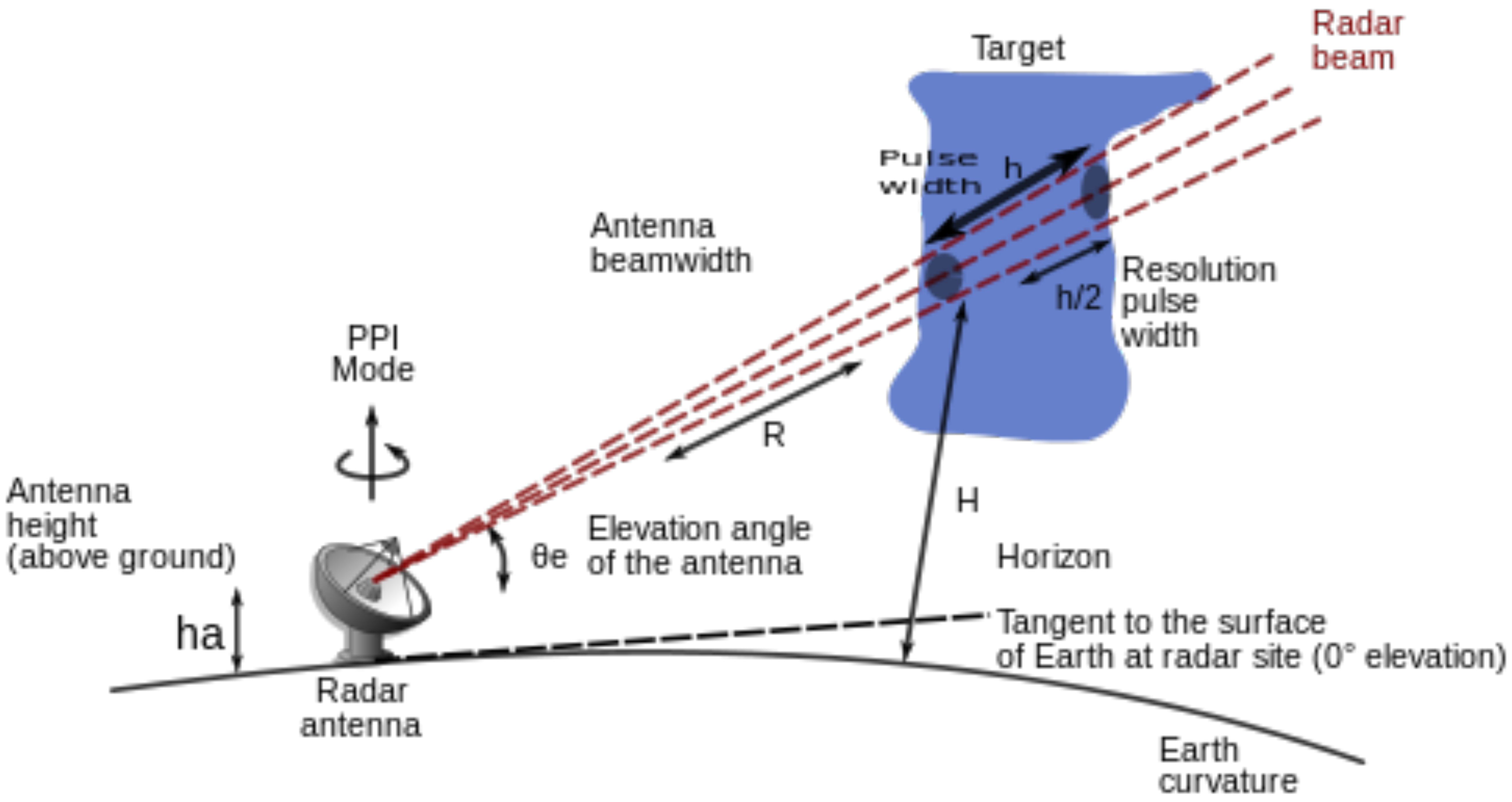
Ground Penetrating Radar (GPR)

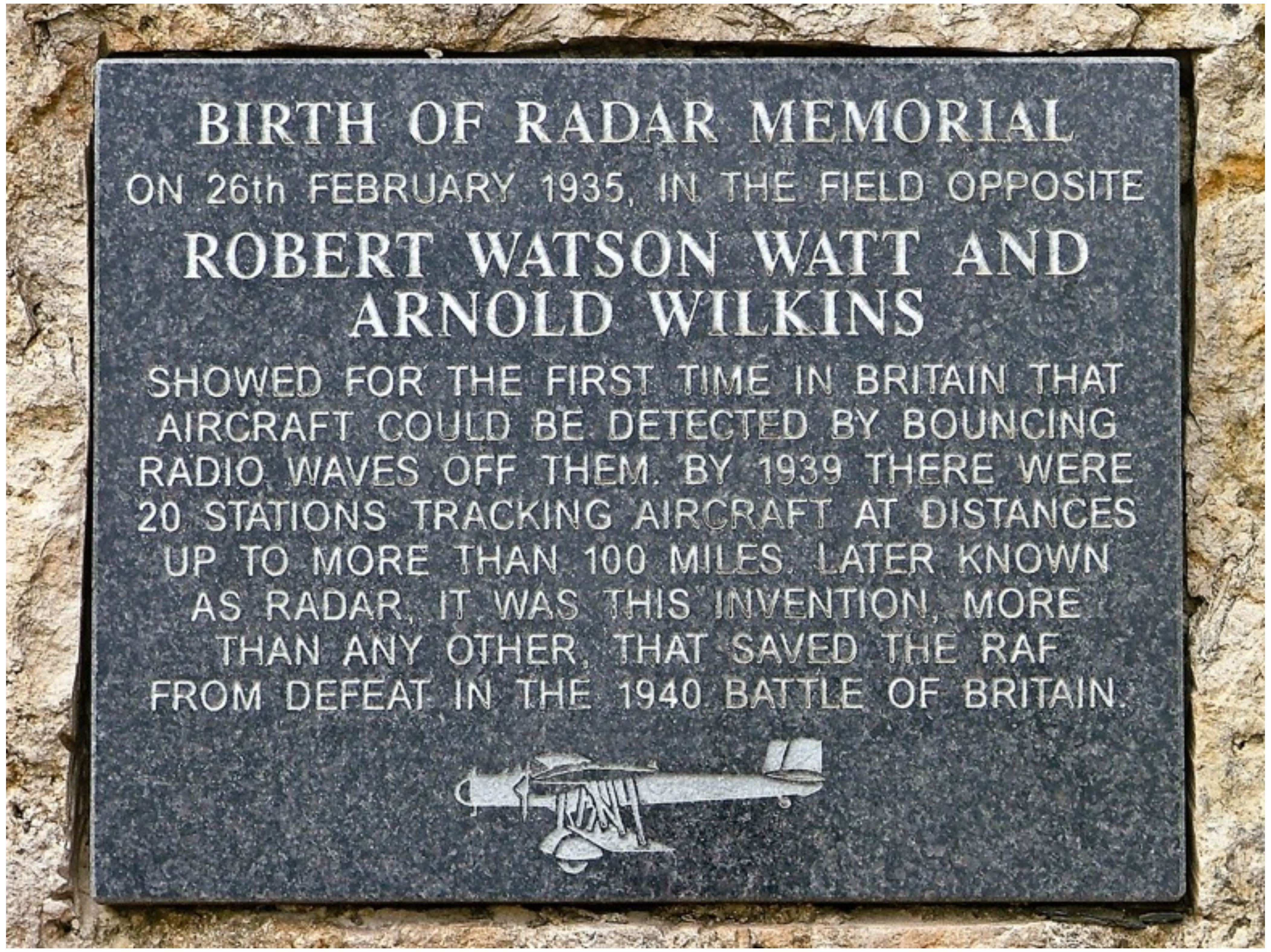
Steven Constable

RADAR: RAdio Detection And Ranging.

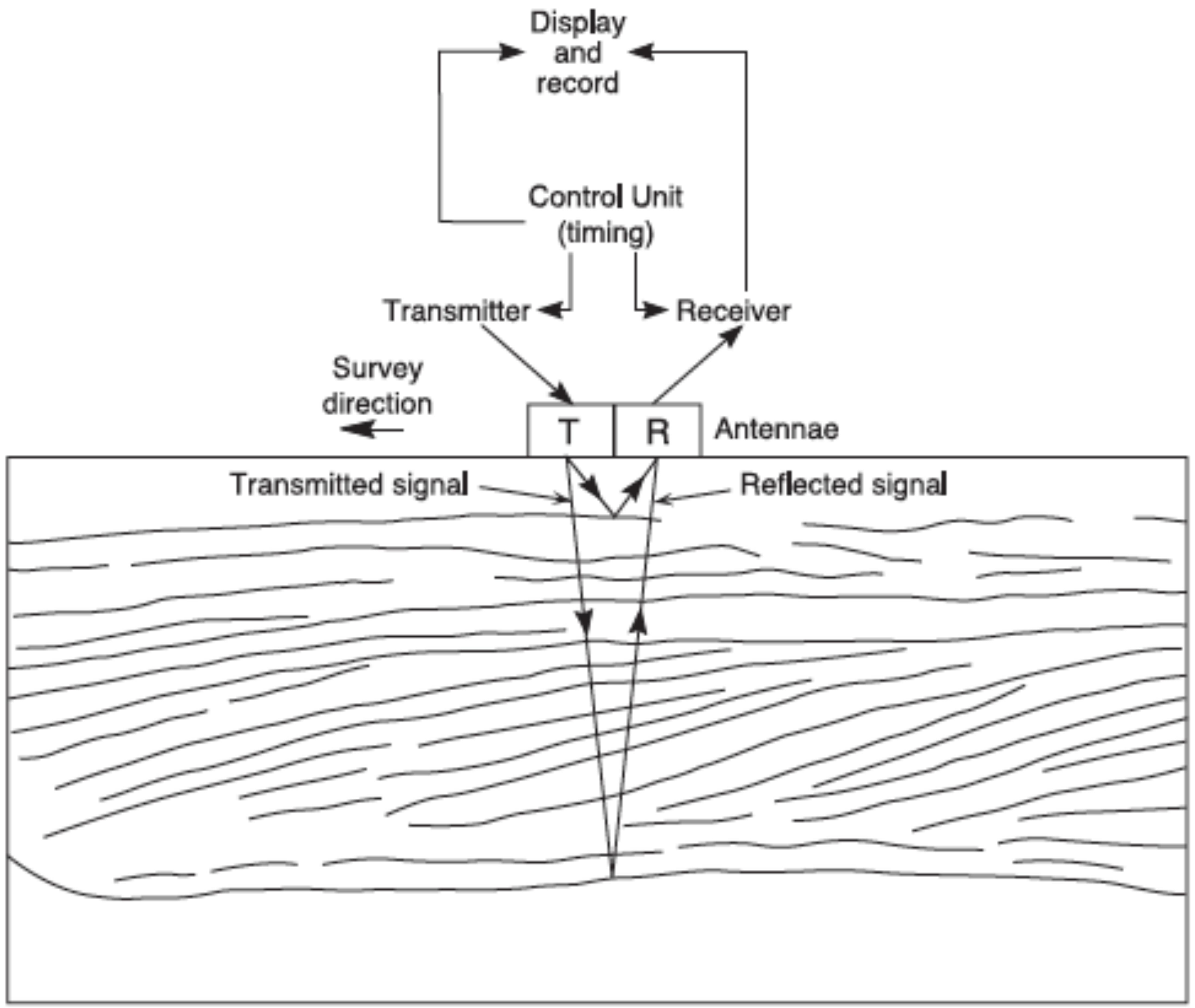
Basically developed during WWII to track aircraft. Now broadly used in meteorology, navigation, surveillance, and in ground imaging (ground penetrating RADAR, or GPR).

Frequencies of 10 MHz to 100 GHz.

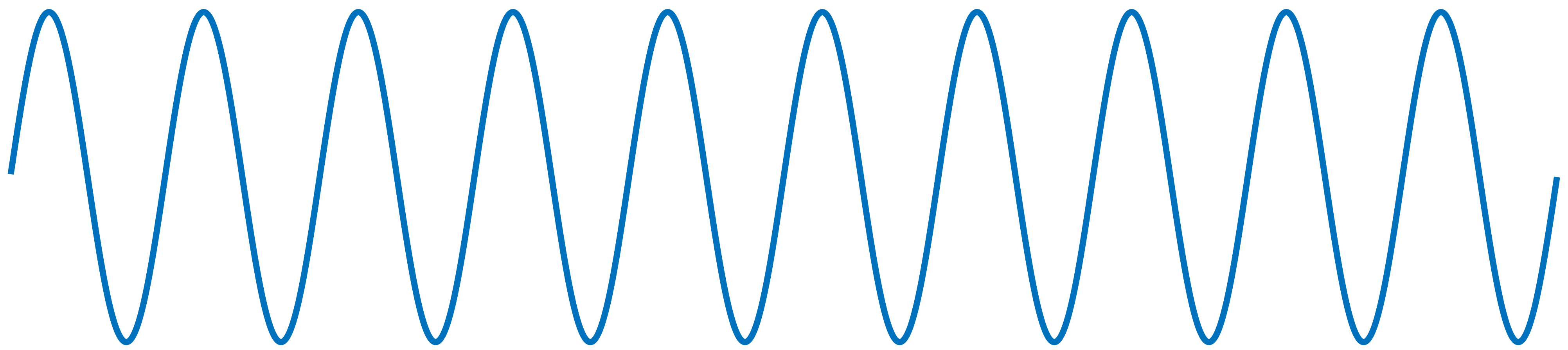




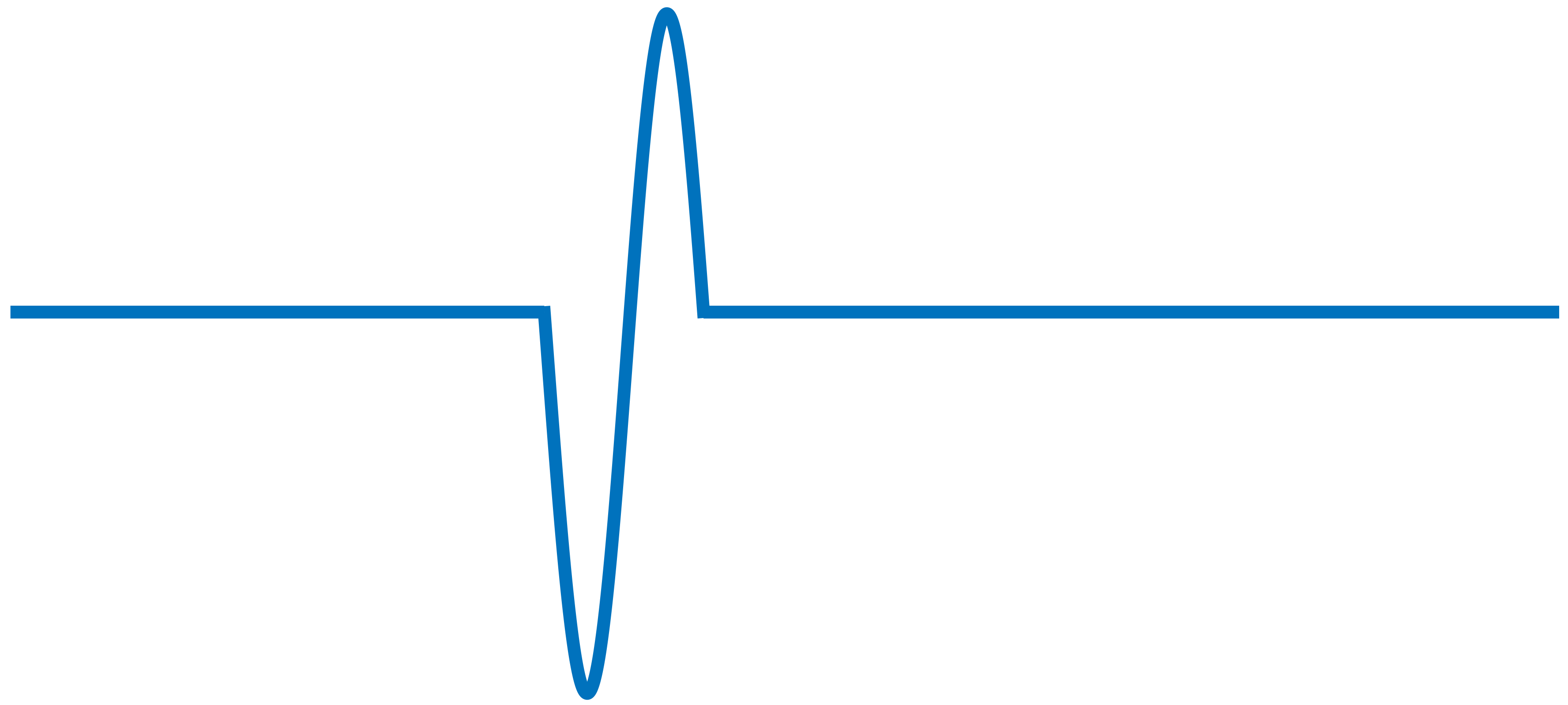
GPR uses frequencies of 50 — 200 MHz to image reflections in the upper few meters of soil/rock.



