o)$. But this term is small in MT.



Maxwell

Oliver Heaviside

In Matter we need constitutive relationships:

$$\mathbf{M} = \chi_M \mathbf{H}$$

$$\mathbf{P} = \epsilon_o \chi_E \mathbf{E} \qquad \mathbf{M} = \frac{\chi_M}{\mu_o}\mathbf{B} \qquad \mathbf{J} = \sigma \mathbf{E}$$

P is electric polarization density, C/m²

M is magnetization, A/m

J is electric current density, S/m

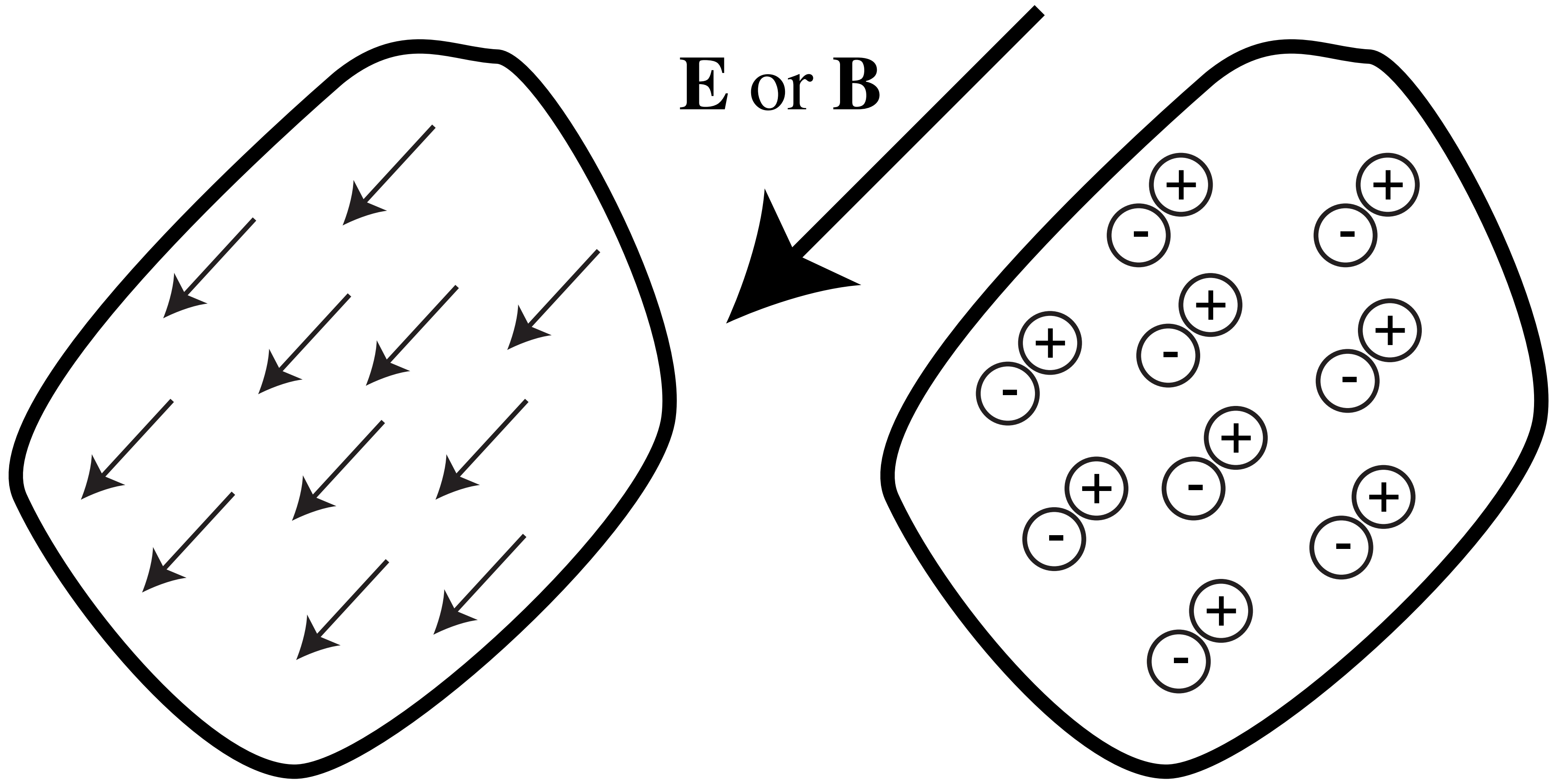
χ_E is electric susceptibility (dimensionless)

χ_M is magnetic susceptibility (dimensionless)

σ is electrical conductivity, S/m

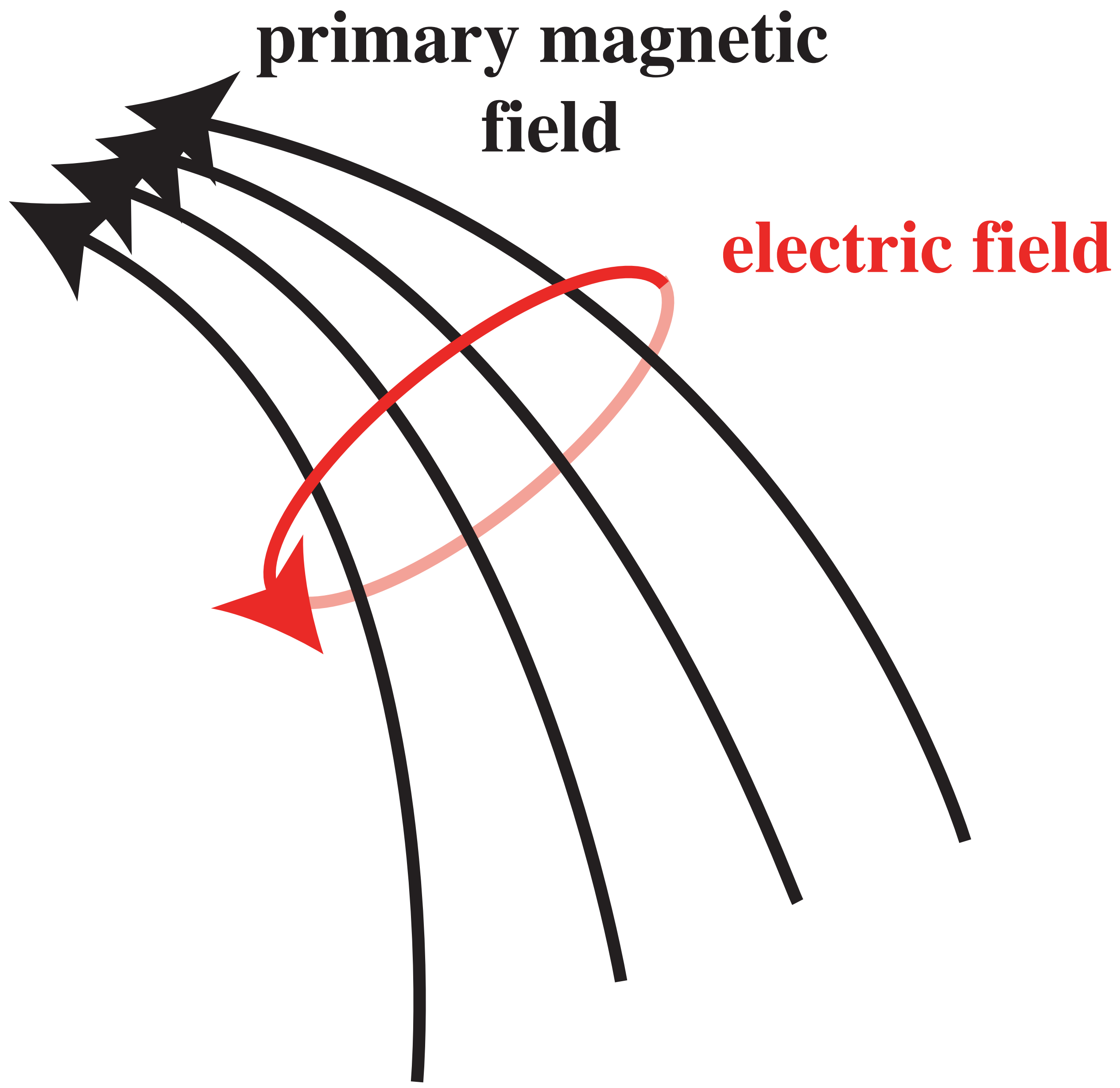
The last equation is Ohm's Law, but these are approximations! Matter does not have to be linear and isotropic. For example, there will be saturation phenomena.

Ohm



Electromagnetic induction in pictures and equations:

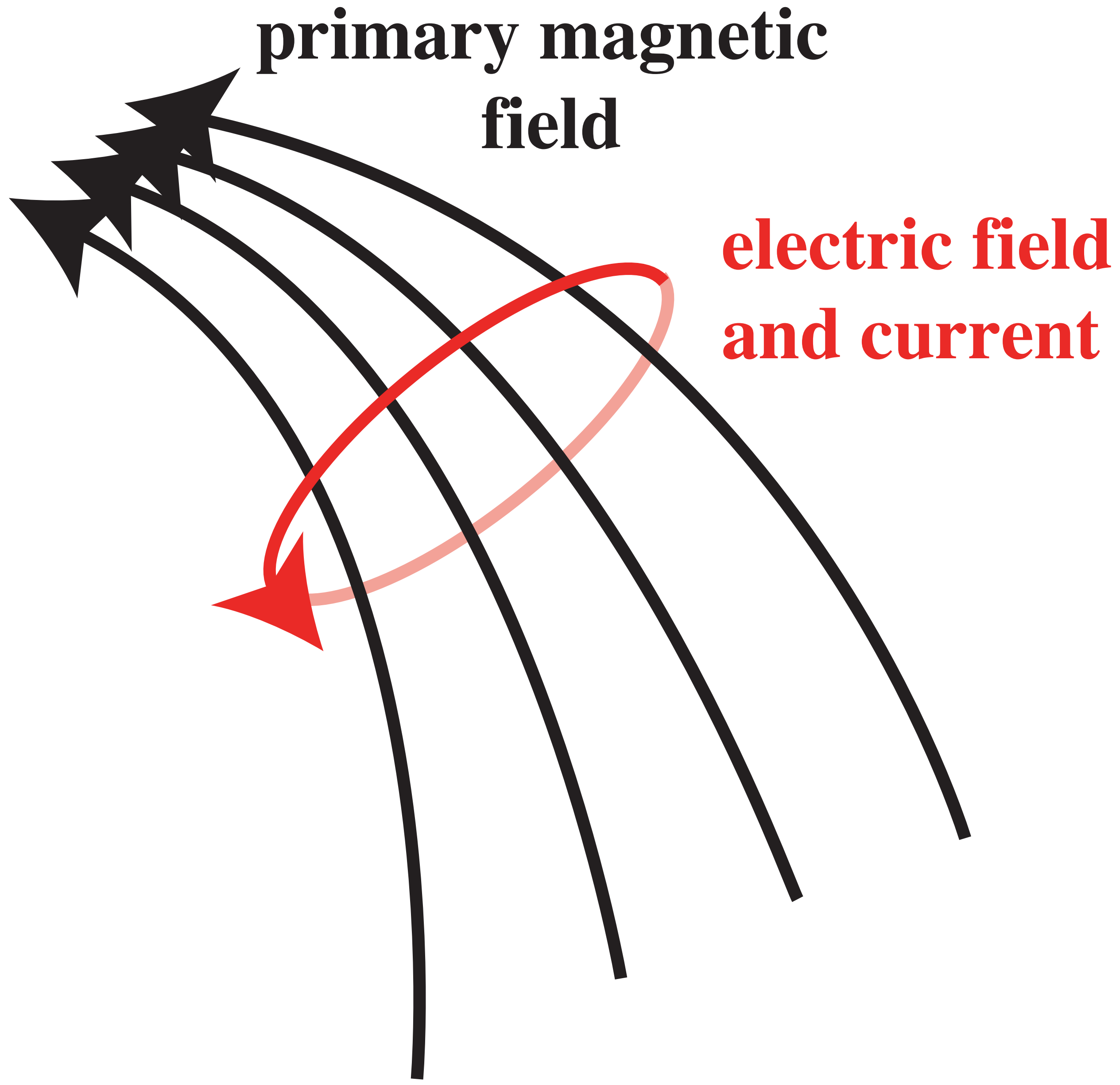
A time varying magnetic field will generate an electric field.



Faraday's Law

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

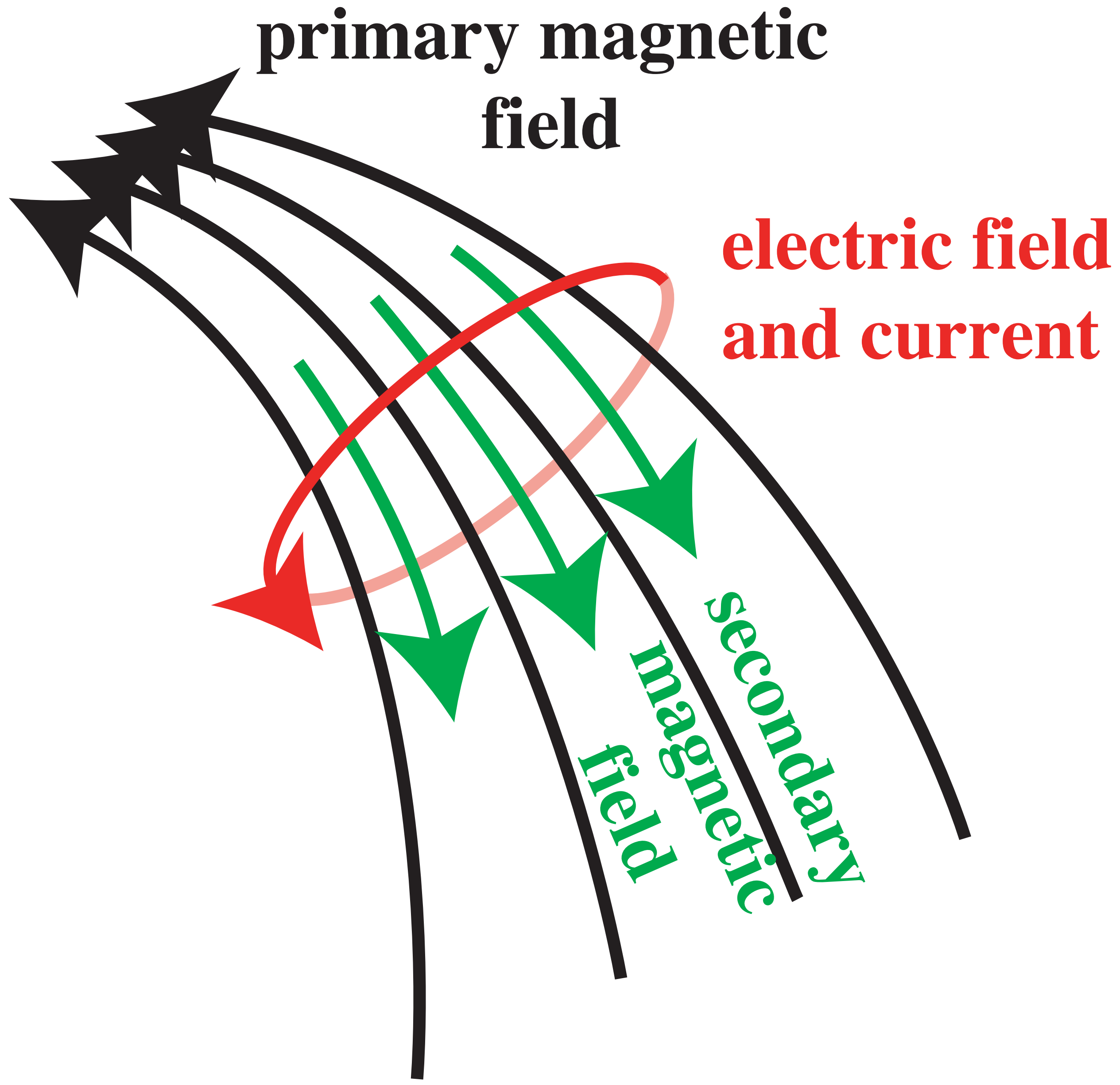
In a conductor this drives an electric current.



Ohm's Law

$$\mathbf{J} = \sigma \mathbf{E}$$

Which generates another magnetic field.



Ampere's Law

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu I$$

Because of the minus sign in Faraday's Law, the secondary field opposes the changes in the primary field. The consequence of this is that conductive rocks absorb variations in EM fields more than resistive rocks.