Neal (2004)



Continuous wave



Pulsed

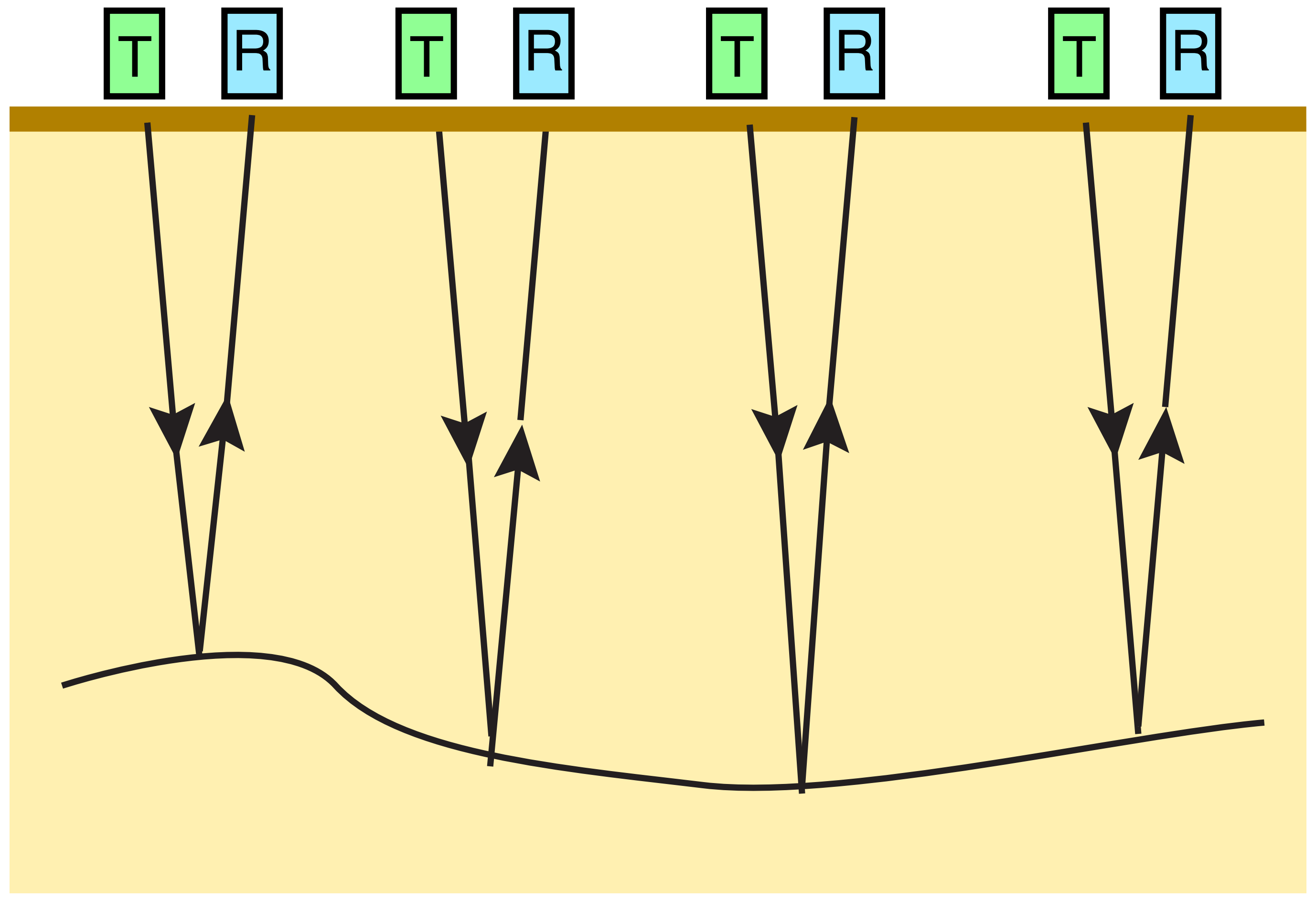
Mono-static: Same antenna for transmitter and receiver (or at least in the same housing)

Bi-static: Separate transmitter and receiver antennas — allows variable spacing, common mid-point, or transillumination

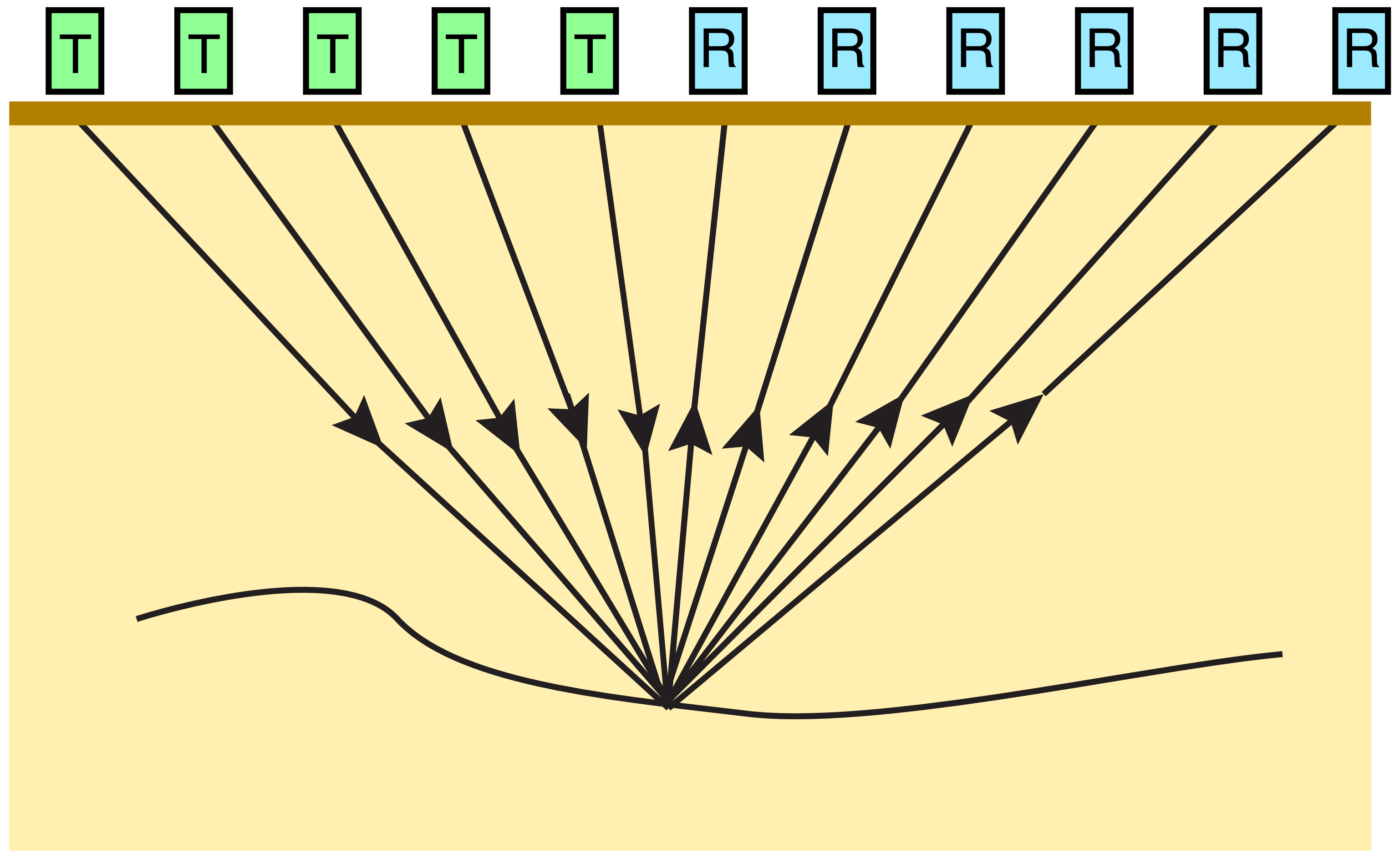
Sensors and Software

Various geometries can be used:

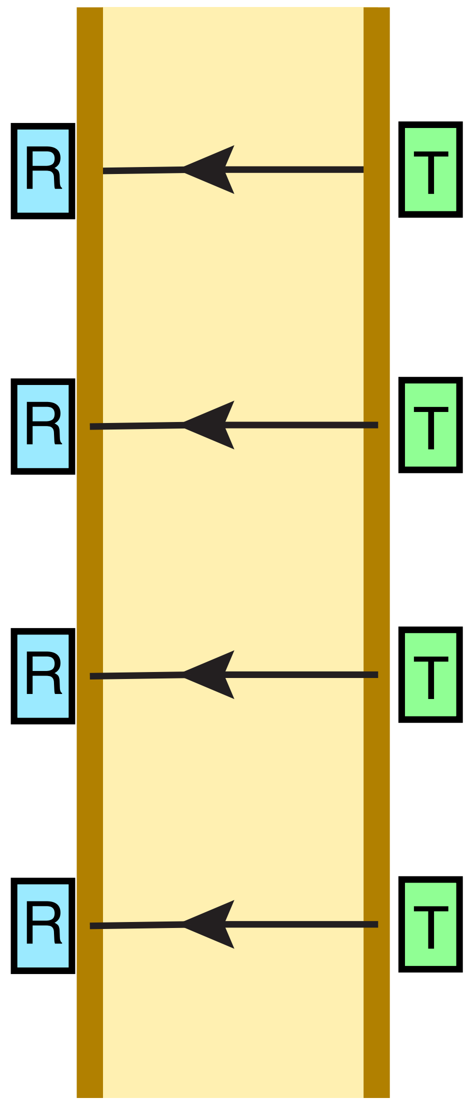
Reflection profiling. Most common method to map subsurface structure.



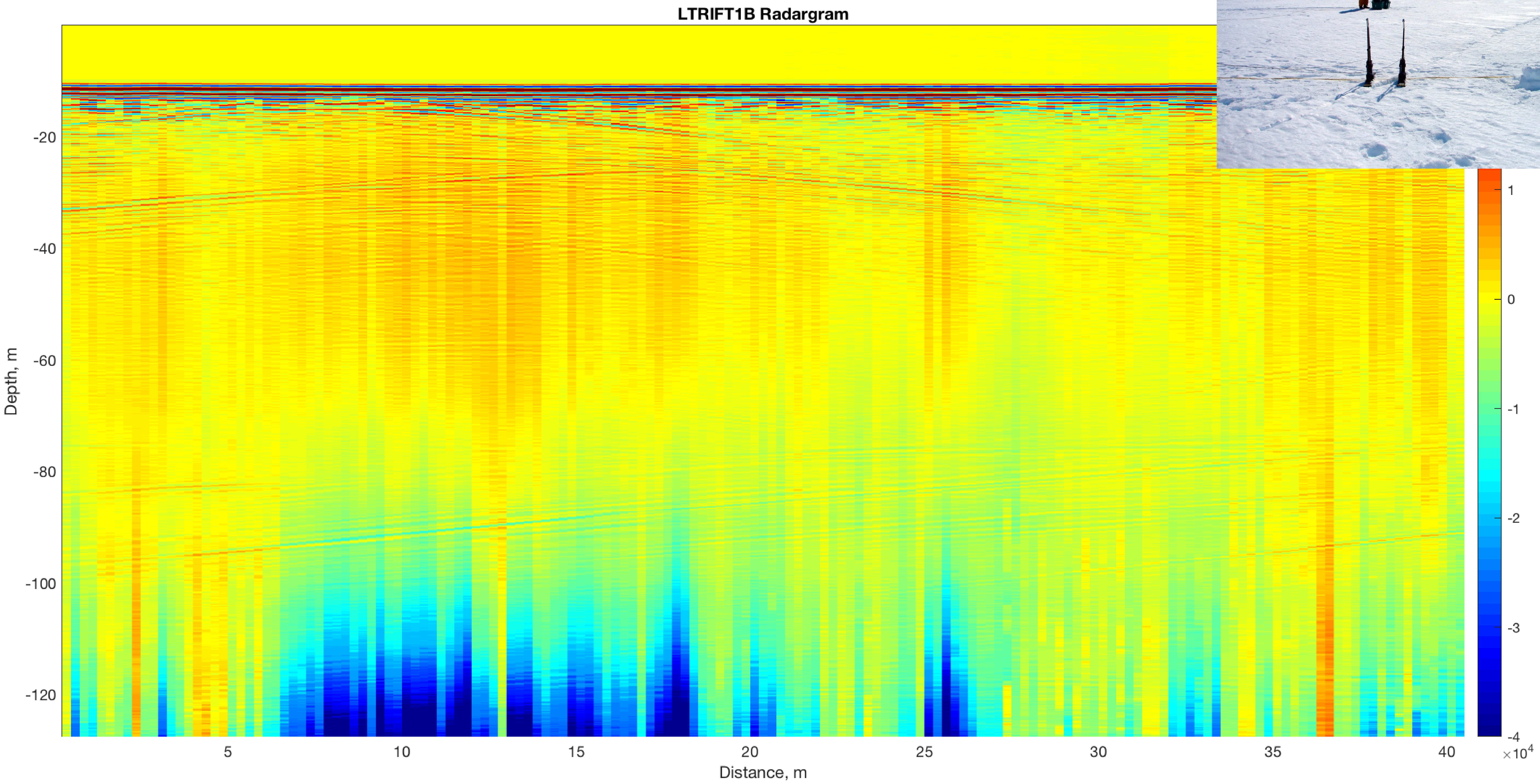
Common mid-point: used to recover velocities.



Transillumination . Used for concrete structures, mine walls, etc.



100 MHz Radargram over Antarctic sea ice, collected by Jim Behrens

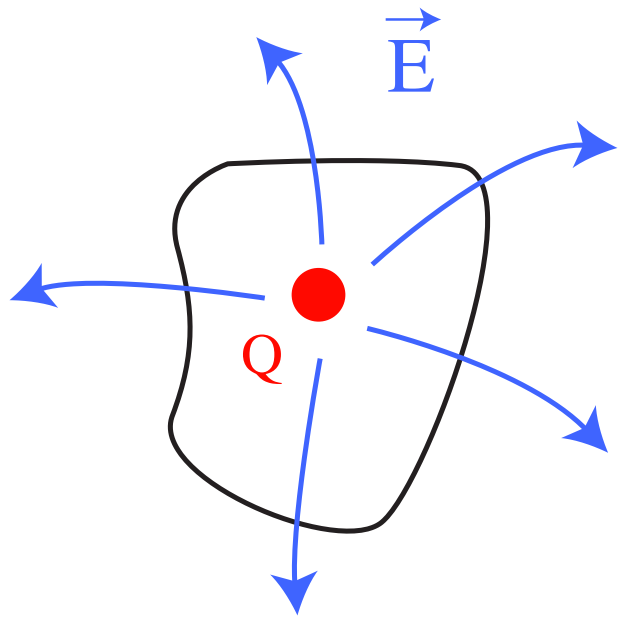


Theory and Physics

Pre-Maxwell equations in free space:

Gauss' Law for charge. The electric field leaving a volume is proportional to the enclosed charge.

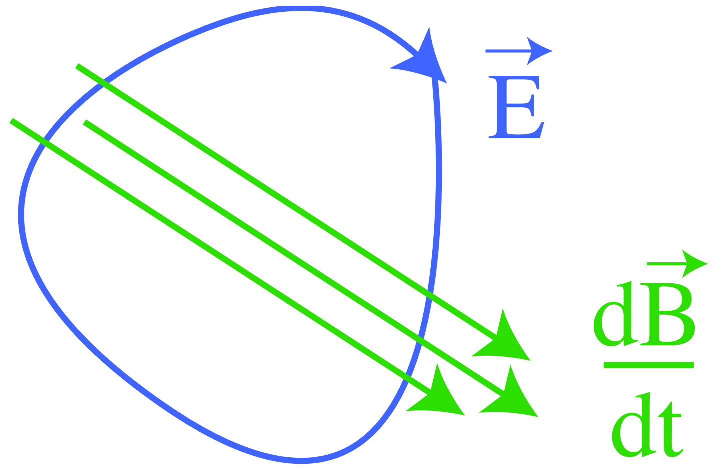
$$\int_{\Omega} \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_o}$$



$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

Faraday's Law. The electric field integrated around a loop is given by the time rate of change of the enclosed magnetic flux.

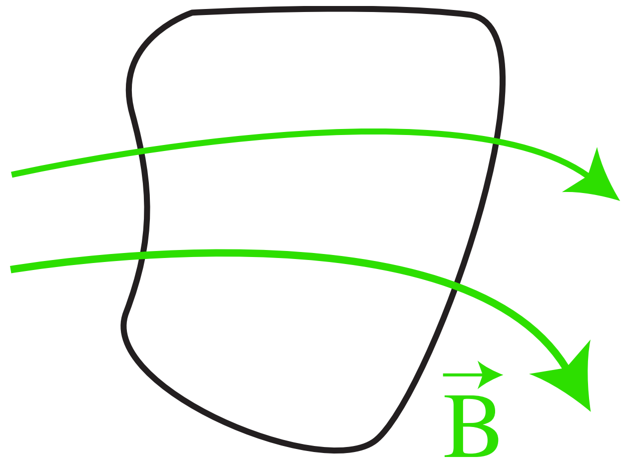
$$\oint_{c} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{\mathrm{B}}}{dt}$$



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Gauss' Law for magnetism. Any magnetic flux that enters a volume must leave it (no monopoles!).