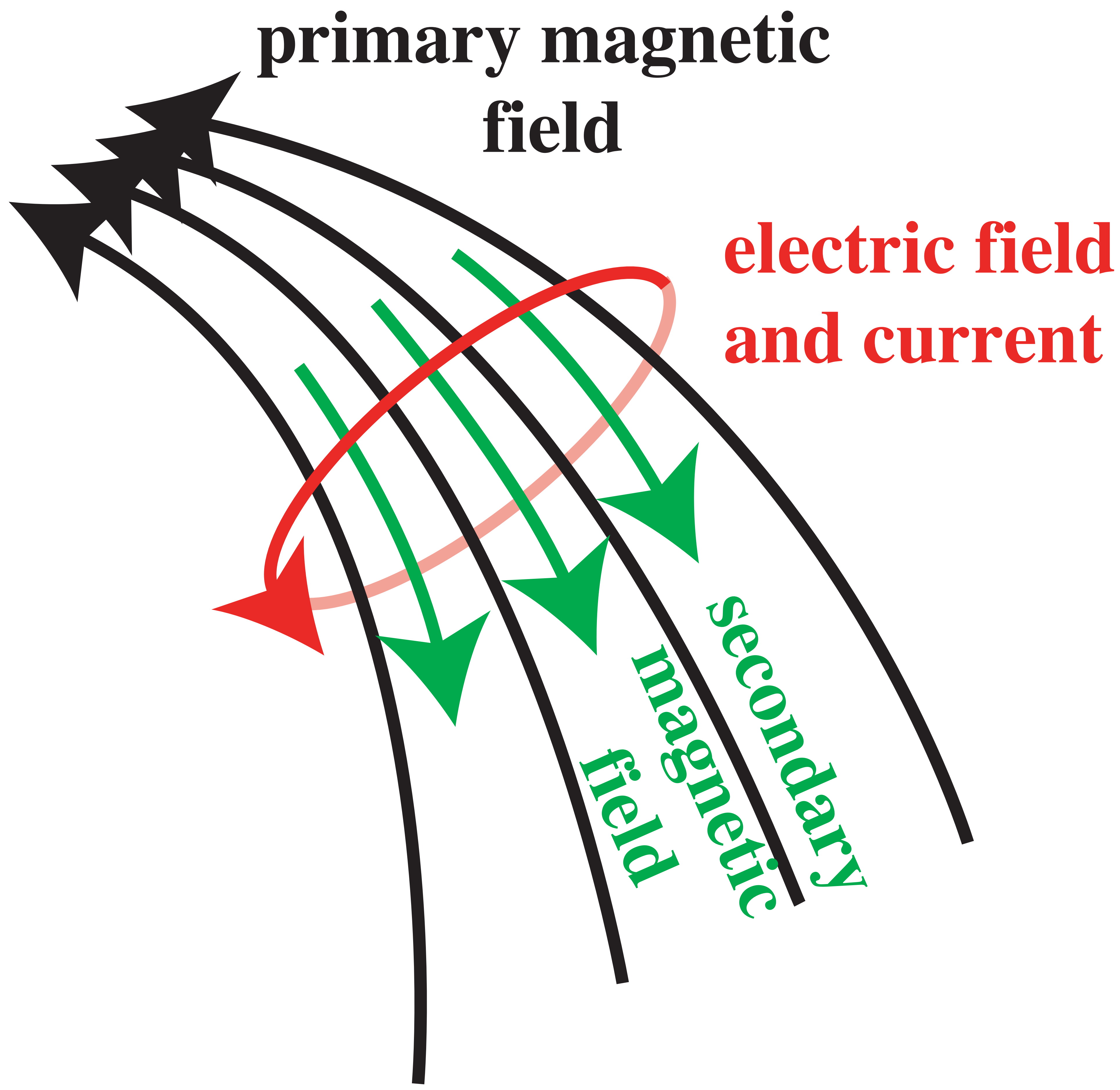
This absorption is exponential:

$$E(z) = E_o e^{-z/z_s}$$

The rate of absorption is given by the skin depth, z_s, which depends on rock resistivity and period: High resistivity, long periods = large skin depths, greater penetration.

$$z_s = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{\rho T}{\pi\omega\mu}}$$

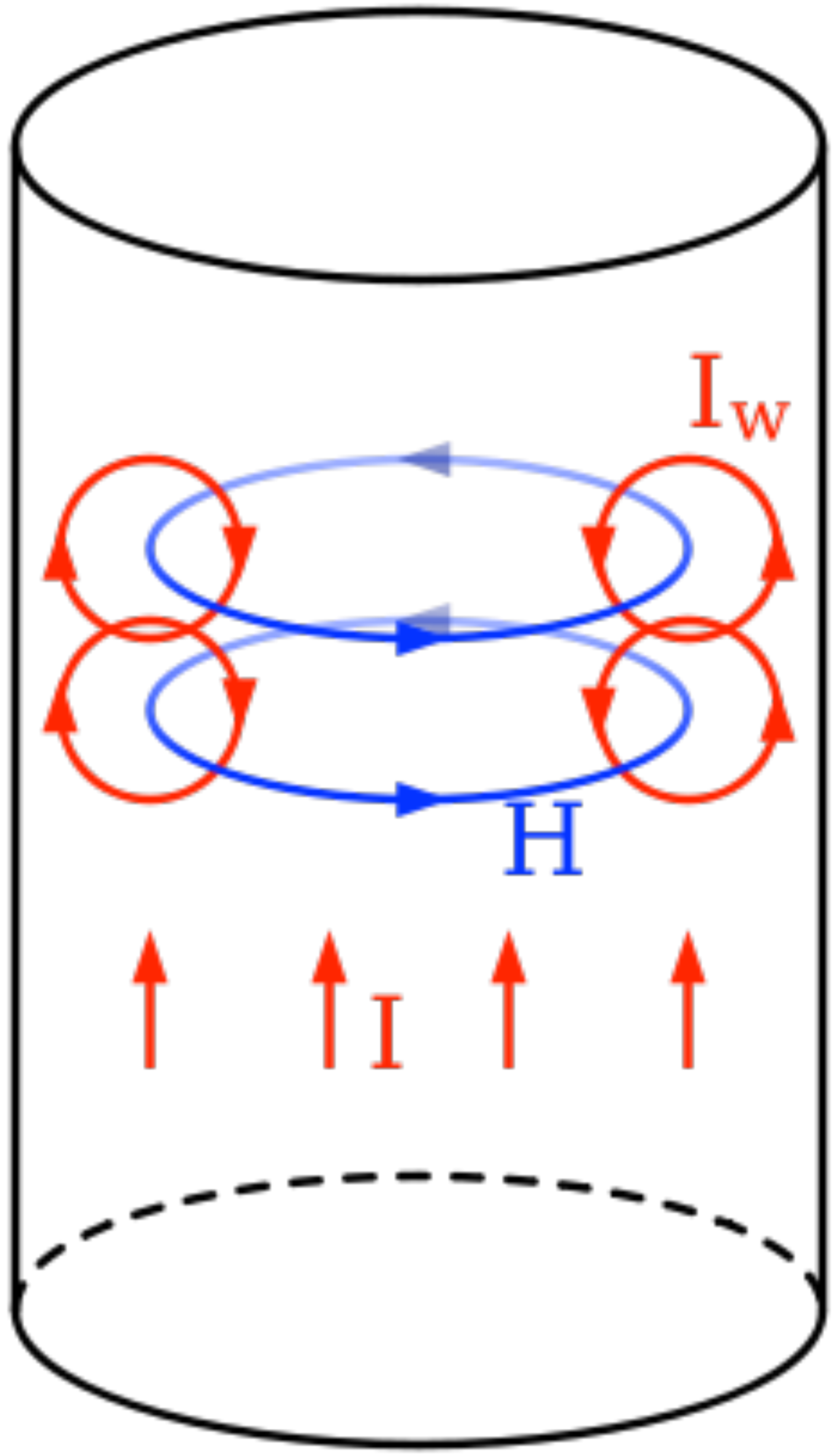
$\omega = 2\pi f$ is angular frequency.



$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

"Skin effect" describes the tendency for current to flow in the skin of a conductor. I (and many others) used to think that it was the reason that the high voltage from an RF Tesla coil didn't kill people, but that's not the case. The skin depth in people at RF frequencies is 25 cm or more.

Turns out that RF the nervous system is insensitive to RF currents.



Wikipedia commons

Let's derive the skin depth equation:

Substitute Ohm's Law into Ampère's Law:

$$\mathbf{J} = \sigma \mathbf{E} \qquad \nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

to get

$$\nabla \times \mathbf{B} = \mu_o \sigma \mathbf{E}$$

Take the curl of this and use Faraday's Law ($\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$):

$$\nabla \times \nabla \times \mathbf{B} = \mu_o \sigma \nabla \times \mathbf{E} \quad \rightarrow \quad \nabla \times \nabla \times \mathbf{B} = -\mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

Now we need the vector identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ to get

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

But, the no-monopoles law says that $\nabla \cdot \mathbf{B} = 0$, so ...

$$\boxed{\nabla^2 \mathbf{B} = \mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Similarly, we can take the curl of Faraday's Law and substitute Ampère's and Ohm's Laws to get

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) \quad \rightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\mu_o \sigma \frac{\partial \mathbf{E}}{\partial t}$$

To pull the same vector identity trick we need to use $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ on Ampère's Law ($\nabla \times \mathbf{B} = \mu_o \mathbf{J}$) to get

$$\nabla \cdot \mathbf{J} = 0 \quad \text{which for constant } \sigma_o \text{ gives} \quad \nabla \cdot \mathbf{E} = 0$$

so

$$\nabla^2 \mathbf{E} = \mu_o \sigma_o \frac{\partial \mathbf{E}}{\partial t}$$

Similarly, we can take the curl of Faraday's Law and substitute Ampère's and Ohm's Laws to get

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) \quad \longrightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\mu_o \sigma \frac{\partial \mathbf{E}}{\partial t}$$