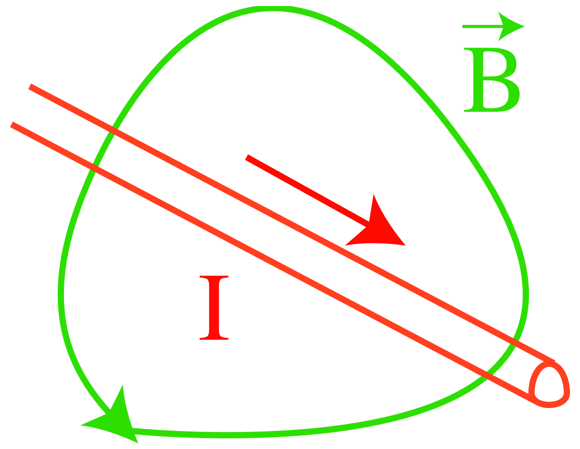
$$\int_{\Omega} \mathbf{B} \cdot d\mathbf{s} = 0$$



$$\nabla \cdot \mathbf{B} = 0$$

Ampère's Law. An electric current will generate a circulating magnetic field.

$$\oint_{c} \mathbf{B} \cdot d\mathbf{l} = \mu_o I$$



$$\nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

E is electric field in V/m **B** is magnetic field in T **J** is current density in A/m2 ρ is charge density in C/m^3

Maxwell's equations in free space:

The extra term in Ampère's Law was added by James Clerk Maxwell to allow magnetic and electric fields to exist without charges or currents. It allows electromagnetic radiation to propagate in a vacuum at speed c:

$$c^2 = \frac{1}{\mu_o \epsilon_o}$$

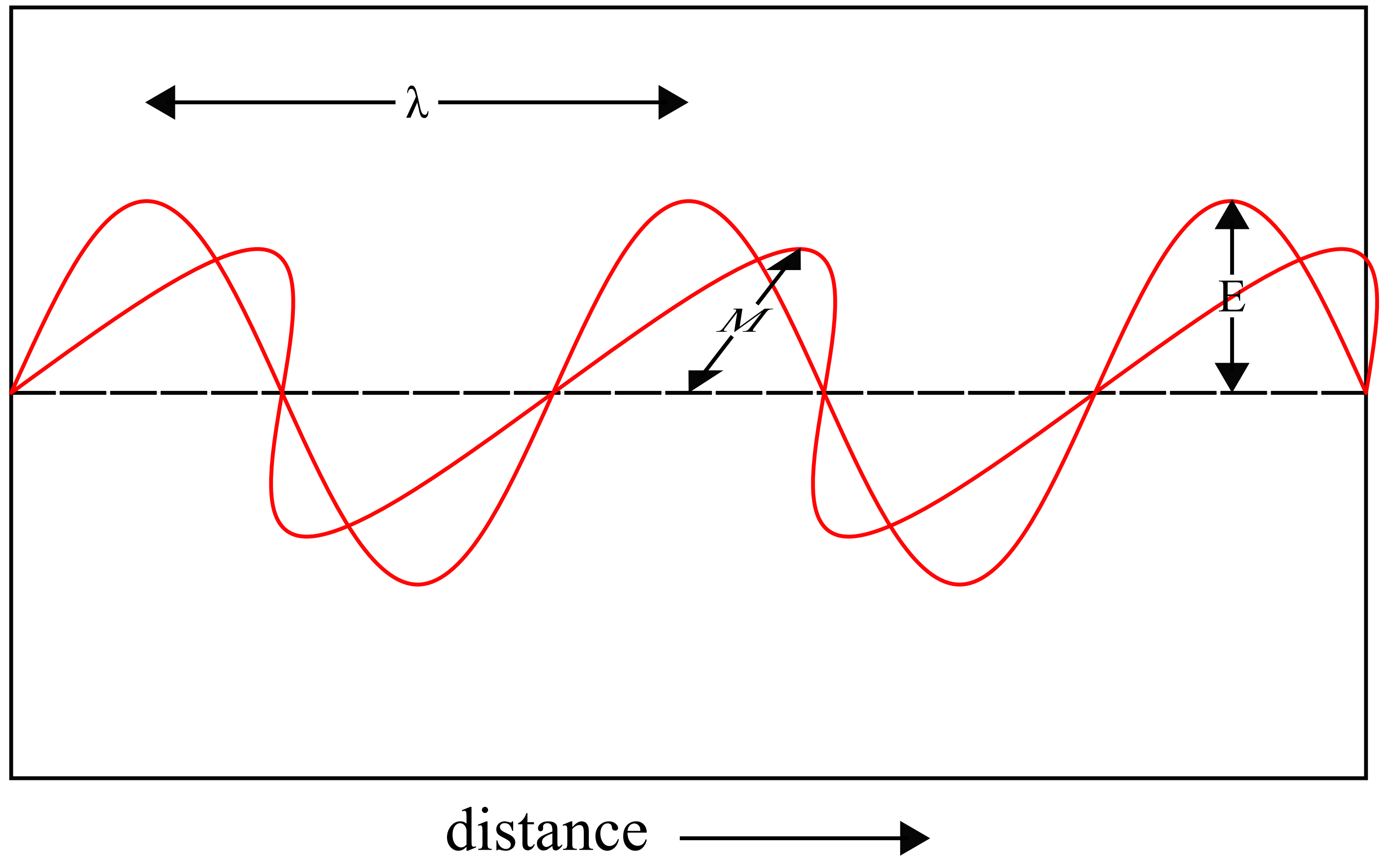
roughly 3×10^8 m/s. In GPR work, this is usually expressed as 0.30 m/ns

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_o \left(\mathbf{J} + \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \right)$$



(These are actually Oliver Heaviside's equations)

$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0$	(1)	Gauss' Law
$\mu\alpha=\frac{d\mathrm{H}}{dy}-\frac{d\mathrm{G}}{dz}$ $\mu\beta=\frac{d\mathrm{F}}{dz}-\frac{d\mathrm{H}}{dx}$ $\mu\gamma=\frac{d\mathrm{G}}{dx}-\frac{d\mathrm{F}}{dy}$	(2)	Equivalent to Gauss' Law for magnetism
$\mathrm{P}=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{d\mathrm{F}}{dt}-\frac{d\Psi}{dx}$ $\mathrm{Q}=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{d\mathrm{G}}{dt}-\frac{d\Psi}{dy}$ $\mathrm{R}=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{d\mathrm{H}}{dt}-\frac{d\Psi}{dz}$	(3)	Faraday's Law (with the Lorentz Force and Poisson's Law)
$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'$ $\quad p'=p+\frac{df}{dt}$ $\frac{d\alpha}{dz}-\frac{d\gamma}{dx}=4\pi q'$ $\quad q'=q+\frac{dg}{dt}$ $\frac{d\beta}{dx}-\frac{d\alpha}{dy}=4\pi r'$ $\quad r'=r+\frac{dh}{dt}$	(4)	Ampère-Maxwell Law
$\mathrm{P}=-\xi p \quad \mathrm{Q}=-\xi q \quad \mathrm{R}=-\xi r$		Ohm's Law
$\mathrm{P}=kf \quad \mathrm{Q}=kg \quad \mathrm{R}=kh$		The electric elasticity equation (**E** = **D**/ε)
$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$		Continuity of charge

Maxwell's equations

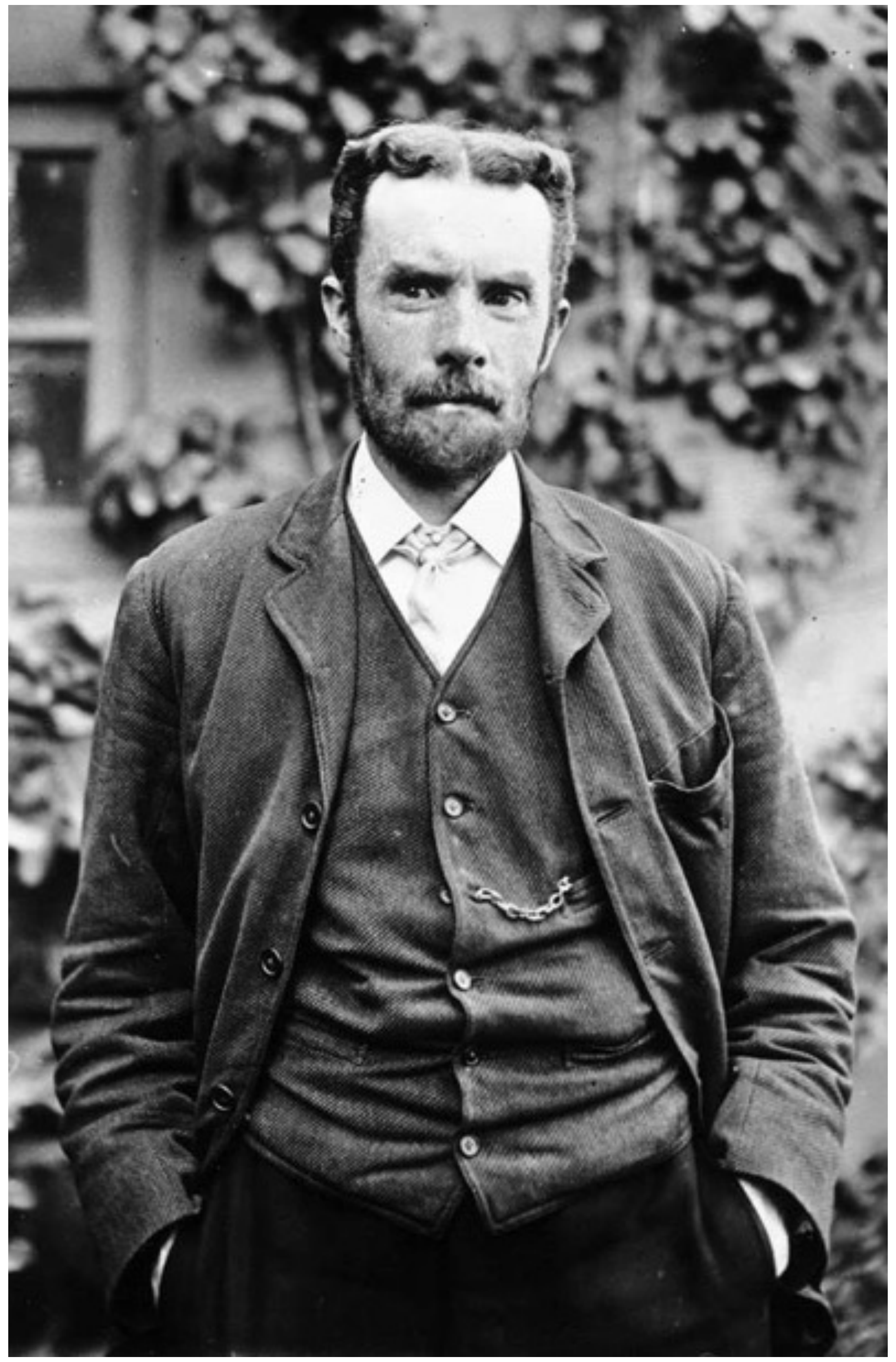
$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_o}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B}=0$$

$$\nabla\times\mathbf{B}=\mu_o\left(\mathbf{J}+\epsilon_o\frac{\partial\mathbf{E}}{\partial t}\right)$$

Heaviside's equations



$$\nabla \times \mathbf{B} = \mu_o \left(\mathbf{J} + \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \right)$$

ϵ_o is permittivity of free space, $= 8.895 \times 10^{-12}$ F/m

μ_o is permeability of free space, $= 4\pi \times 10^{-7}$ H/m

This is as far as you need to go for RADAR in air, but for GPR we have to worry about what happens in matter. Here we need to introduce the **constitutive relations**:

$$\mathbf{J} = \sigma \mathbf{E}$$

where σ is electrical conductivity, S/m. This is Ohm's Law: an electric field in matter will move charges to create an electric current. (V = IR maps to $\mathbf{E} = \mathbf{J}\rho$)

$$\mathbf{M} = \frac{\chi_m}{\mu_o} \mathbf{B}$$

where χ_m is magnetic susceptibility. In the presence of a magnetic field, the magnetic moment of spinning electrons align to create an internal magnetization **M**.

$$\mathbf{P} = \epsilon_o \chi_e \mathbf{E}$$

where χ_e is electric susceptibility. In the presence of an electric field, the electric charges in atoms polarize to reduce the electric field but without steady current flow. P is dielectric polarization density.

$$\mathbf{J} = \sigma \mathbf{E} \qquad \mathbf{M} = \frac{\chi_m}{\mu_o}\mathbf{B} \qquad \mathbf{P} = \epsilon_o \chi_e \mathbf{E}$$

These linear, isotropic, relations are clearly only approximations. Saturation can occur in all cases, the properties can be frequency dependent, and vary with direction (think of a diode, for example). From these we can define

$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P}$ the electric displacement field in C/m^2 $\qquad$ $\mathbf{D} = \epsilon_o \mathbf{E}$ in a vacuum

$\mathbf{H} = \mathbf{B}/\mu_o - \mathbf{M}$ the magnetizing field in A/m $\qquad$ $\mathbf{B} = \mu_o \mathbf{H}$ in a vacuum

So finally, Maxwell's equations in matter can be written as

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

For most GPR applications, we can ignore magnetic effects. The important rock properties are the conductivity σ and permittivity ϵ.

Permittivity is related to electric polarization:

$$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P} = \epsilon_o (1 + \chi_e \mathbf{E}) = \epsilon \mathbf{E}$$

$$\chi_e = \frac{\epsilon}{\epsilon_o} - 1 = \epsilon_r - 1$$

where ϵ_r is relative permittivity. The most polarizable material in geology is water, with a relative permittivity of 80, so water content mostly determines ϵ_r.

In sediments and soils, conductivity is also driven by the water content, but now the salinity of the water also determines how conductive it is. Geophysicists will also talk about resistivity, the reciprocal of conductivity, with units of Ωm:

$$\rho = \frac{1}{\sigma}$$

material	ϵ_r	conductivity, S/m	resistivity, Ωm
air	1	0	large
fresh water	80	< 0.01	> 100
sea water	80–88	3–5	0.2–0.3
dry sand	3–10	$\approx$0.001	$\approx$1,000
wet sand	10–30	$\approx$0.1	$\approx$ 10
granite	5–8	$<$0.001	$>$1,000

In GPR, the velocity of the radio waves is largely determined by permittivity.

Velocity is given by $$v = \frac{c}{\sqrt{\epsilon_r \frac{1+\sqrt{P^2+1}}{2}}}$$

where P is a "loss factor" $P = \frac{\sigma}{\omega\epsilon}$ and $\omega = 2\pi f$ is angular frequency. This trade-off between conductivity and frequency times permittivity is fundamental. Substituting Ohm's Law into Maxwell's version of Ampère's Law, and replacing the time derivative with frequency we have

$$\nabla \times \mathbf{B} = \mu_o \mathbf{E}(\sigma + \epsilon\omega)$$

$$\sigma \approx 10^{-4}\ \mathrm{S/m} \qquad \epsilon_o \approx 10^{-11}\ \mathrm{F/m}$$

so unless the frequency is very high, > 10 MHz, the second term is negligible even for low conductivities and the equation collapses back to the pre-Maxwell form with no wave propagation (it becomes the diffusion equation, important for electromagnetic methods in geophysics, but that's another story).

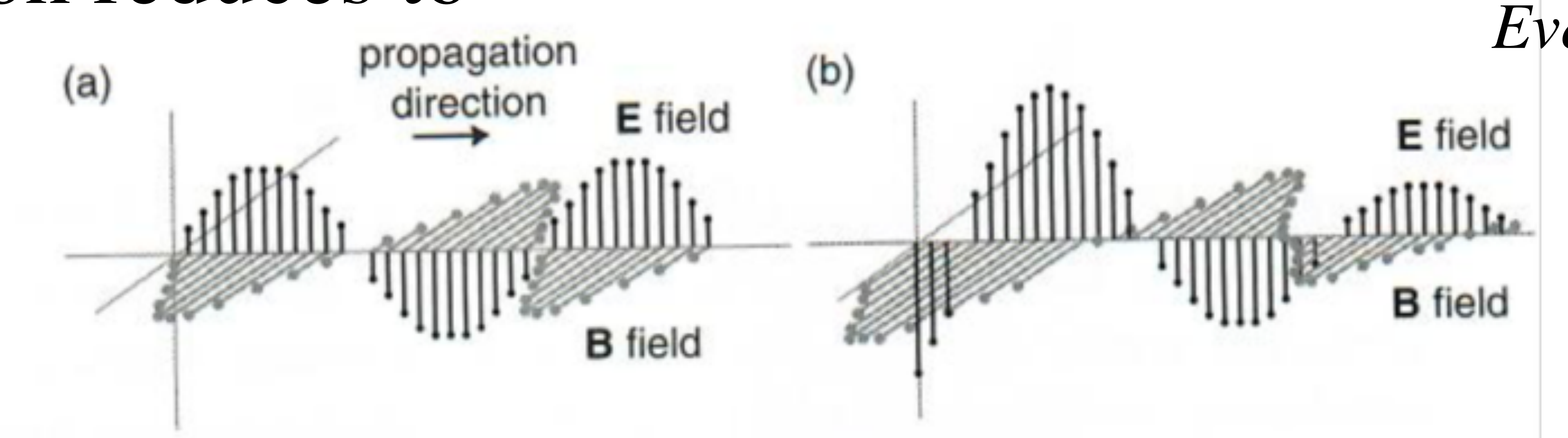
When P is small, the above equation reduces to $v = \frac{c}{\sqrt{\epsilon_r}}$ and in water the velocity is about $\frac{c}{9}$.