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up until this point everything was general

so

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up until this point everything was general

so

$$\nabla^2 \mathbf{E} = \mu_o \sigma_o \frac{\partial \mathbf{E}}{\partial t}$$

now we need the idea of a half-space

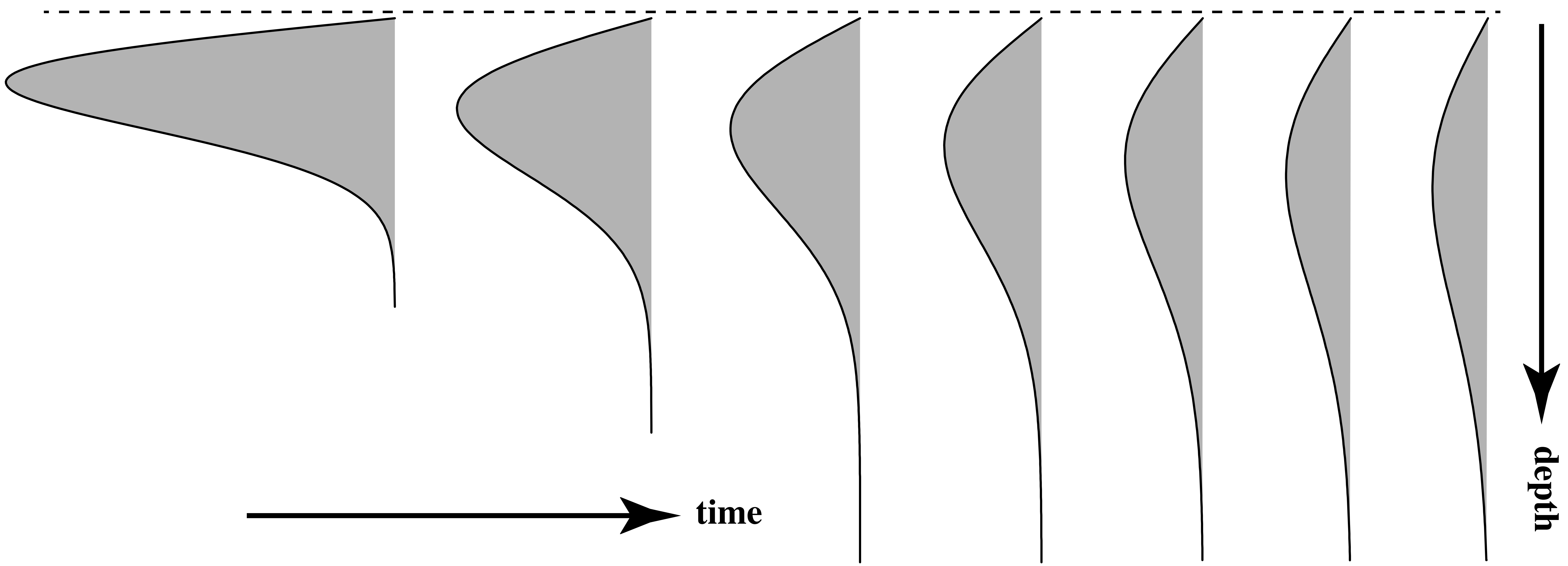
air, $\sigma = 0$

earth, $\sigma = \sigma_o$

These equations in E and B are Diffusion Equations :

$$\nabla^2 \mathbf{B} = \mu_o \sigma \frac{\partial \mathbf{B}}{\partial t} \qquad \nabla^2 \mathbf{E} = \mu_o \sigma \frac{\partial \mathbf{E}}{\partial t}$$

We can define a diffusivity $\eta = 1/(\mu_o \sigma_o)$ so that $\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}$



Amplitude decays as t^{-1} and depth as $t^{1/2}$. Not as bad as heat flow: We can choose time dependence through frequency to alter the decay rate through skin depth.

Now it is time to consider a single frequency ω, so

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$\mathbf{B}(t) = \mathbf{B}e^{i\omega t} \qquad \text{and} \qquad \frac{\partial \mathbf{B}}{\partial t} = i\omega\mathbf{B}$$

$$\nabla^2\mathbf{B} = \mu_o\sigma_o\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla^2\mathbf{E} = \mu_o\sigma\frac{\partial \mathbf{E}}{\partial t}$$

and the same for **E**, so our diffusion equations become

$$\nabla^2\mathbf{E} = i\omega\mu_o\sigma_o\mathbf{E} \qquad \text{and} \qquad \nabla^2\mathbf{B} = i\omega\mu_o\sigma_o\mathbf{B}$$

For external sources of **B**, at Earth's surface **B** is purely horizontal and uniform. Then

$$\nabla^2\mathbf{B} = \cancel{\frac{\partial^2 B_x}{\partial x^2}} + \cancel{\frac{\partial^2 B_y}{\partial y^2}} + \frac{\partial^2 B_z}{\partial z^2} \longrightarrow \frac{d^2 B}{dz^2} = i\omega\mu_o\sigma_o B(z)$$

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$$\nabla^2\mathbf{E} = \mu_o\sigma\frac{\partial \mathbf{E}}{\partial t}$$

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We define a *complex wavenumber* $k^2 = i\omega\mu_o\sigma_o$ so $\dfrac{d^2B}{dz^2} = k^2B(z)$

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$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$\mathbf{B}(t) = \mathbf{B}e^{i\omega t} \quad \text{and} \quad \frac{\partial \mathbf{B}}{\partial t} = i\omega\mathbf{B}$$

$$\nabla^2\mathbf{B} = \mu_o\sigma_o\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla^2\mathbf{E} = \mu_o\sigma\frac{\partial \mathbf{E}}{\partial t}$$

and the same for **E**, so our diffusion equations become

$$\nabla^2\mathbf{E} = i\omega\mu_o\sigma_o\mathbf{E} \quad \text{and} \quad \nabla^2\mathbf{B} = i\omega\mu_o\sigma_o\mathbf{B}$$

For external sources of **B**, at Earth's surface **B** is purely horizontal and uniform. Then

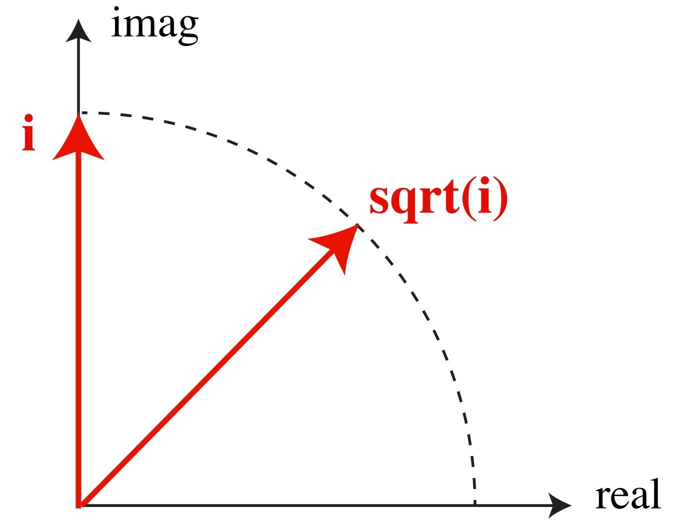
$$\nabla^2\mathbf{B} = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2} \longrightarrow \frac{d^2B}{dz^2} = i\omega\mu_o\sigma_o B(z)$$

We define a *complex wavenumber* $k^2 = i\omega\mu_o\sigma_o$ so $\frac{d^2B}{dz^2} = k^2B(z)$

This is a second order linear ODE with solutions of the form $B(z) = c_1e^{kz} + c_2e^{-kz}$

The first term grows with depth so $c_1 = 0$ and setting $z = 0$ we can infer that $c_2 = B_oe^{i\omega t}$.

This all gives $B(z) = B_o e^{i\omega t} e^{-kz}$



Recalling that $k^2 = i\omega\mu_o\sigma_o$ we can write k as

$$k = \sqrt{i\omega\mu_o\sigma_o} = (1+i)\sqrt{\frac{\omega\mu_o\sigma_o}{2}} = \frac{1+i}{z_o} \quad \text{where} \quad z_o = \sqrt{\frac{2}{\omega\mu_o\sigma_o}}$$

to get

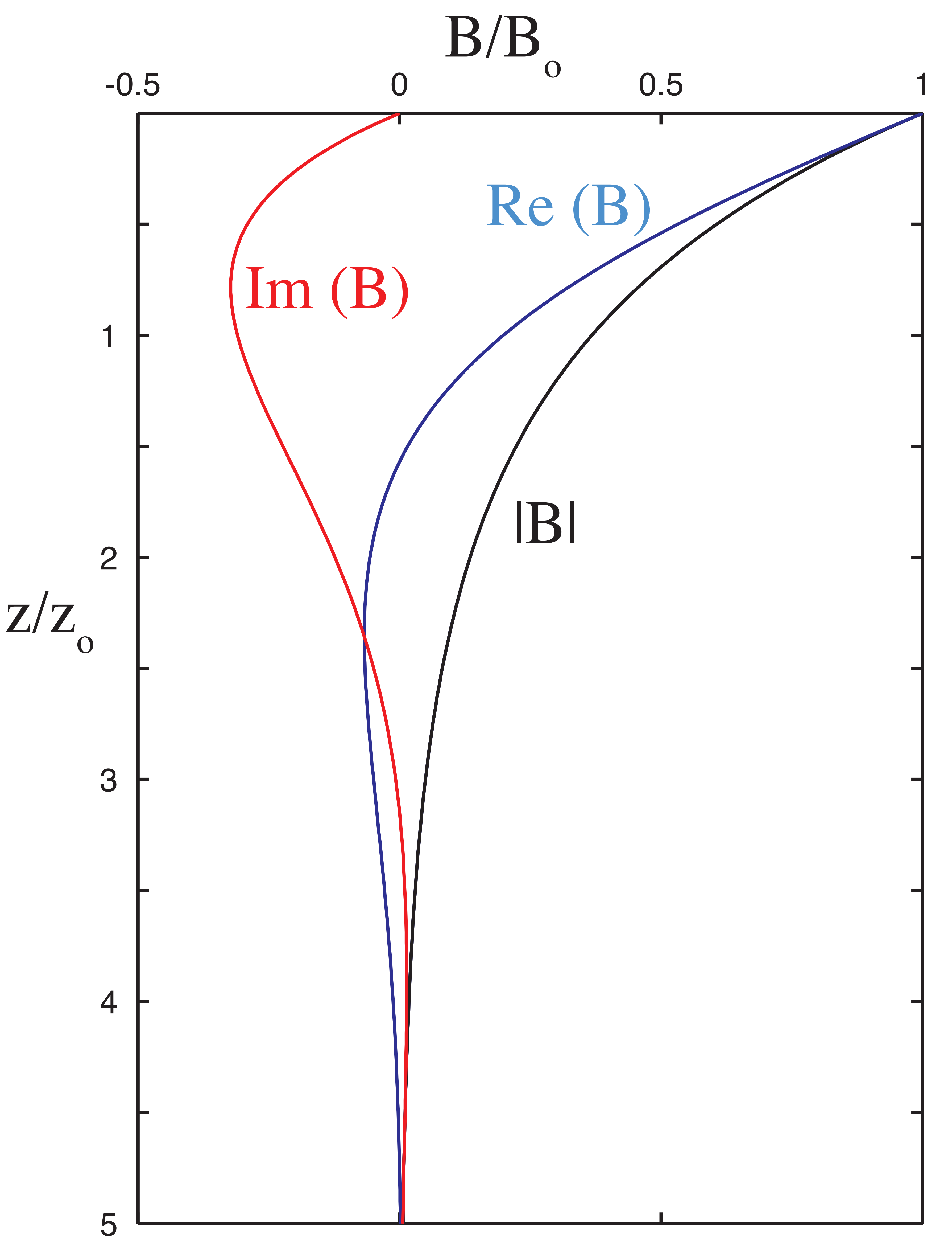
$$B(z) = B_o e^{i\omega t} e^{-z(1+i)/z_o}$$

So $B(z)$ falls off exponentially with z_o, which is the **skin depth**.

$z_o \approx 500\sqrt{\rho T}$ m where $\rho = 1/\sigma$ and T is period in seconds.

Some typical numbers:

material	σ, S/m	1 day	1 hour	1 sec	1 ms
seawater	3	85 km	17 km	290 m	9 m
sediments	0.1	460 km	95 km	1.6 km	50 m
igneous rock	1×10^{-5}	50000 km	9500 km	160 km	5 km



The resolution of EM induction sits between wave propagation and potential field methods:

Wave equation: Resolution ~ wavelength

High frequency (megahertz)

Radar

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Seismics

$$\nabla^2 u = \epsilon \frac{\partial u}{\partial t} + \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

Diffusion equation: Resolution ~ size/depth

Mid frequency (0.001 - 1000 Hz)

Inductive EM

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t}$$

Heat flow

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Laplace equation: Resolution ~ bounds only

Zero frequency

DC Resistivity

$$\nabla^2 \mathbf{E} = 0$$

Gravity/Magnetism

$$\nabla^2 U = 0$$

σ (electrical conductivity) ~ $3 - 10^{-6}$ S/m >> ϵ (electric permittivity) ~ $10^{-9} - 10^{-11}$ F/m

μ = magnetic permeability ~ $10^{-4} - 10^{-6}$ H/m

The Magnetotelluric Method:

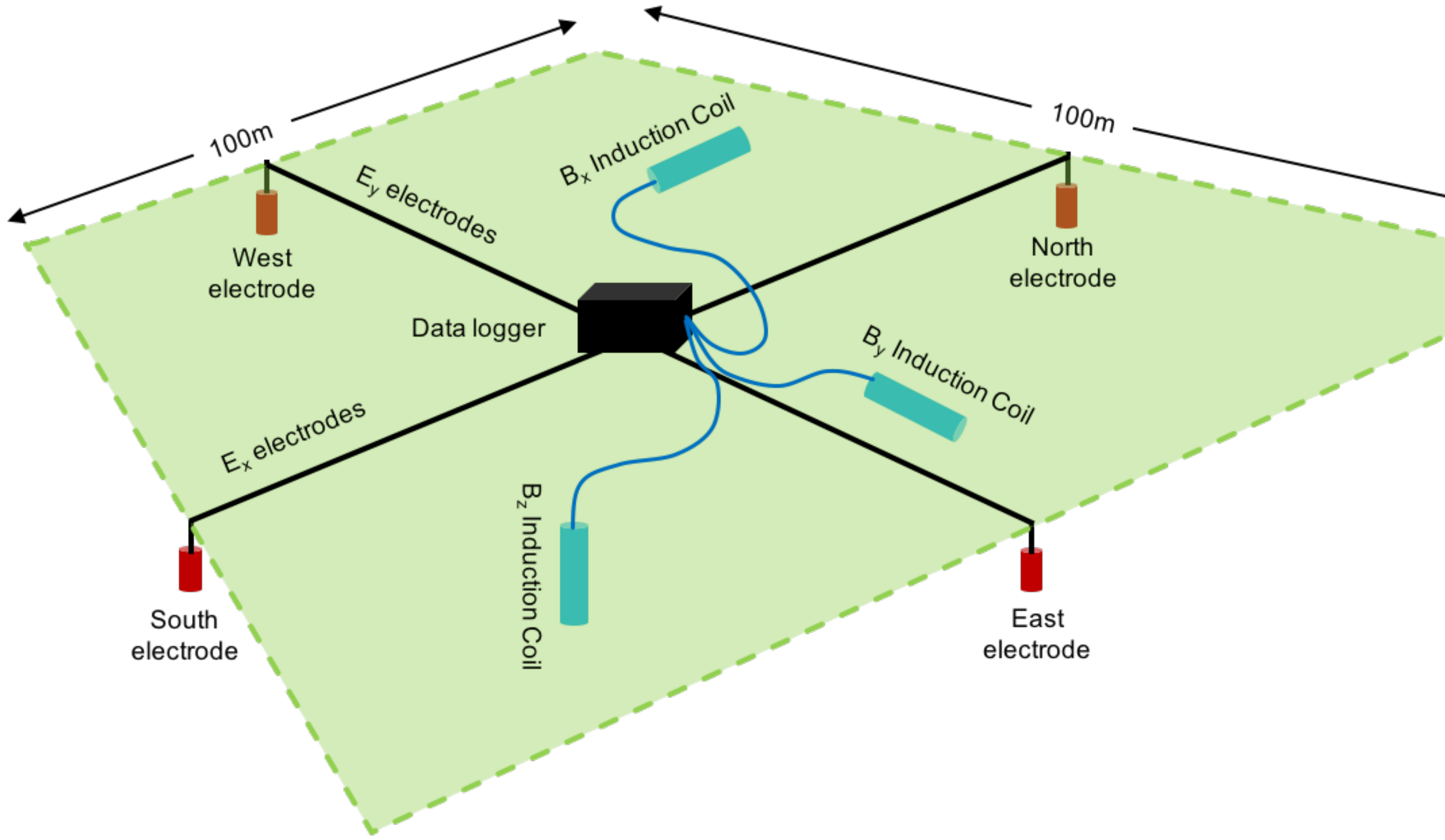
We just showed that $B(z) = B_o e^{i\omega t} e^{-z(1+i)/z_o}$ and recall $\nabla \times \mathbf{B} = \mu_o \sigma \mathbf{E}$

But if **B** is in the *x* direction the only non-zero component of the curl is $\partial B_x / \partial z$ in the y-component:

no *Bz* or *By*

$$\nabla \times \mathbf{B} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \boxed{\frac{\partial B_x}{\partial z}} - \frac{\partial B_z}{\partial x}, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)$$

Bx uniform across surface



so

$$E_y = \frac{1}{\mu_o \sigma_o} \frac{dB_x}{dz} = -\frac{1+i}{\mu_o \sigma_o z_o} B_x = -\frac{k}{\mu_o \sigma_o} B_x$$

and similarly

$$E_x = \frac{1}{\mu_o \sigma_o} \frac{-dB_y}{dz} = \frac{1+i}{\mu_o \sigma_o z_o} B_y = \frac{k}{\mu_o \sigma_o} B_y$$