The attenuation (loss, absorption) is largely determined by conductivity. In a uniform conductor the electric field will decay exponentially with depth x

$$E(x) = E_o e^{-\alpha x} \quad \text{where the attenuation coefficient is given by} \quad \alpha = \omega \sqrt{\frac{\mu_o \epsilon}{2}\left(\sqrt{1+P^2}-1\right)}$$

When the loss factor P is much larger than 1, this equation reduces to

Everett (2013)



$$\alpha = \omega \sqrt{\frac{\mu_o \epsilon}{2} P} = \sqrt{\frac{\omega \mu_o \sigma}{2}}$$

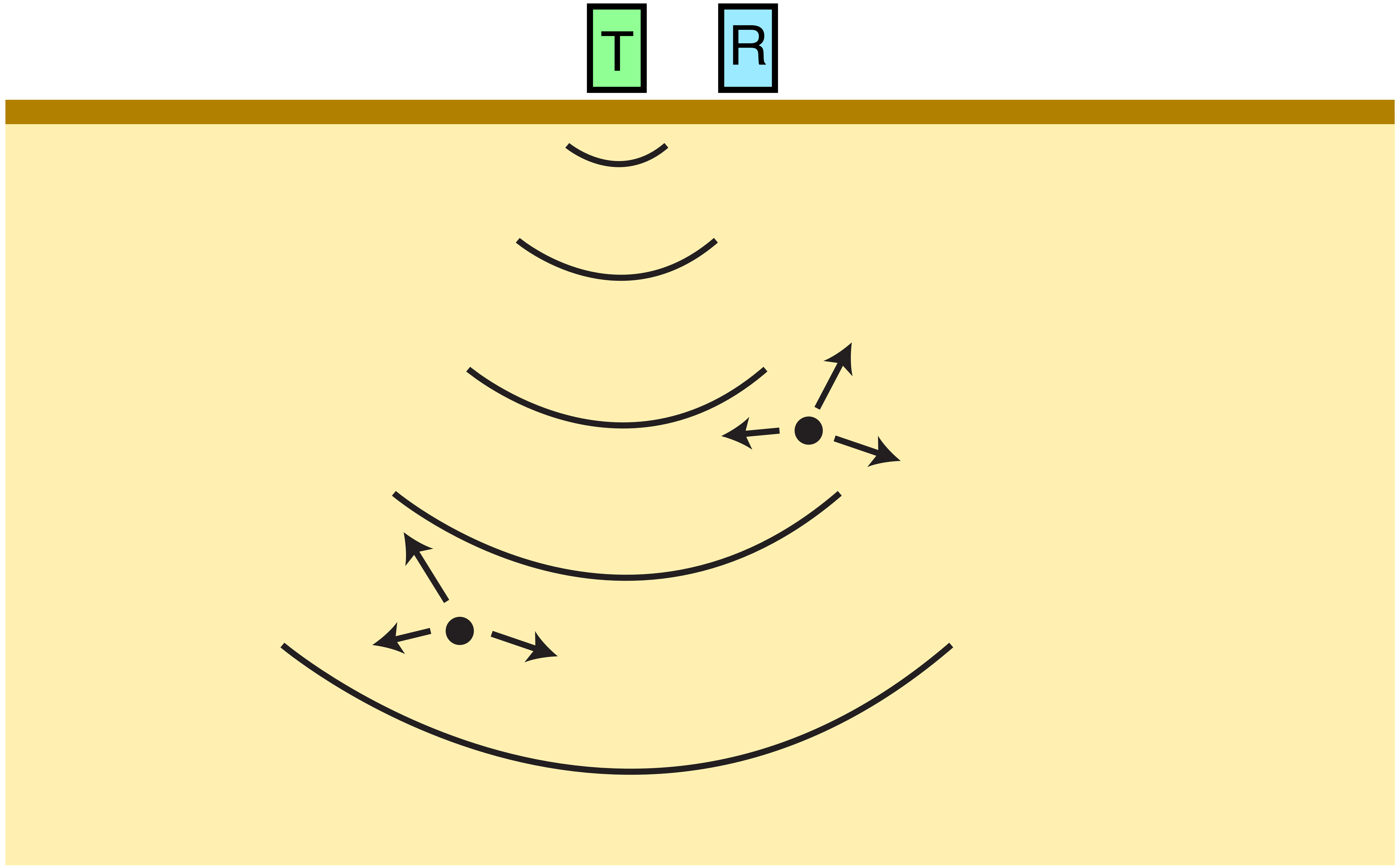
which is the reciprocal of the skin depth, again important in EM methods. When the loss factor P is much smaller than 1, this equation reduces to

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu_o}{\epsilon}}$$

which is frequency independent.

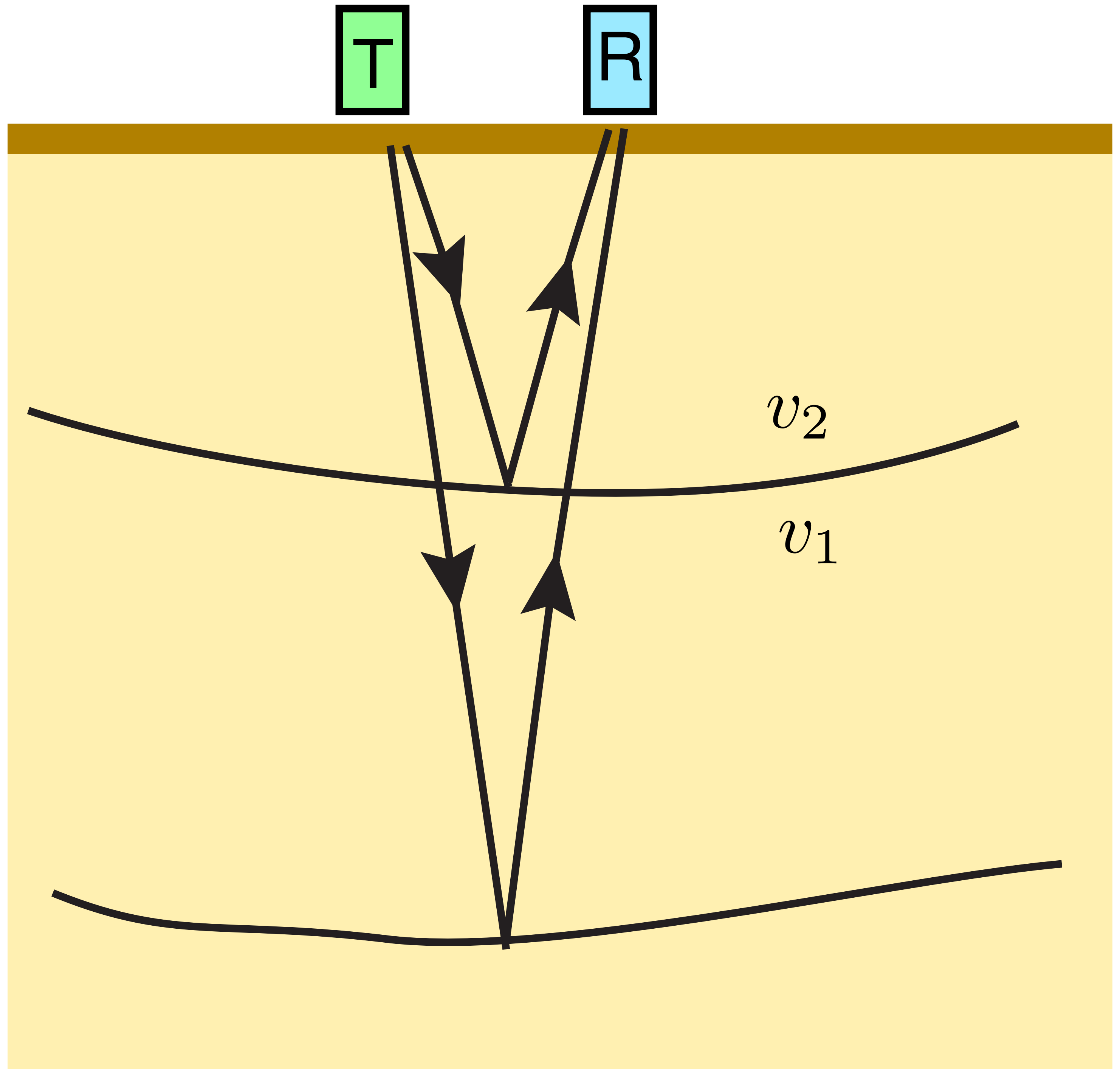
(We have used the binomial theorem: $(1+x)^n = 1 + nx + \text{higher order terms}$.)

Attenuation also comes from geometric spreading and scattering.



Reflection: When a normal-incidence radar wave encounters a change in velocity, some energy will be reflected. The reflection coefficient (the fraction of energy that gets reflected) is given by

$$R = \frac{v_1 - v_2}{v_1 + v_2}$$



which for low loss materials is

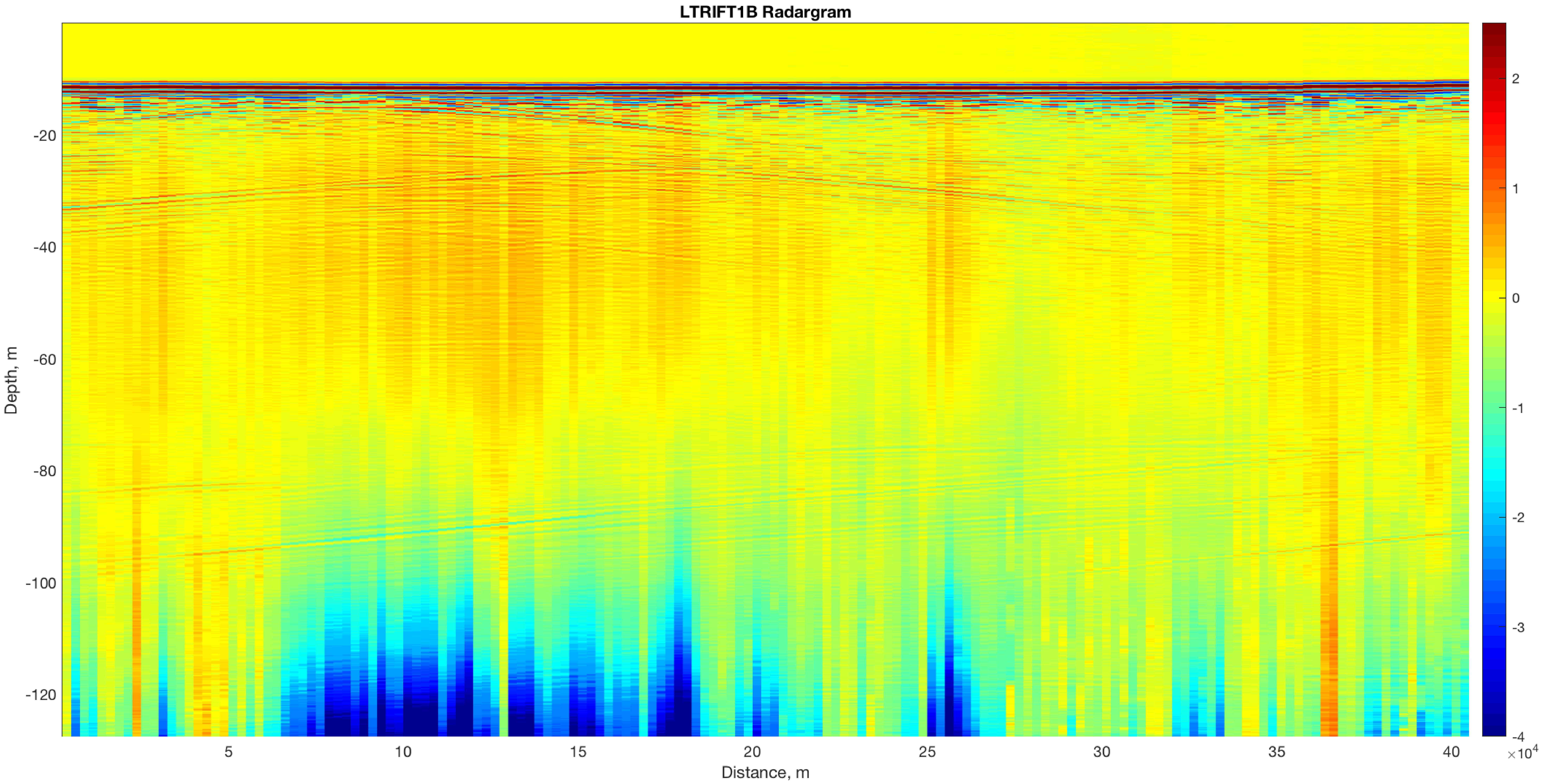
$$R = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

(but remember that velocity also depends on conductivity in high loss materials).

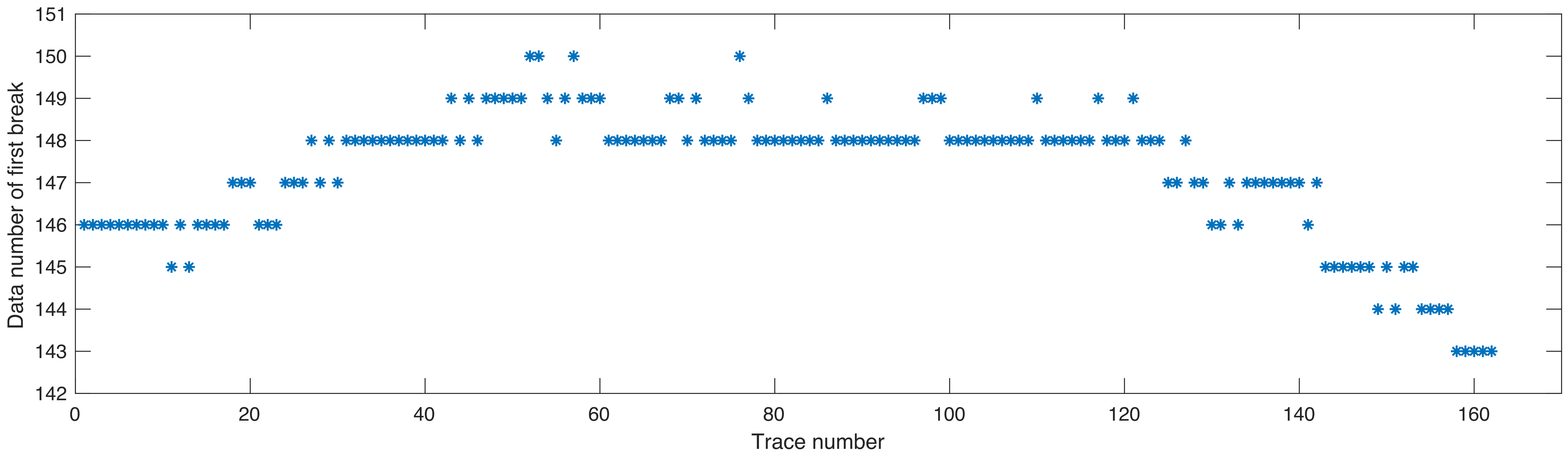
The energy transmitted is (1-R).

Processing

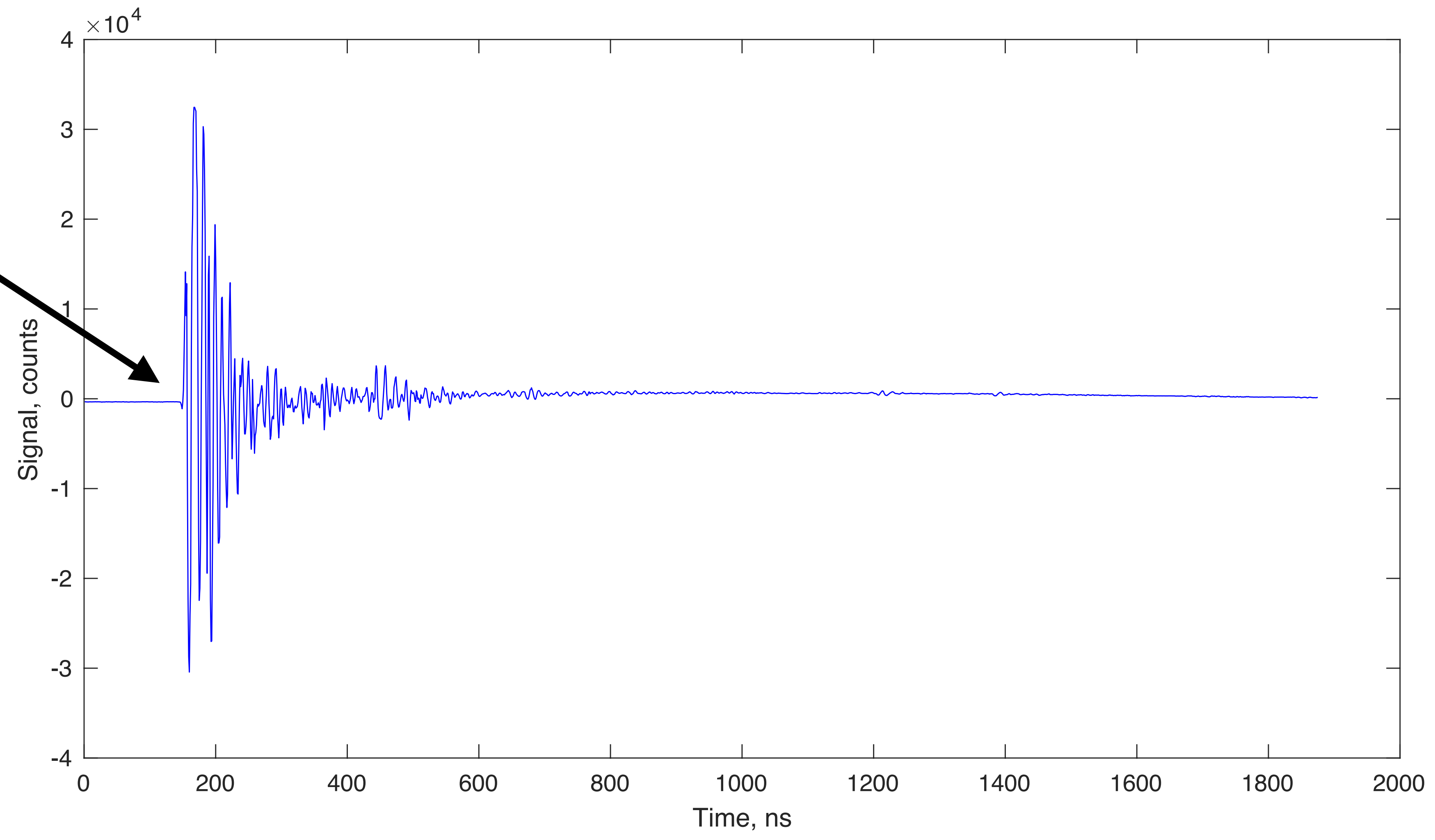
Some basic **processing**, using the ice radargram as an example:



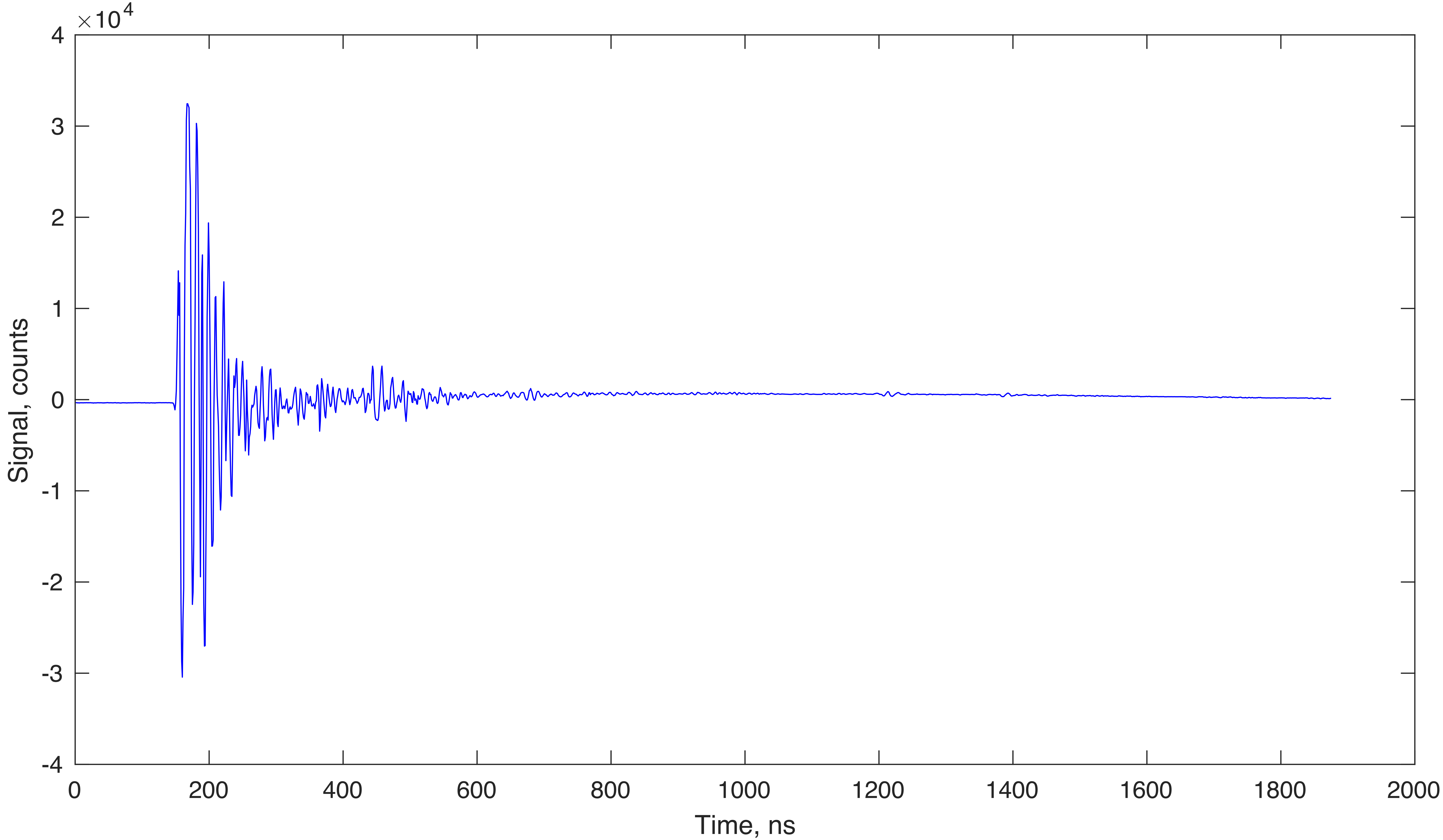
Time zero correction: Tracking signals to a fraction of a nanosecond is very demanding on electronics, and there will be some time drift, often driven by temperature. Here we see a drift of about 5 ns in time zero by looking at the first break for every trace.



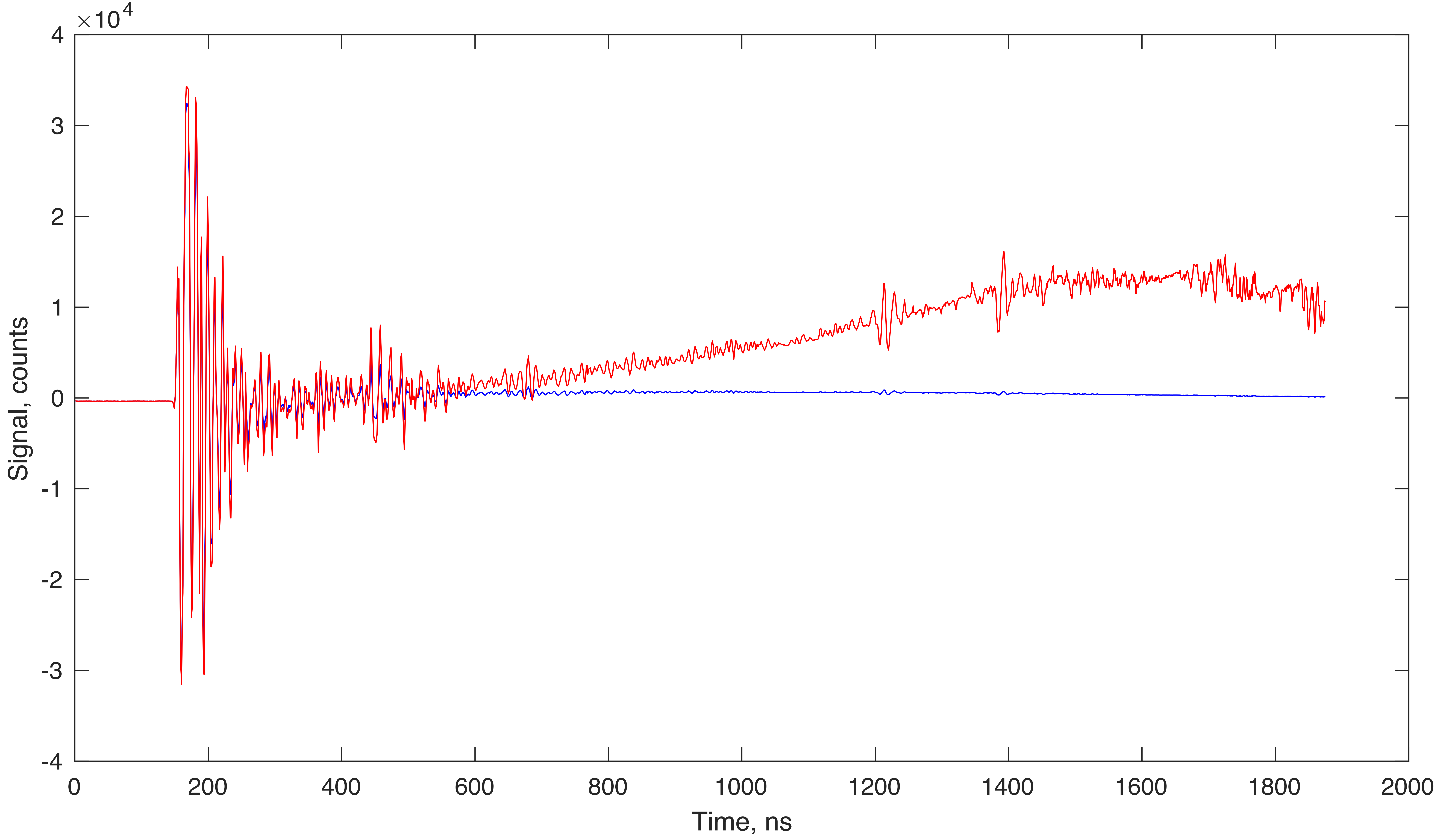
first break



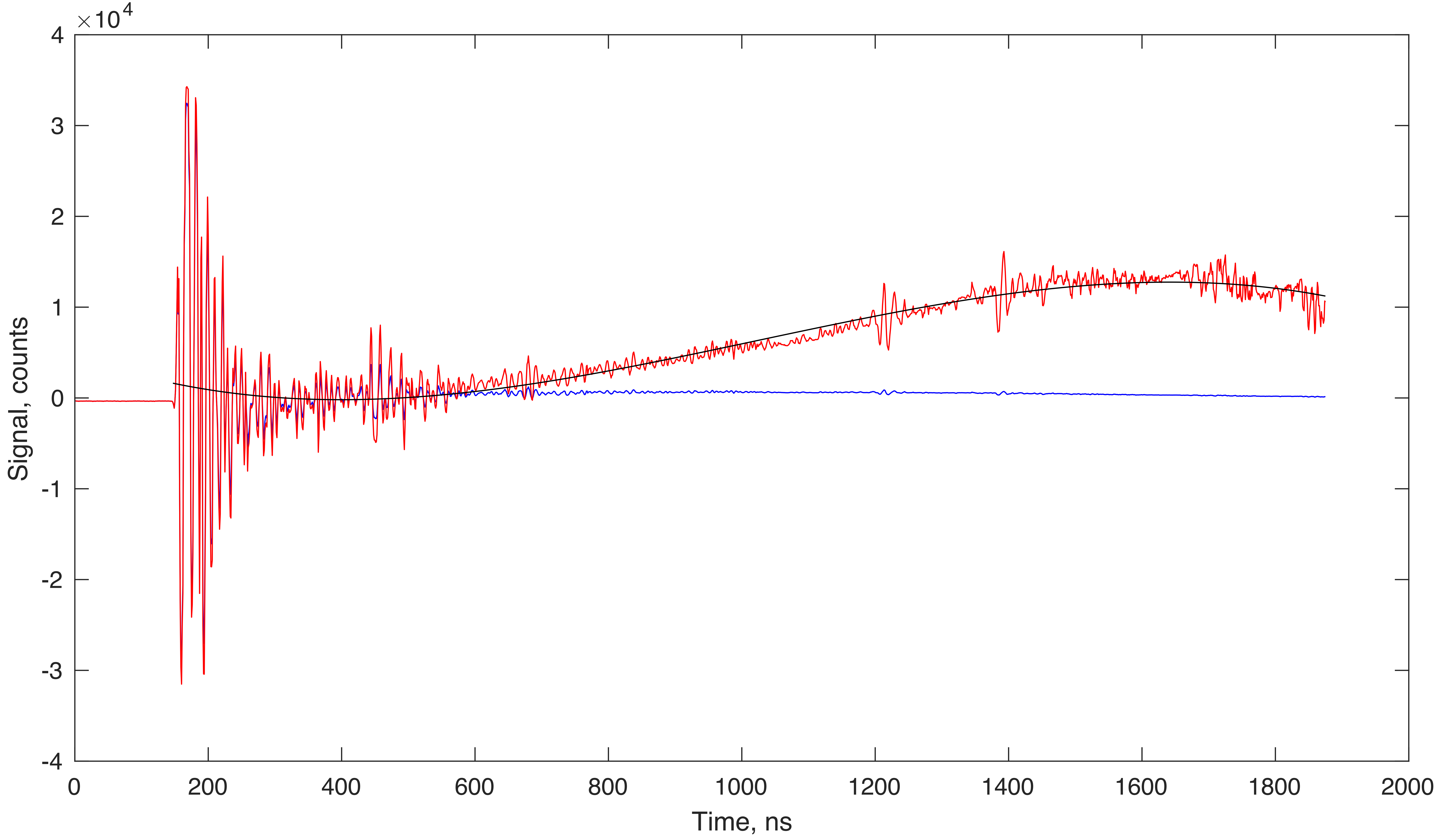
Single trace, raw data. Note loss of signal with depth (= time).



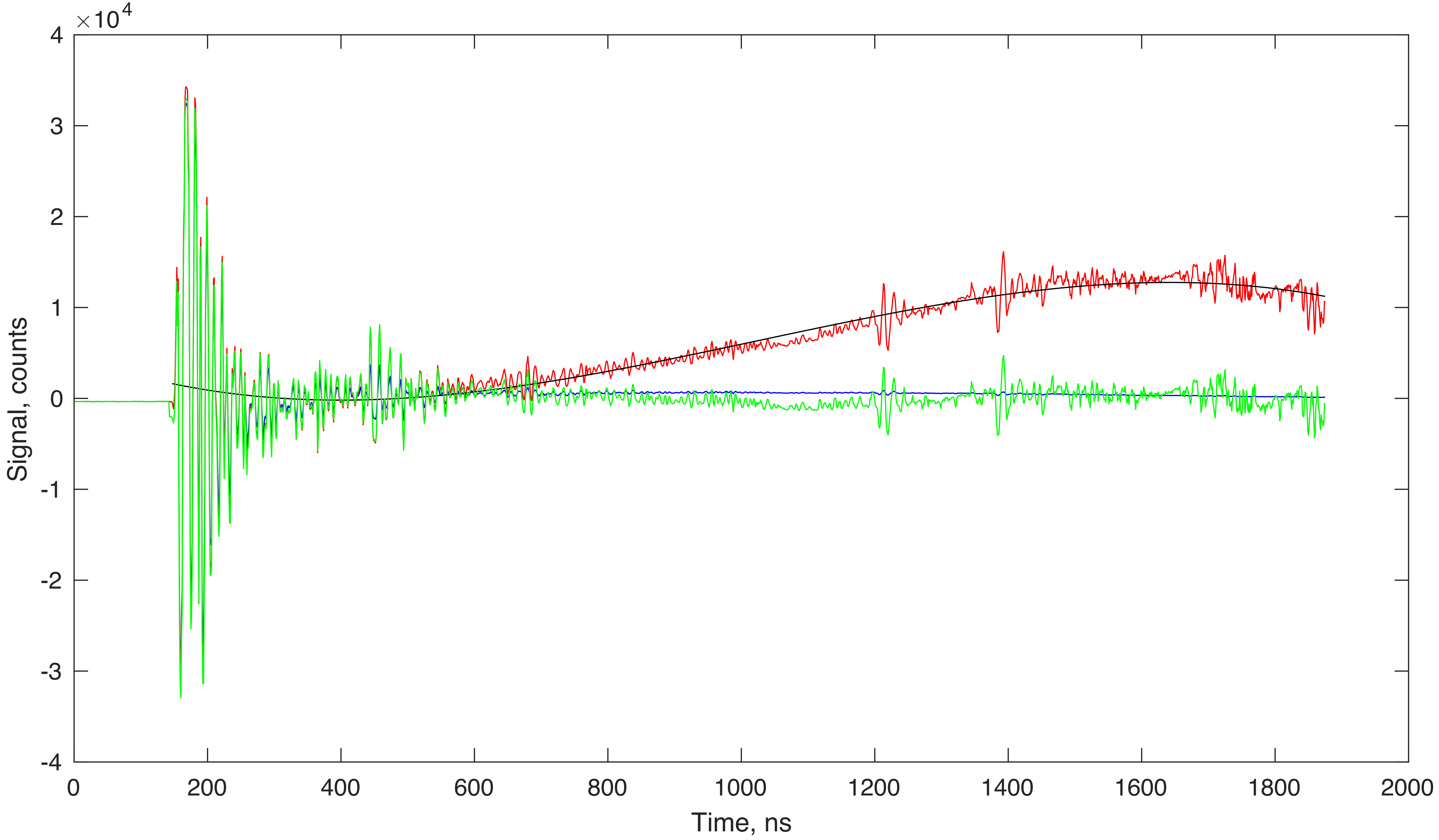
Exponential gain applied. Note low frequency signal/drift.