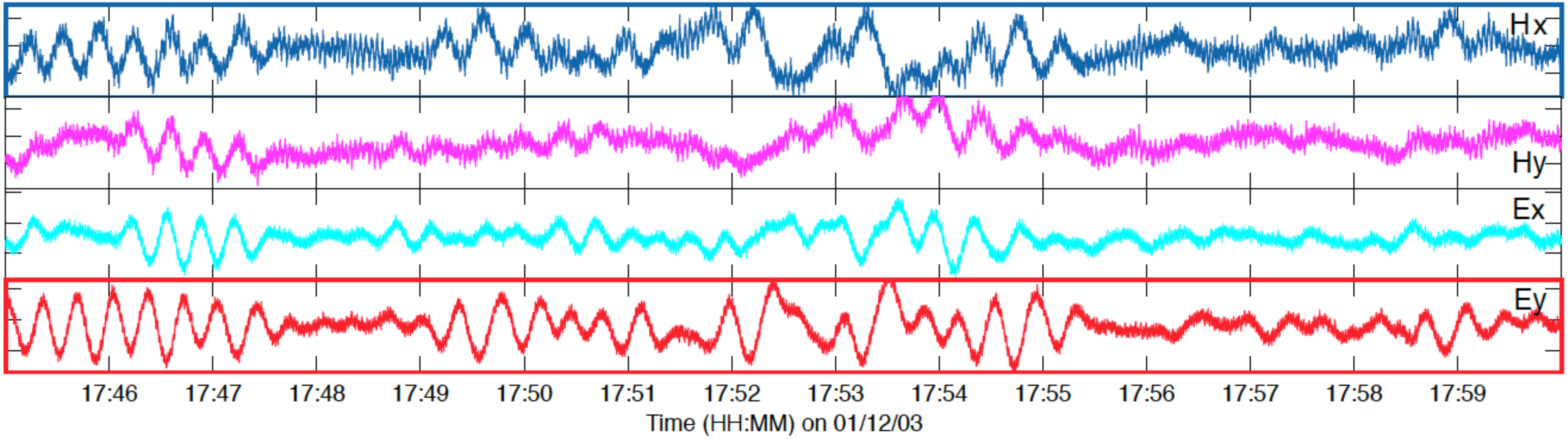
This is valid for any depth *z*, but we are only interested in the surface where $z = 0$ and $B_x = B_o e^{i\omega t}$

We have that

$$E_y = -\frac{k}{\mu_o \sigma_o} B_x \qquad E_x = \frac{k}{\mu_o \sigma_o} B_y \qquad \text{where} \qquad k = \sqrt{i\omega\mu_o\sigma_o}$$

These equations tell us that there is an induced electric field that is linearly proportional to the external magnetic field. The constant of proportionality depends on conductivity and frequency. *Ey* is anti-correlated with *Bx*, and *Ex* is correlated with *By* (but both with a 45° phase shift).

Site t03 from GoM 2003: 15 minutes at 32 Hz sampling

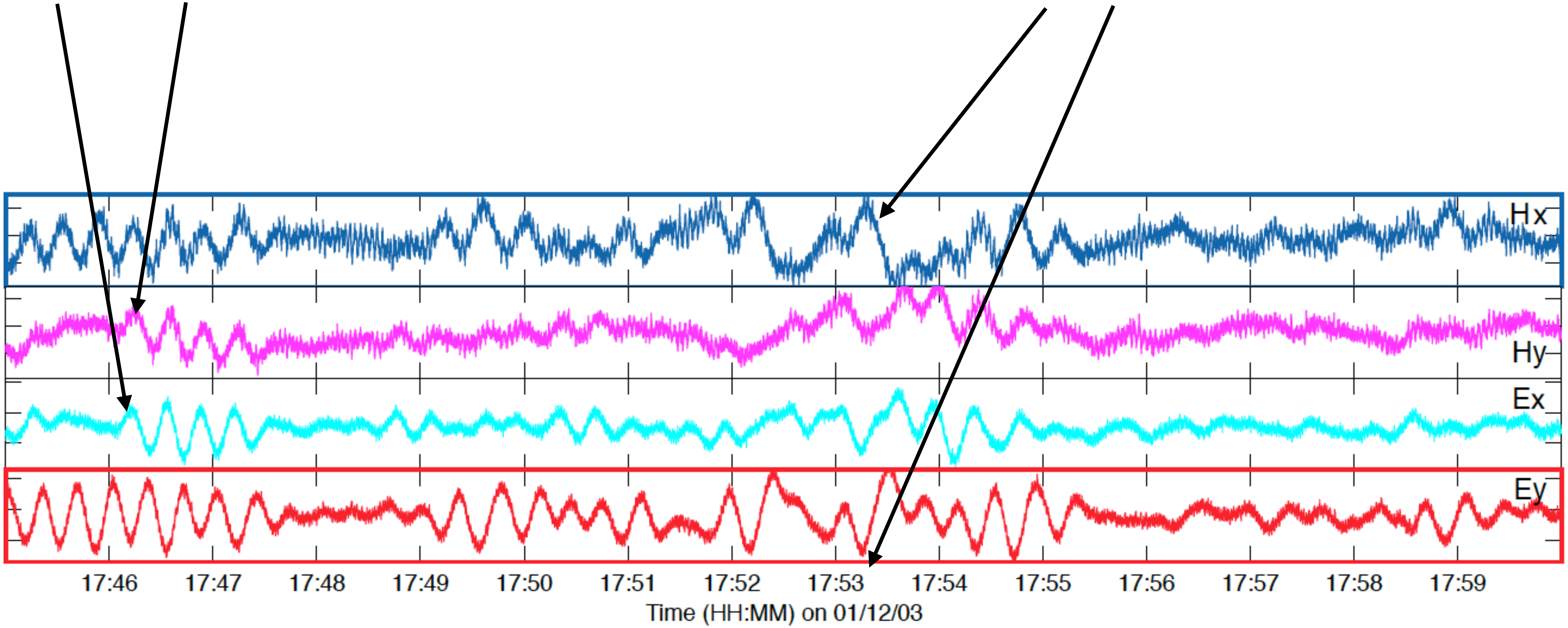


We have that

$$E_y = -\frac{k}{\mu_o \sigma_o} B_x \qquad E_x = \frac{k}{\mu_o \sigma_o} B_y \qquad \text{where} \qquad k = \sqrt{i\omega\mu_o\sigma_o}$$

Ex is correlated with *By*

Ey is anti-correlated with *Bx*



The Magnetotelluric Method continued:

We can take the ratio of the electric to magnetic field at any particular frequency to obtain the half-space resistivity:

$$\left|\frac{E_y}{B_x}\right|^2 = \left(\frac{k}{\mu_o\sigma_o}\right)^2 = \frac{\omega\mu_o\sigma_o}{(\mu_o\sigma_o)^2} = \frac{\omega}{\mu_o\sigma_o}$$

$$\rho = \frac{\mu_o}{\omega}\left|\frac{E_y}{B_x}\right|^2 \qquad\qquad \phi = \tan^{-1}\left(\frac{E}{B}\right)$$

This is the MT equation made famous in Cagniard's 1953 paper.

$$\rho = 0.2T\left(\frac{E}{H}\right)^2.$$

This is all still only true for a half-space, but we can call this apparent resistivity regardless of how complicated the structure is.

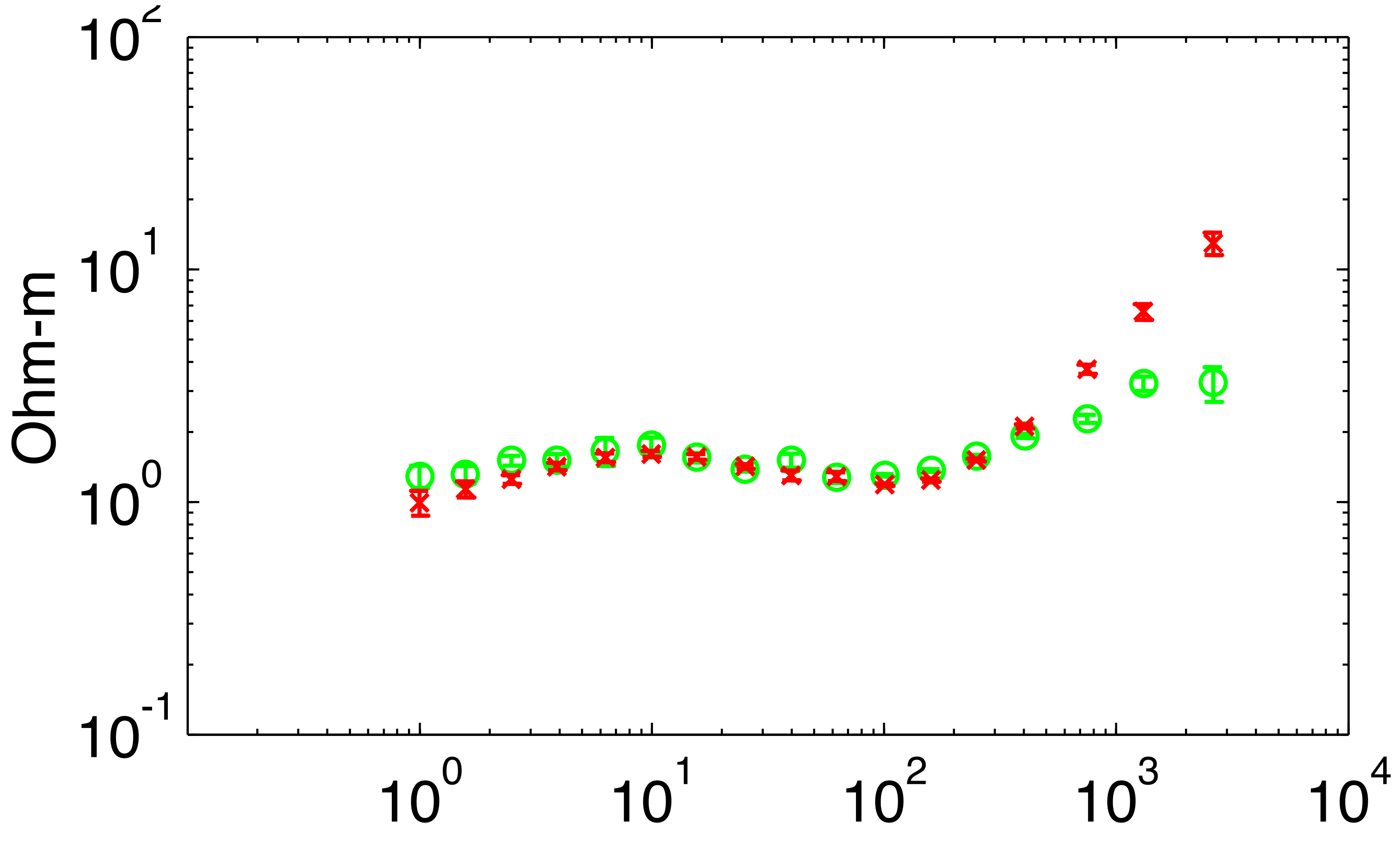
The two components of **E** are related to the two components of **B** through the impedance matrix **Z**

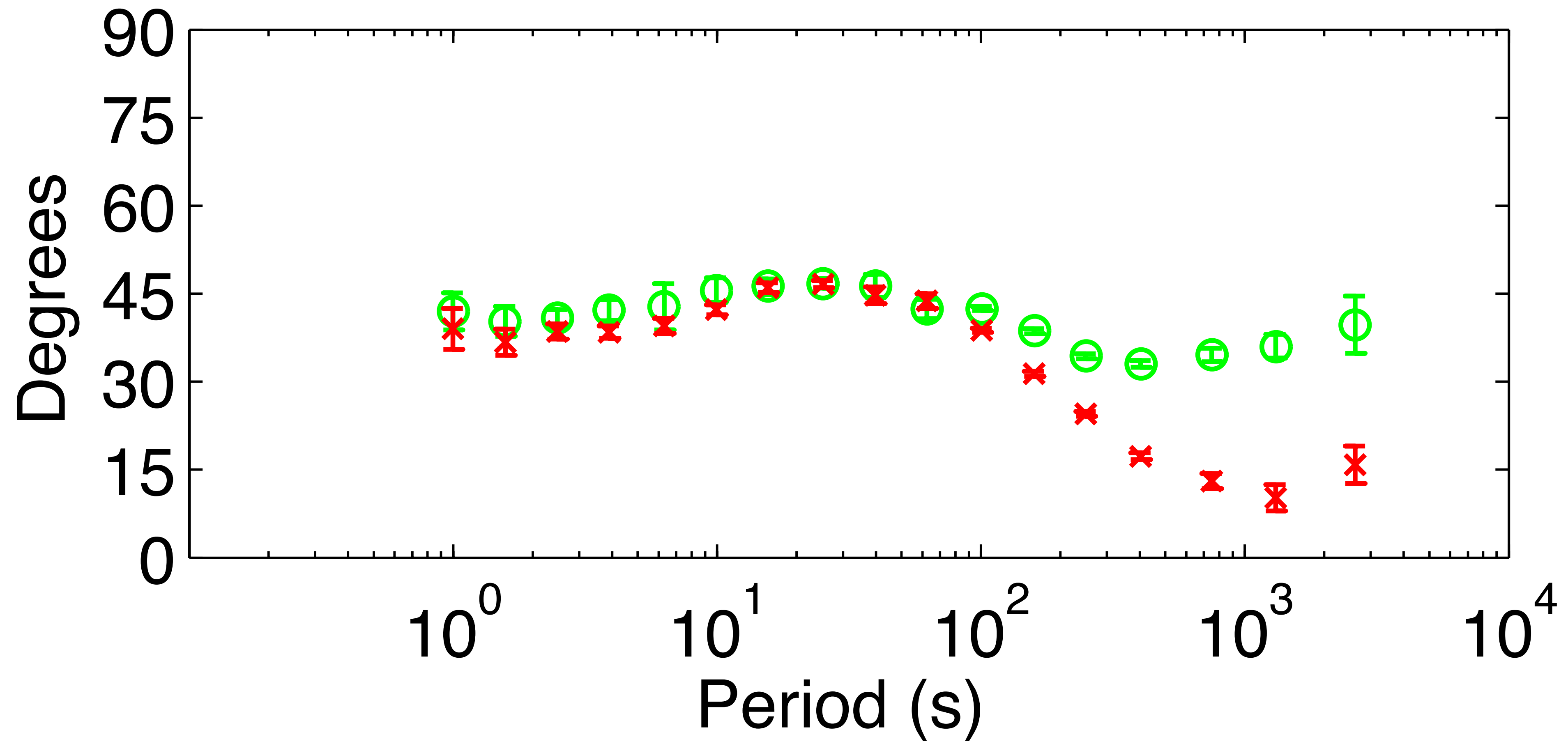
$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

Note that all terms are complex numbers as a function of frequency

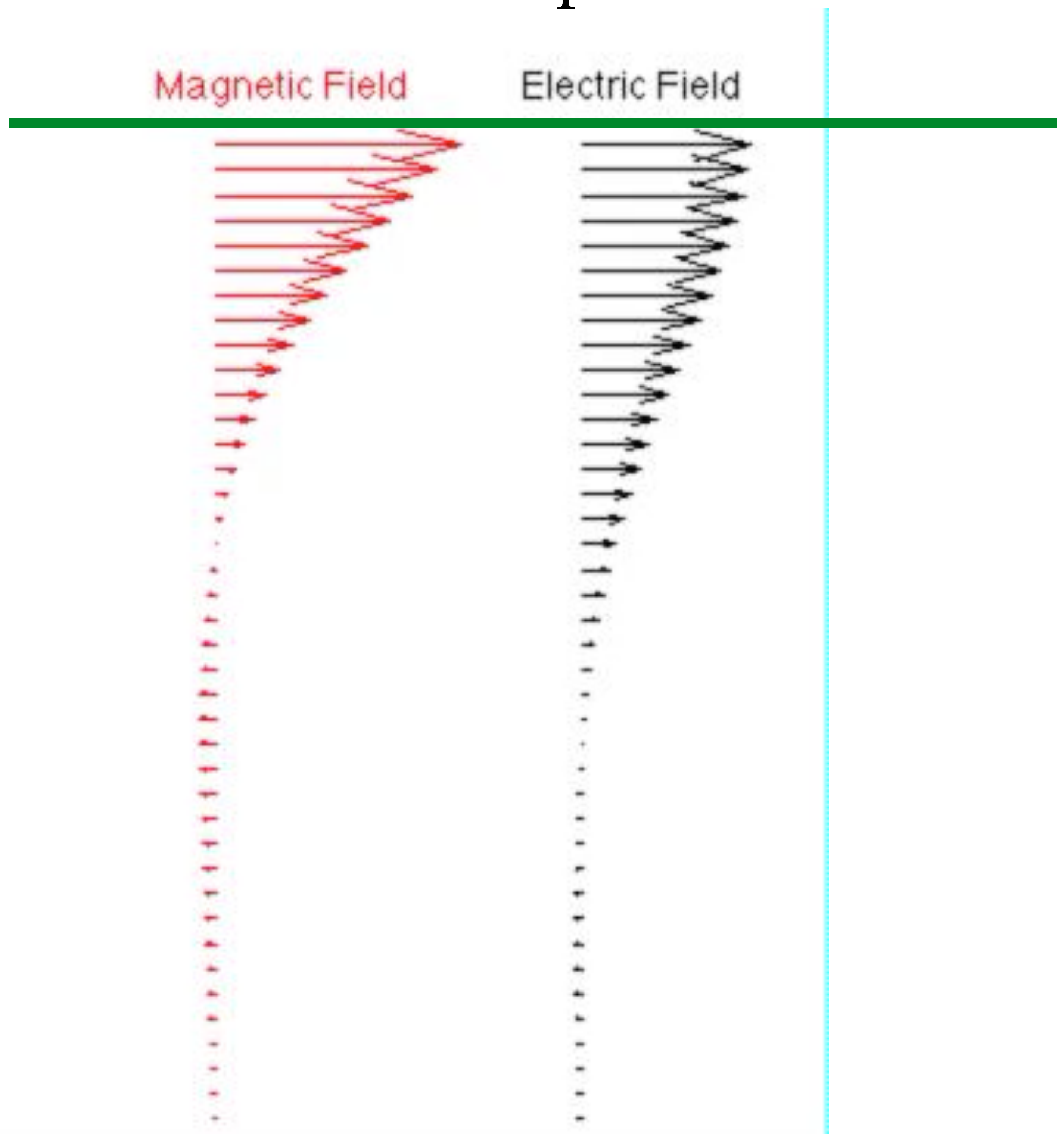
except that most MT people like to use the magnetizing field **H** to define impedance. This slightly changes the apparent resistivity formula (of course, the phase remains the same).