Polynomial (cubic) fit to long period drift:

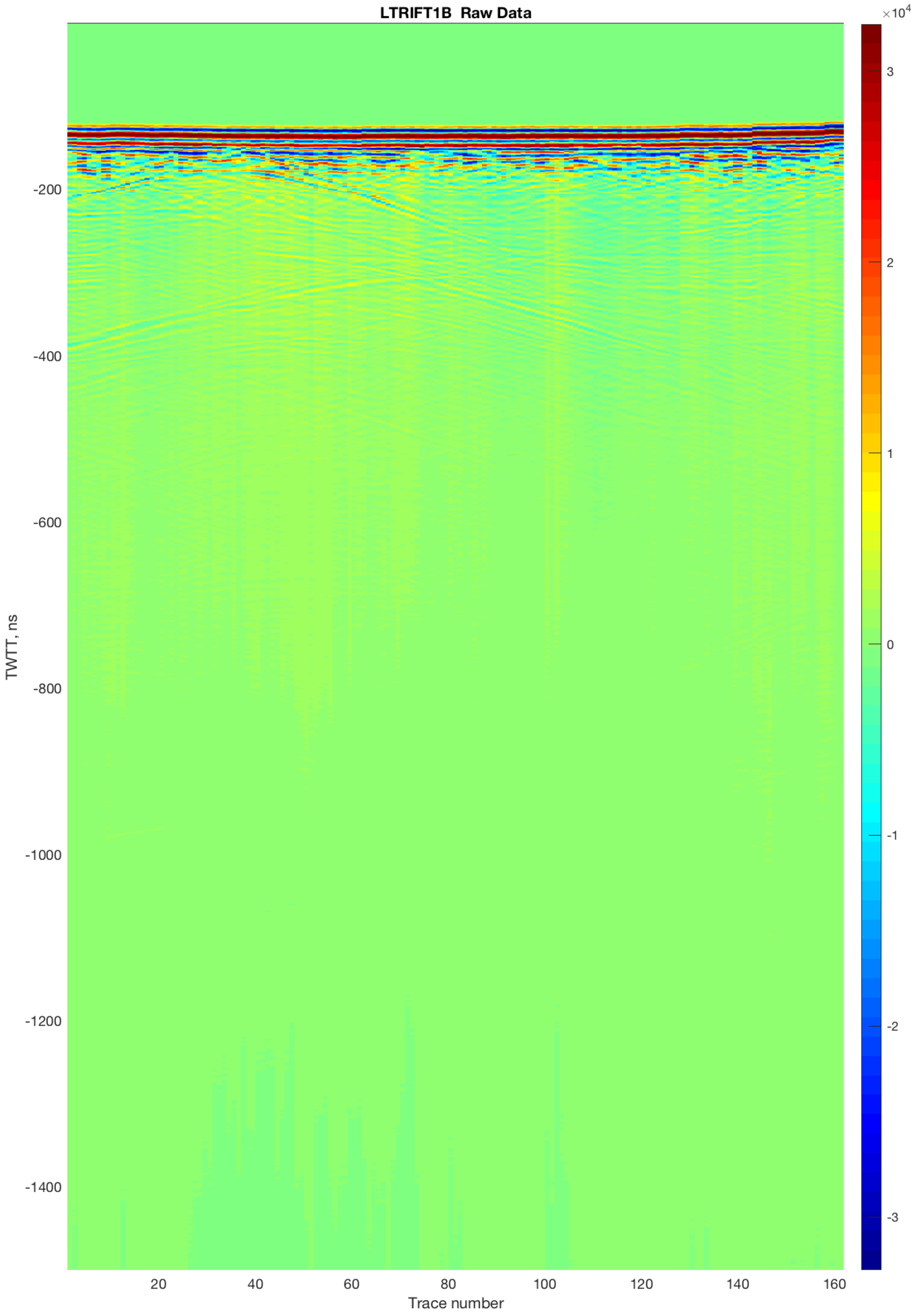


Drift removed (**de-wow**)

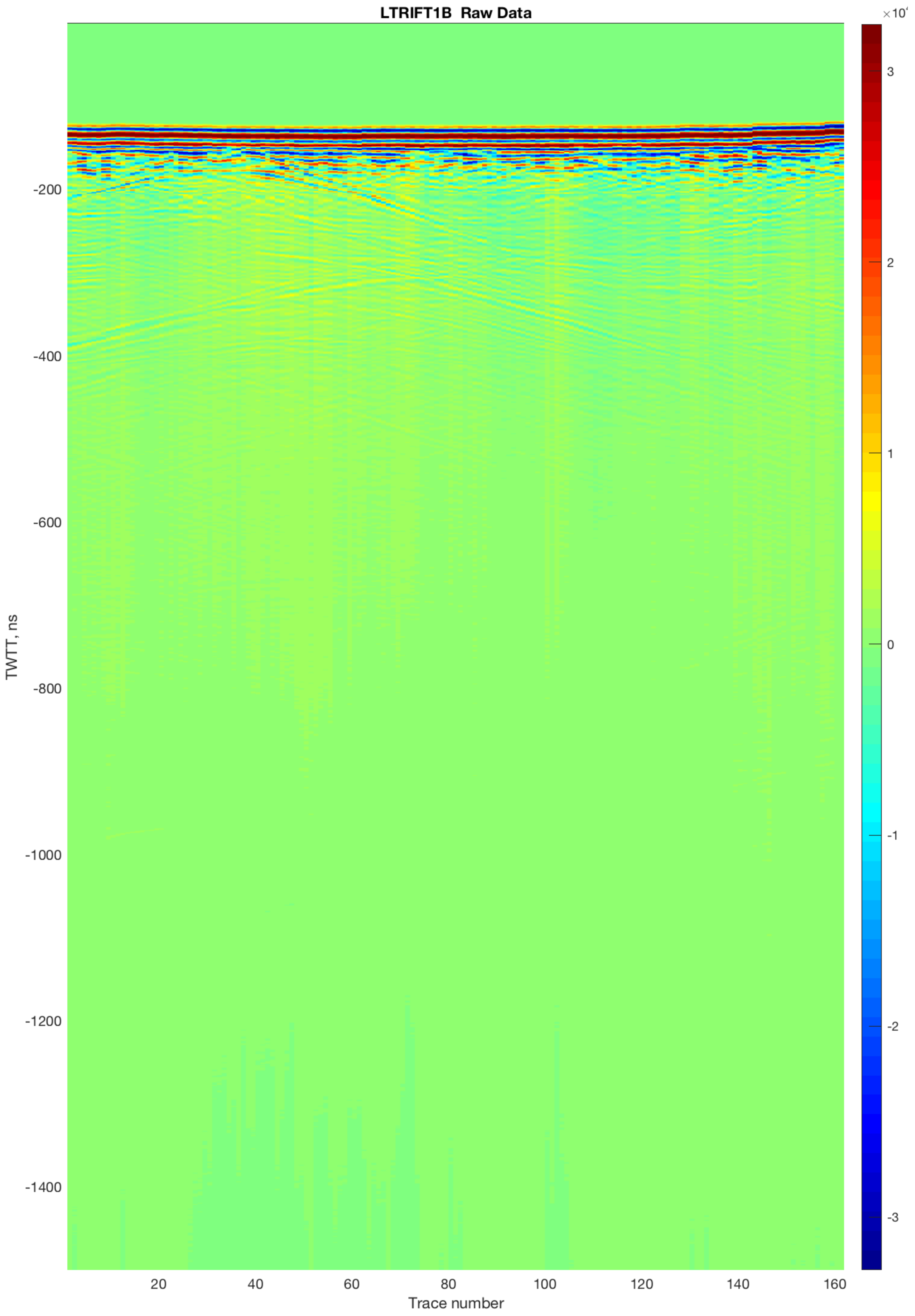


Same steps applied to all traces:

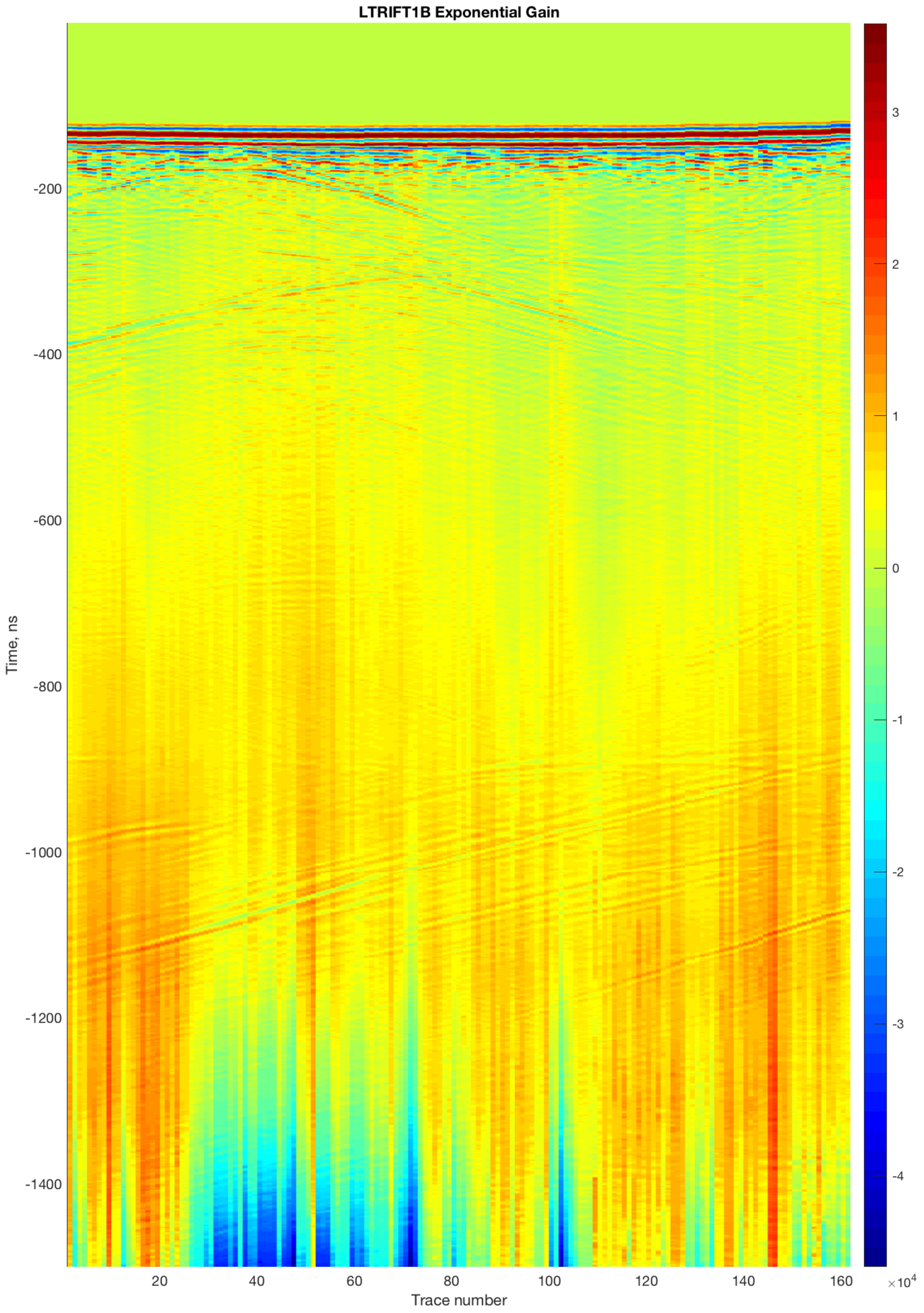
Raw data.



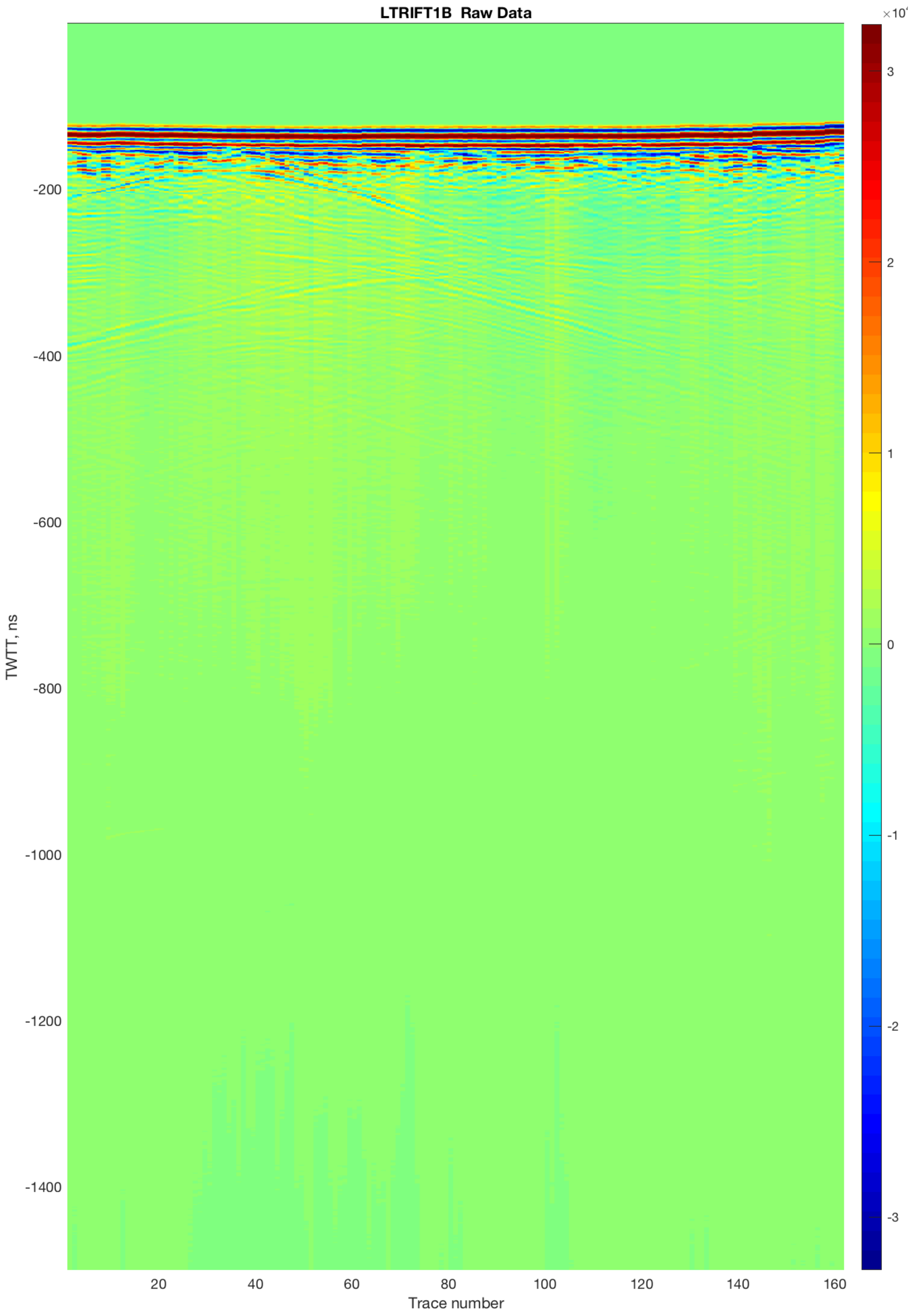
Raw data.



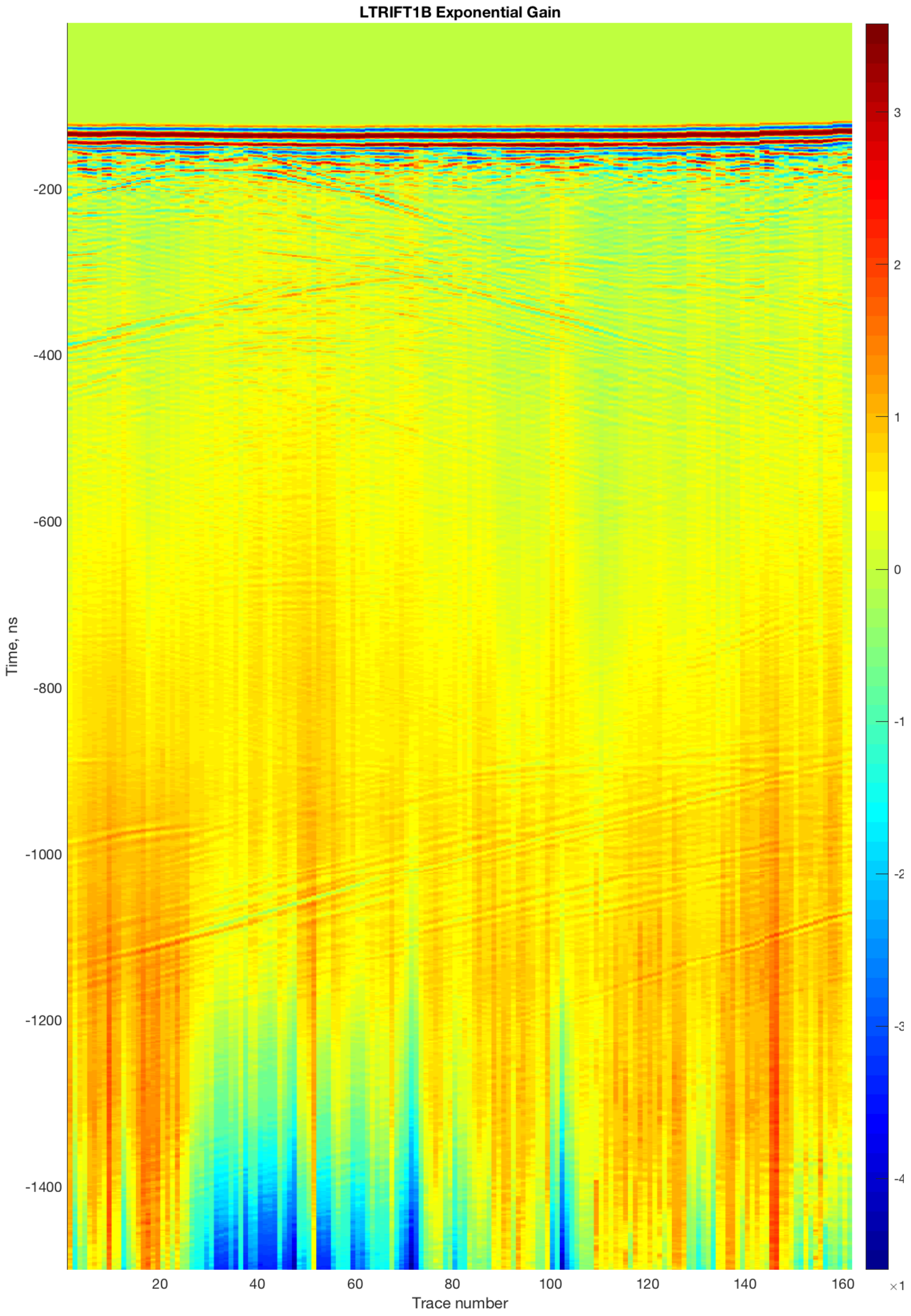
Gain applied.