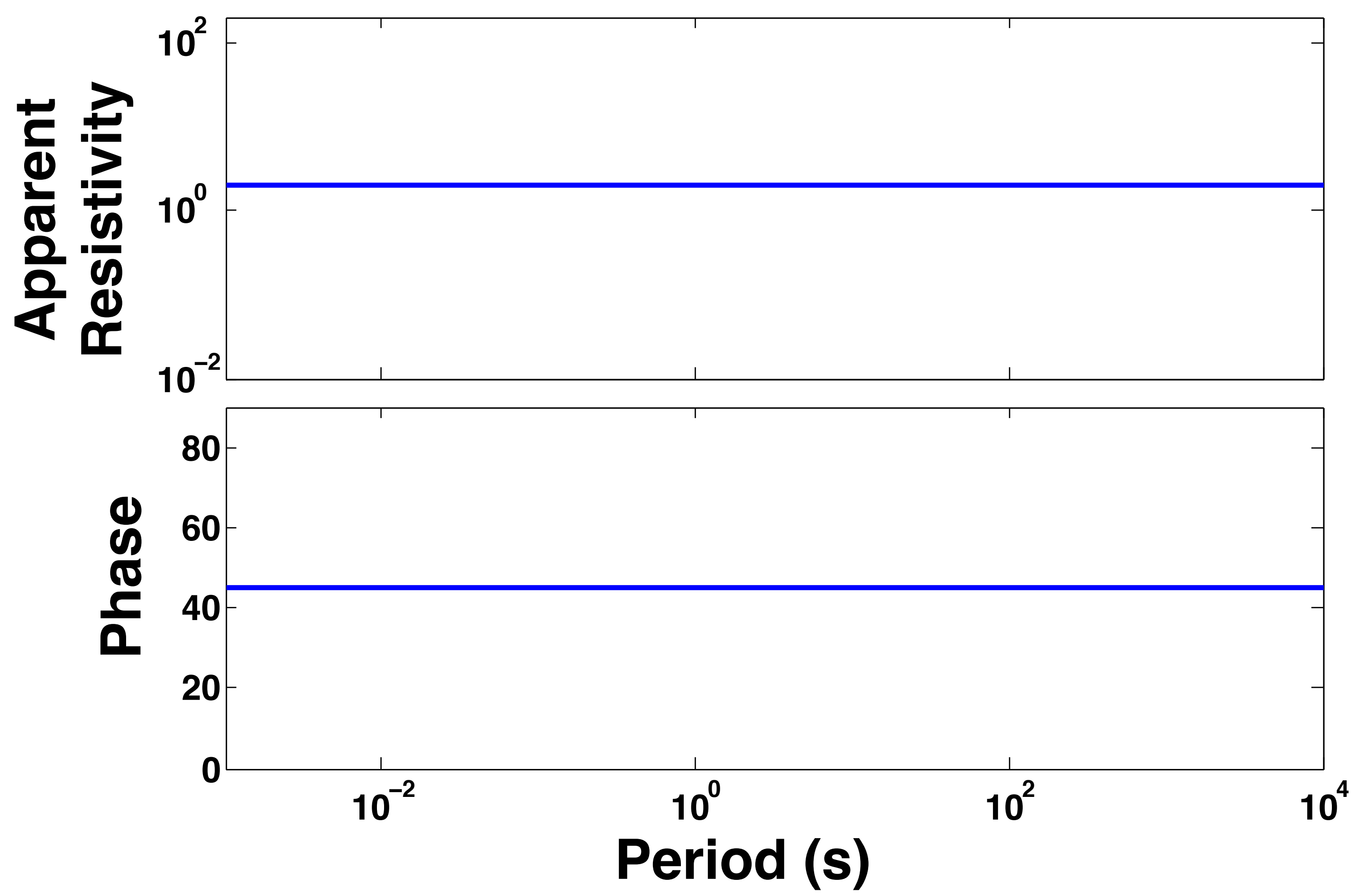
$$\rho_a = \frac{1}{\omega\mu_o}\left|\frac{E}{H}\right|^2 \qquad \phi = \tan^{-1}\left(\frac{E}{H}\right)$$





Half-space





Here is what the MT fields look like in a uniform conductor. The fields decay exponentially with a scale length given by the **skin depth**

$$z_o \approx \frac{500 \text{ m}}{\sqrt{\sigma f}}$$

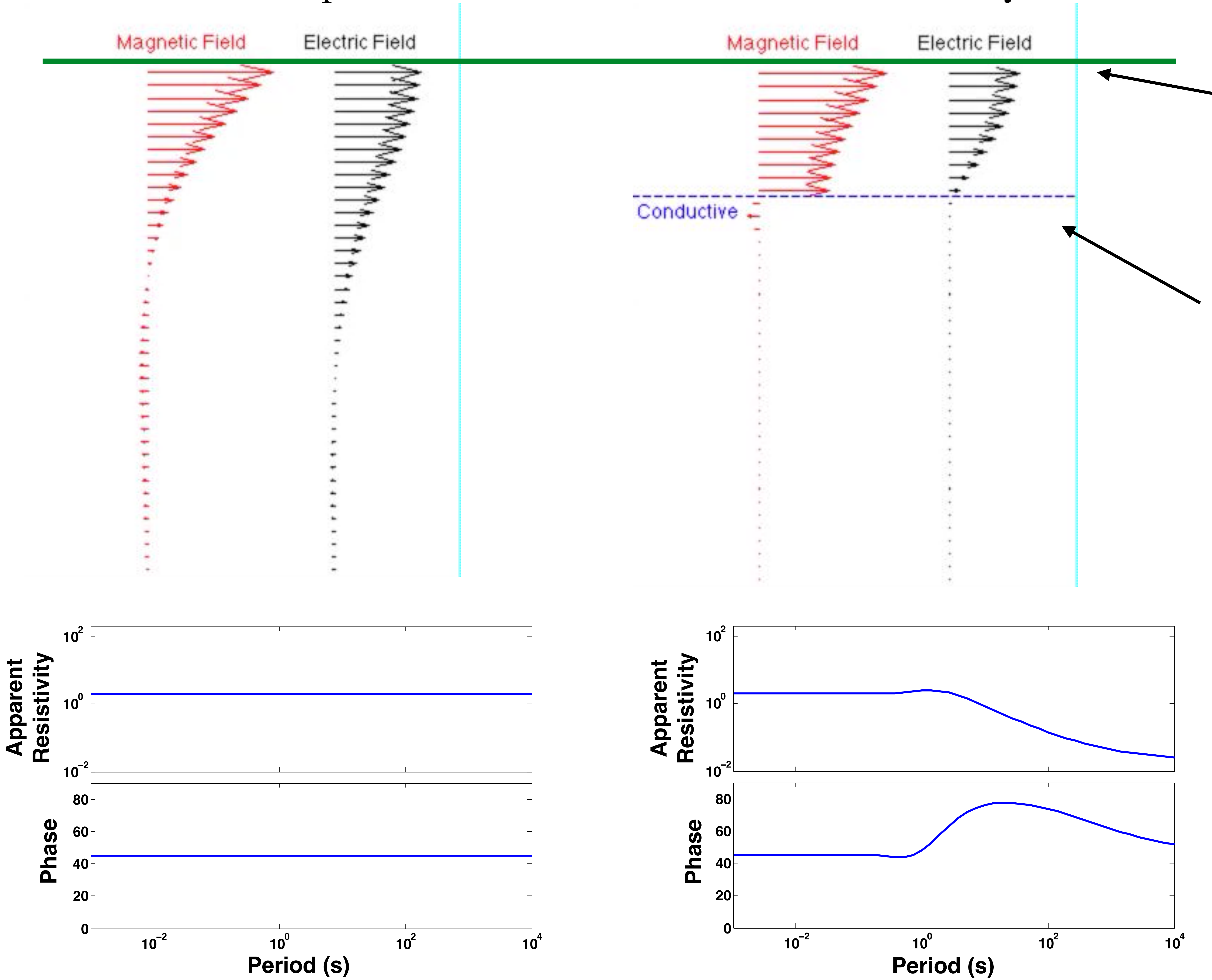
The induced electric field is 45° out of phase with the primary magnetic field.

We can compute a half-space equivalent electrical resistivity (apparent resistivity) at each frequency:

$$\rho_{\mathrm{a}}(\omega) = \frac{\mu_o}{\omega} \left| \frac{E(\omega)}{B(\omega)} \right|^2$$

We can also compute the phase difference between E and B. These become the MT sounding curves.

Half-space | Conductive Layer



E is smaller

We can add a conductive layer at depth and things change at the surface

Half-space | Conductive Layer | Resistive Layer