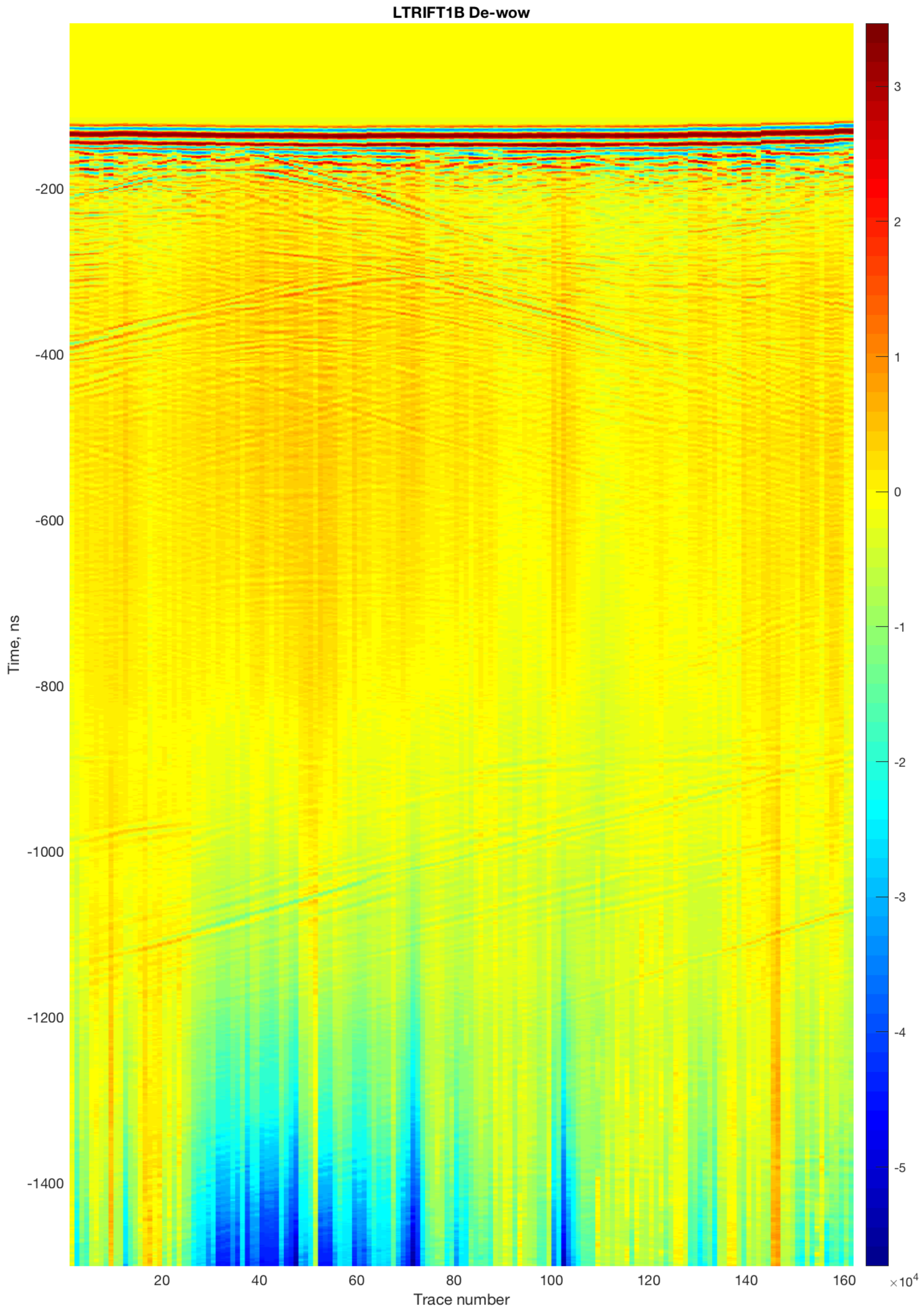
Raw data.

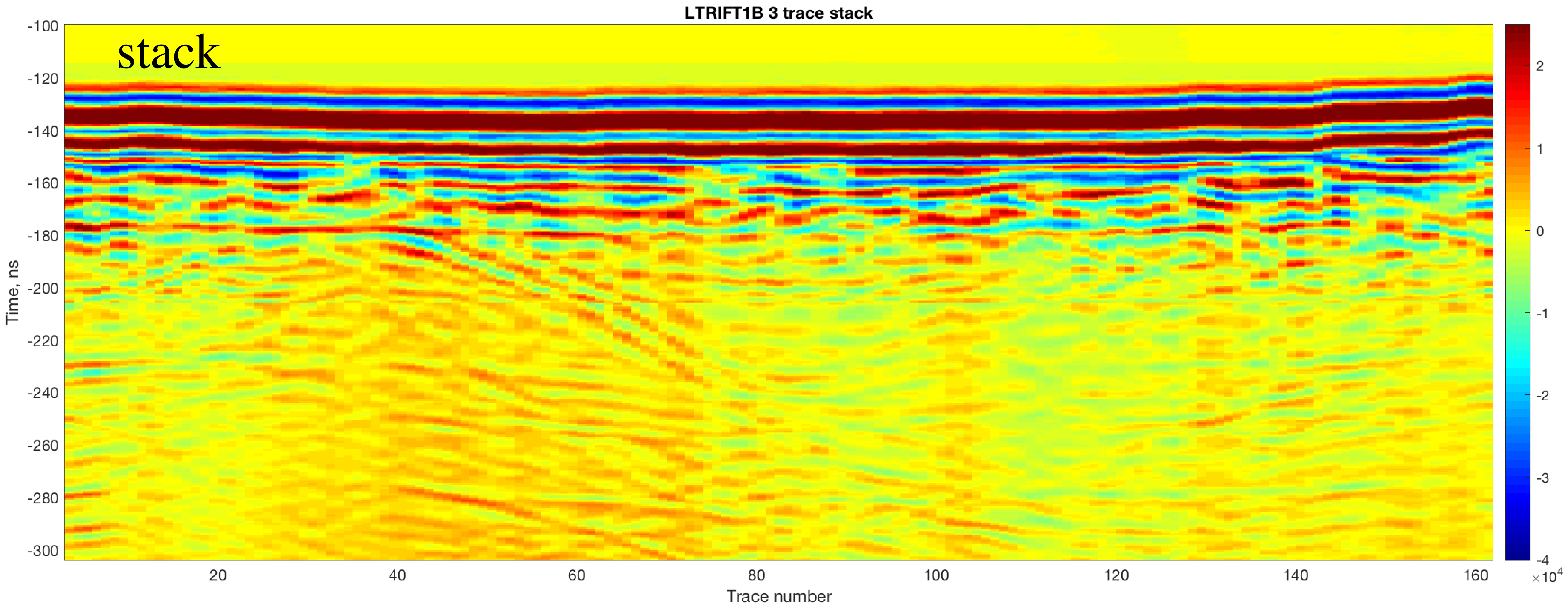


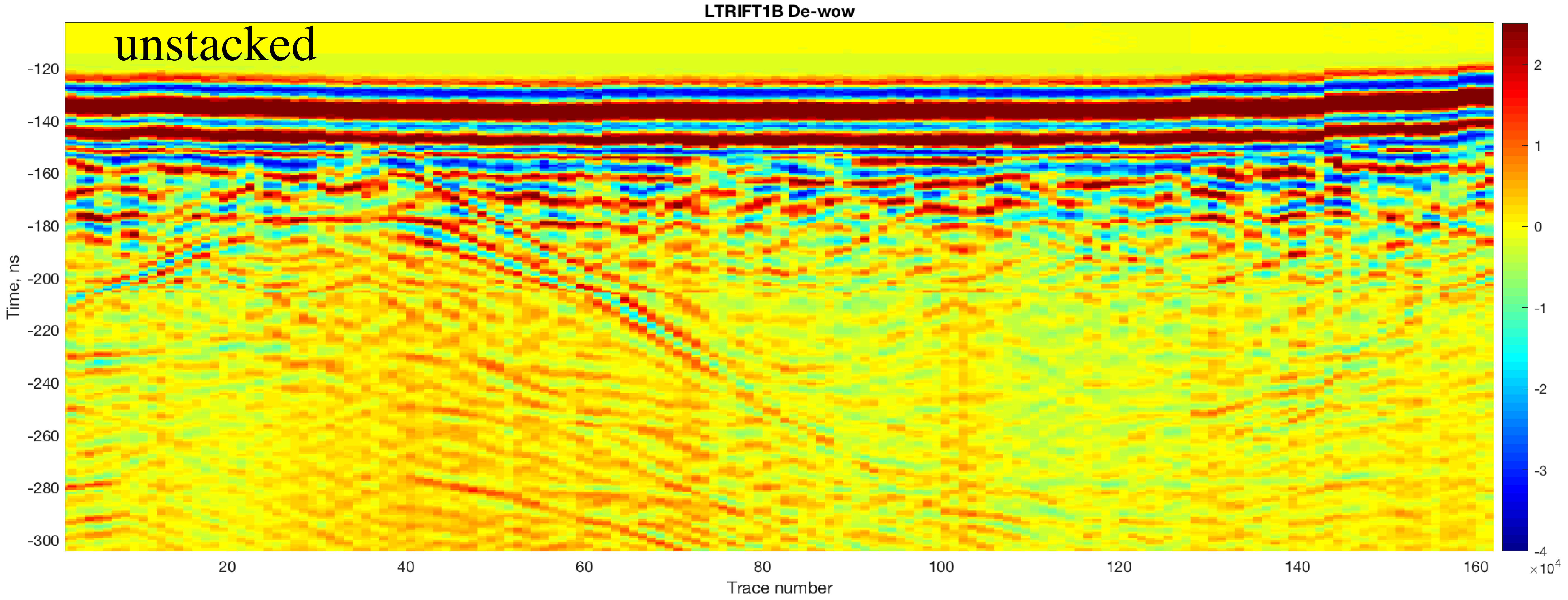
Gain applied.



De-wow



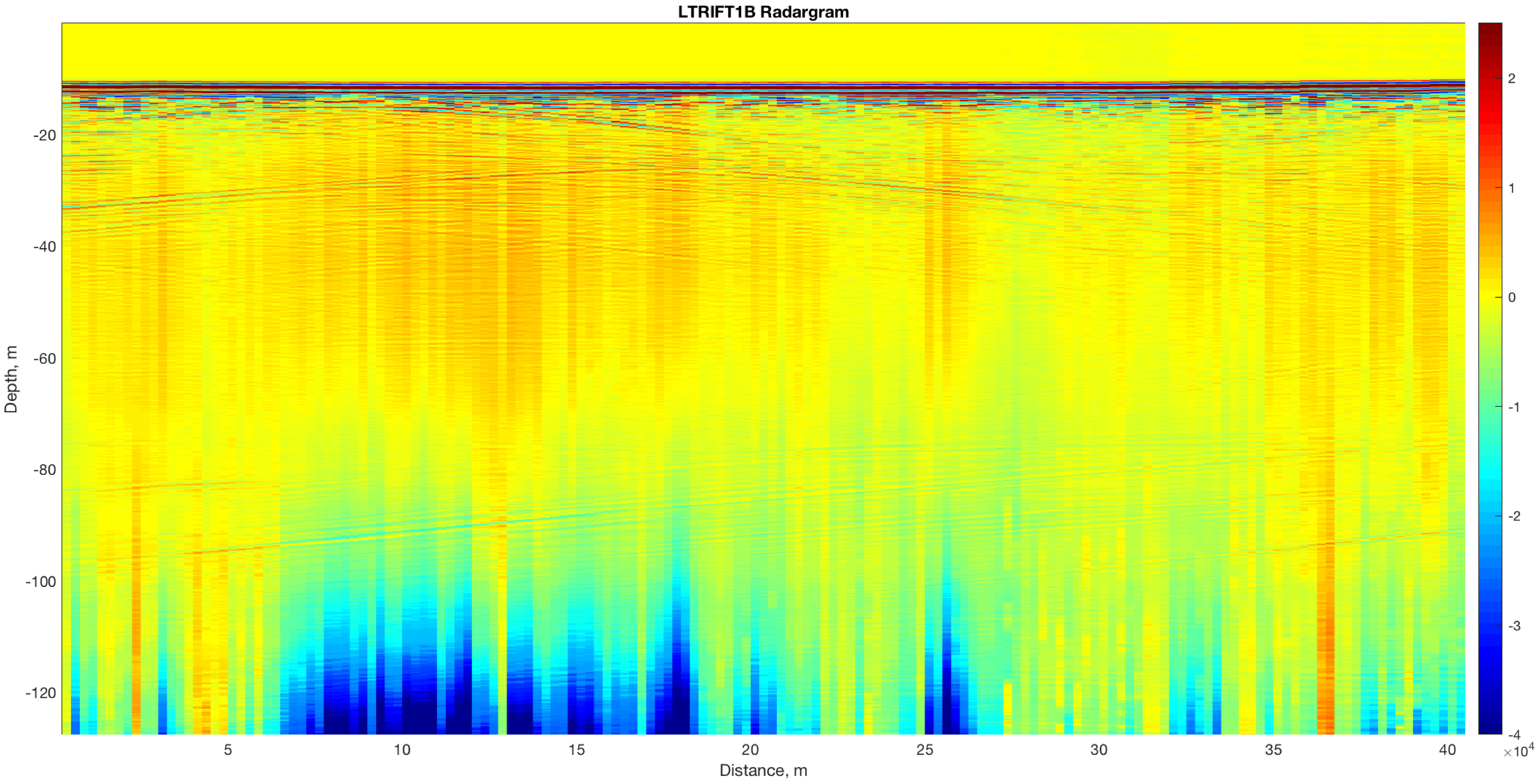




Stacking adjacent traces can improve signal to noise on horizontal beds, but will decrease the signal on dipping beds. Here I have stacked three traces.

Clearly, too much stacking will ruin resolution.

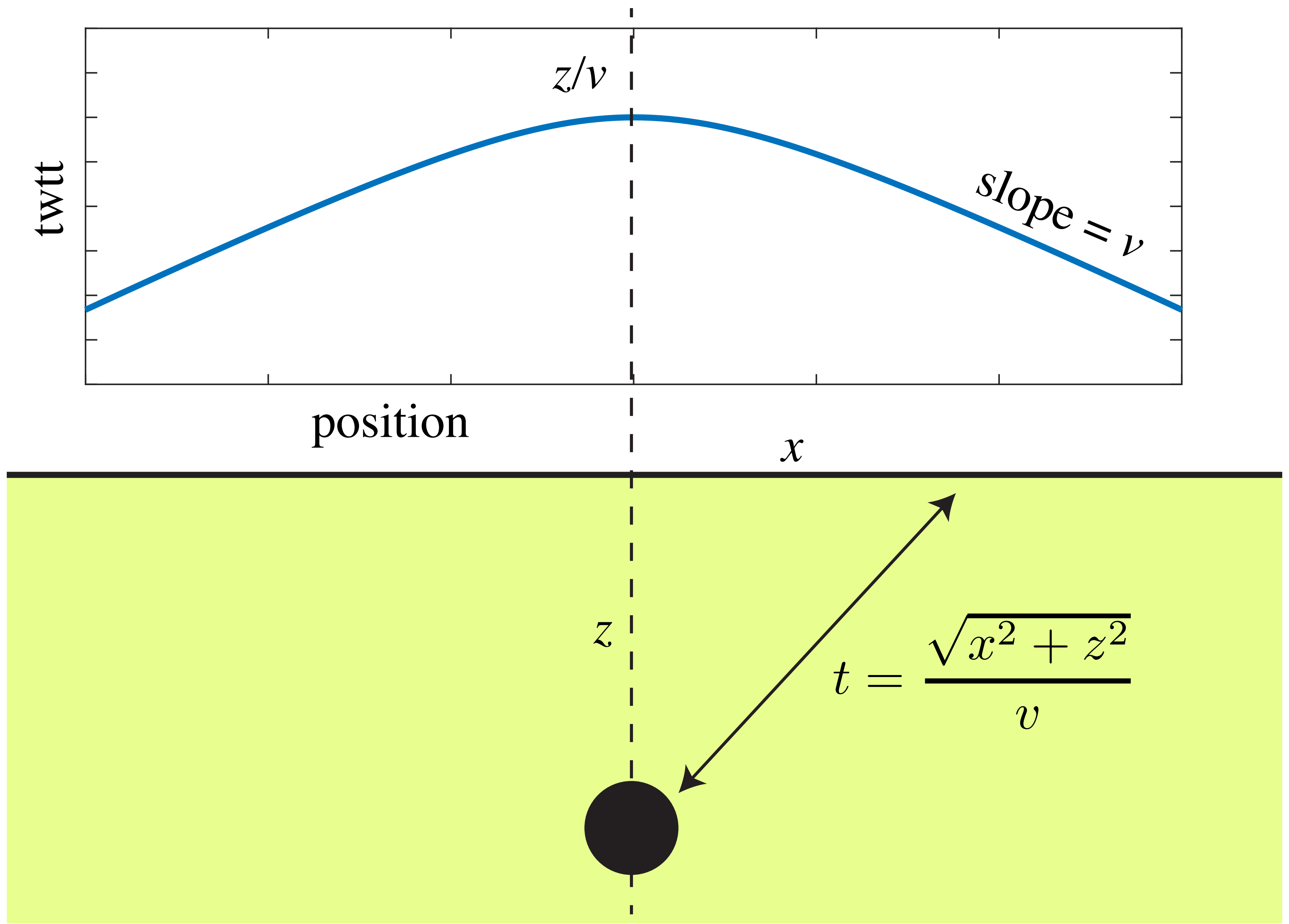
Finally, time and trace number **converted to depth and distance**, using a velocity in ice of 0.085 m/ns and a step size of 0.25 m.

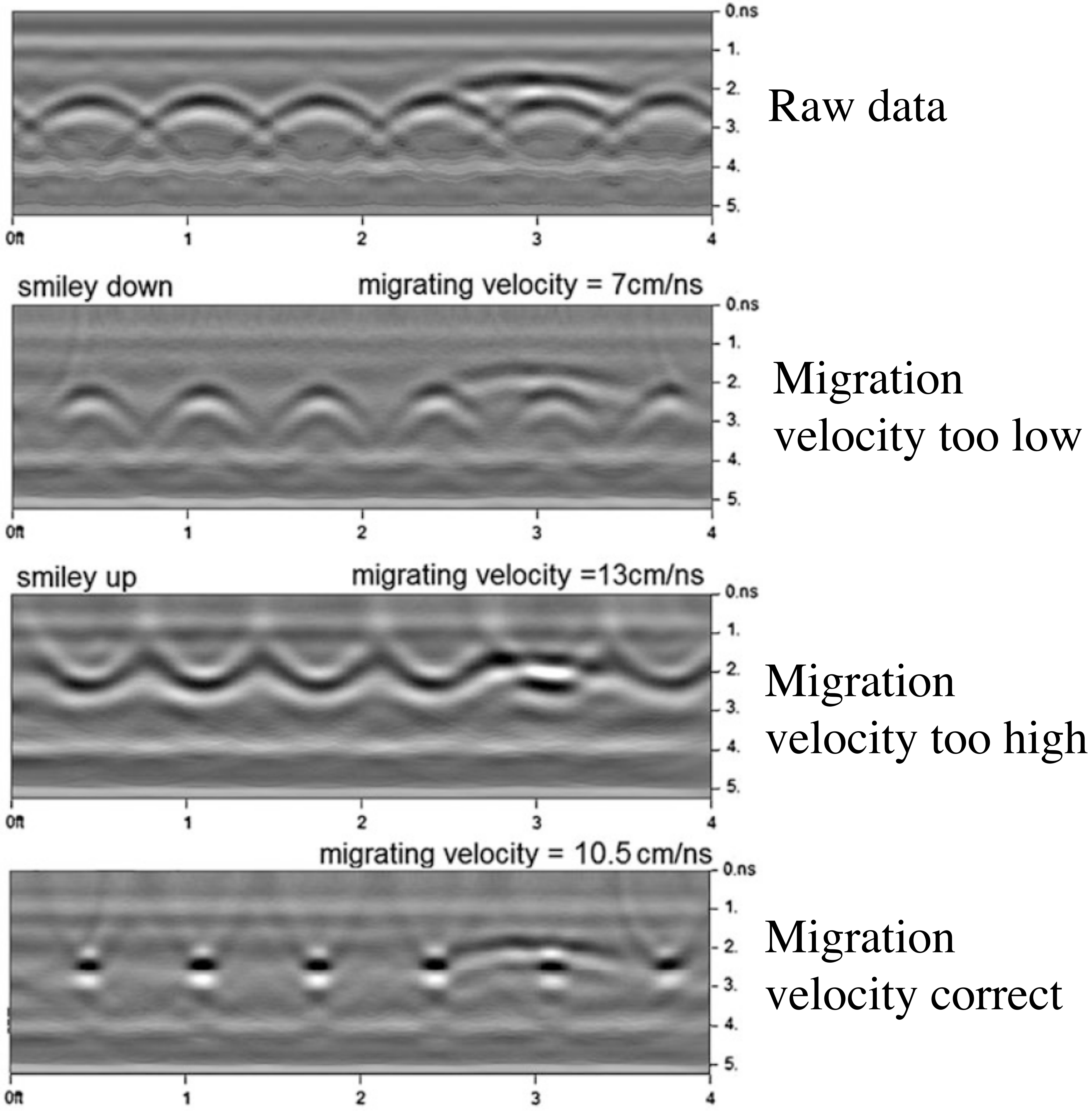


Normal moveout (NMO) and migration.

A compact reflector will create a reflection that has the mathematical form of a hyperbola. The point of closest approach (CPA) will give the depth, if the velocity is known. The asymptotic slope at large x is proportional to velocity.

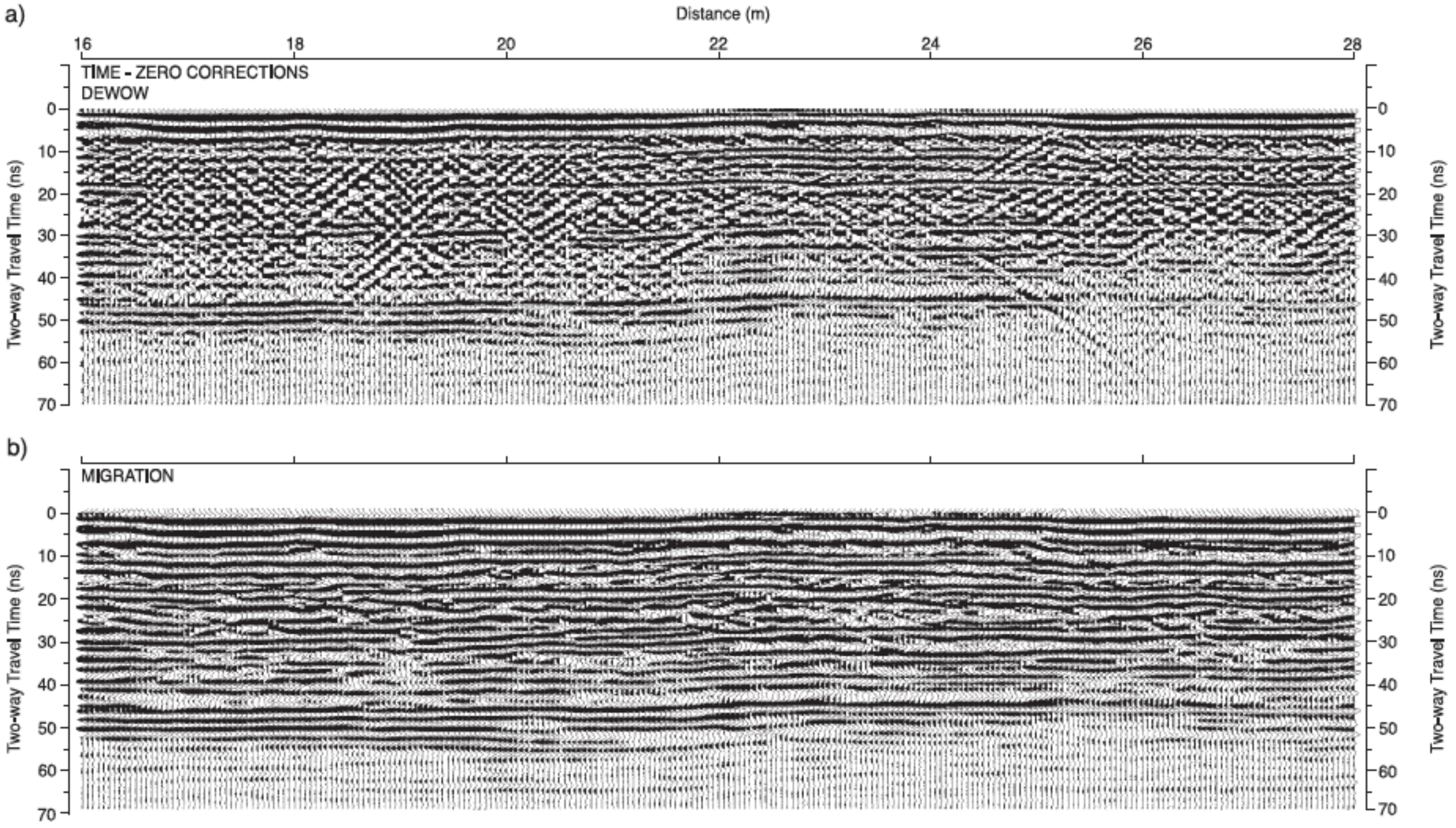
Migration is a processing step that corrects for this, given a velocity estimate.





Goodman and Piro (2013)

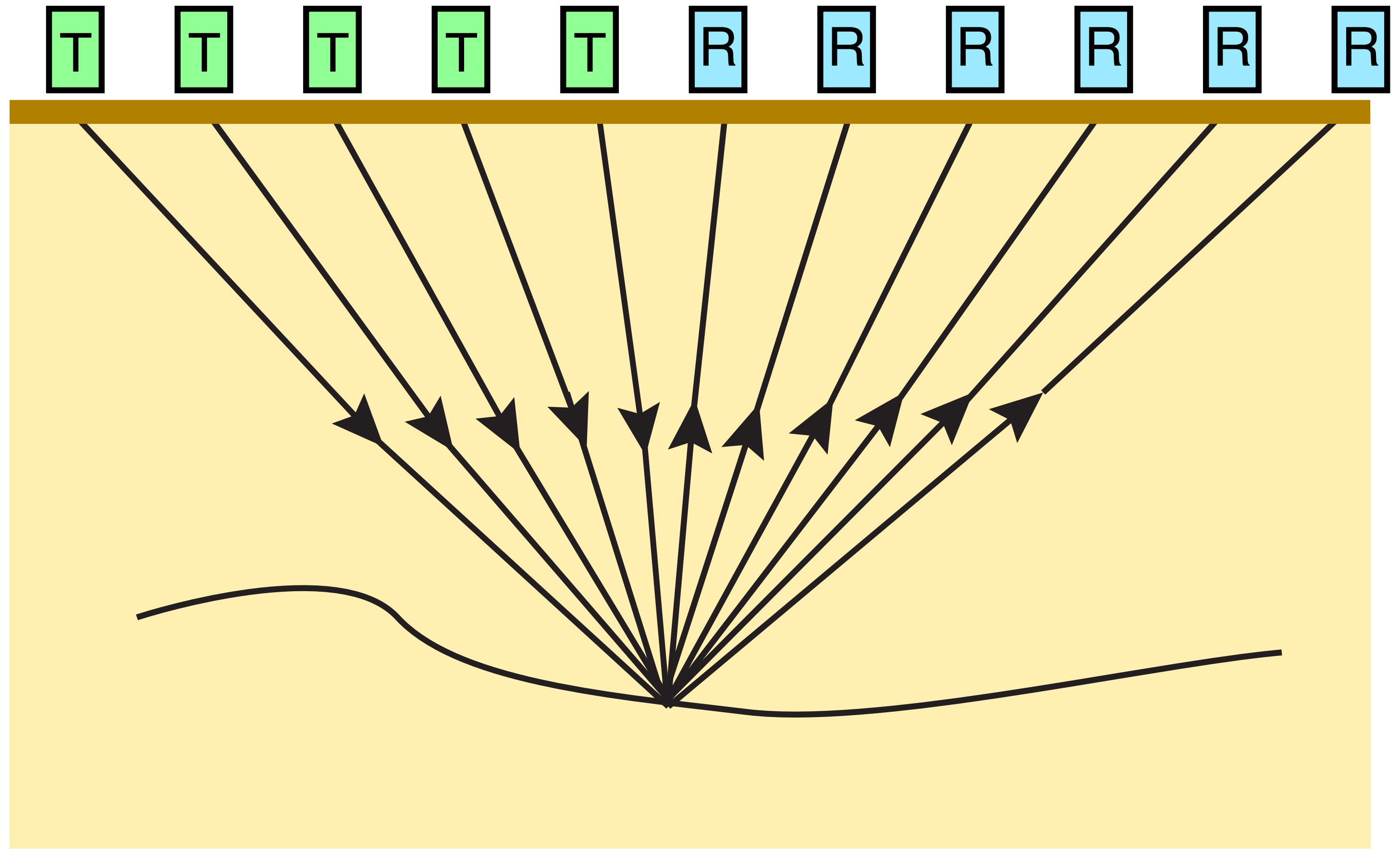
Normal moveout (NMO) and migration.

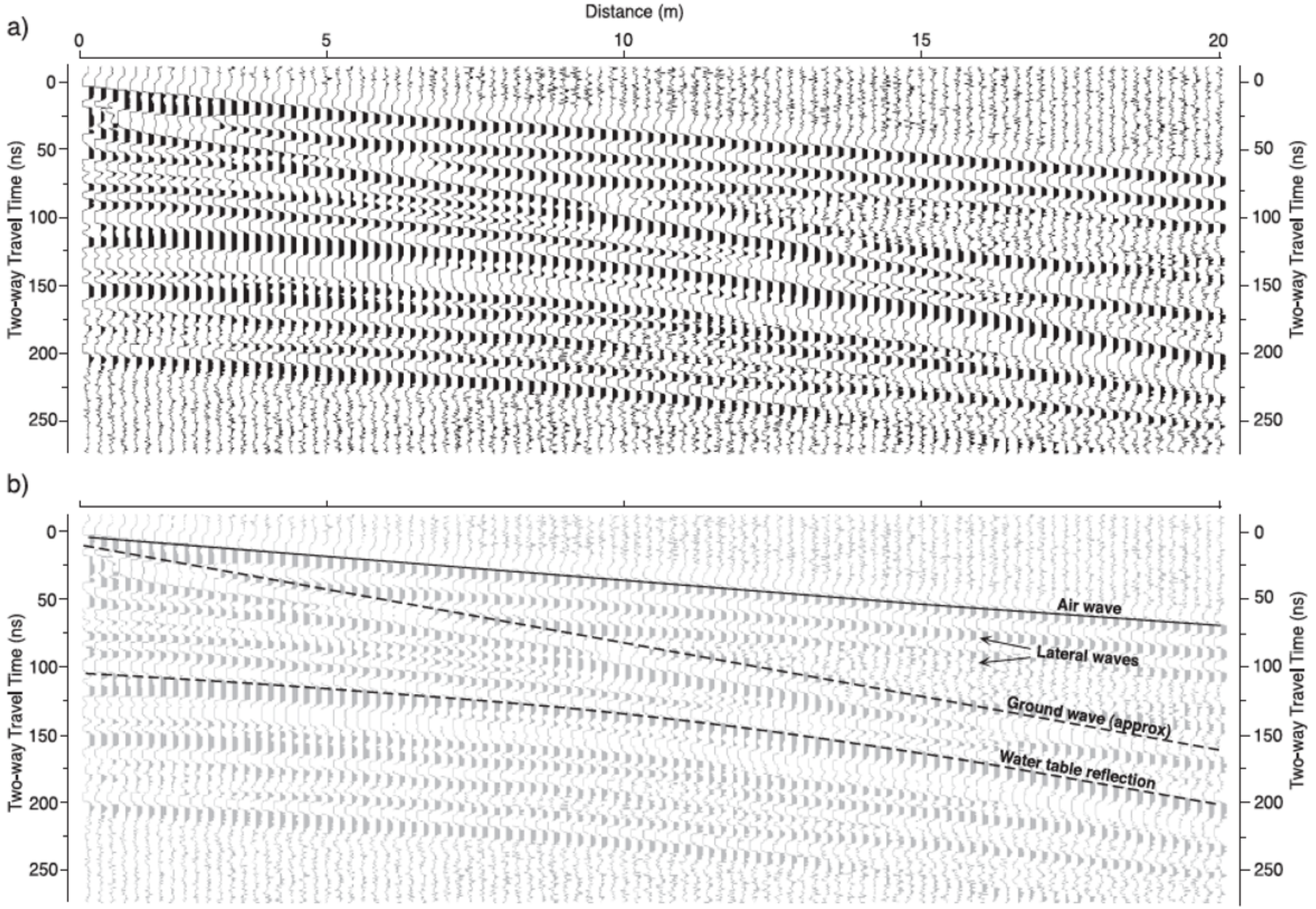


Neal (2004)

Common Mid-Point (CMP)

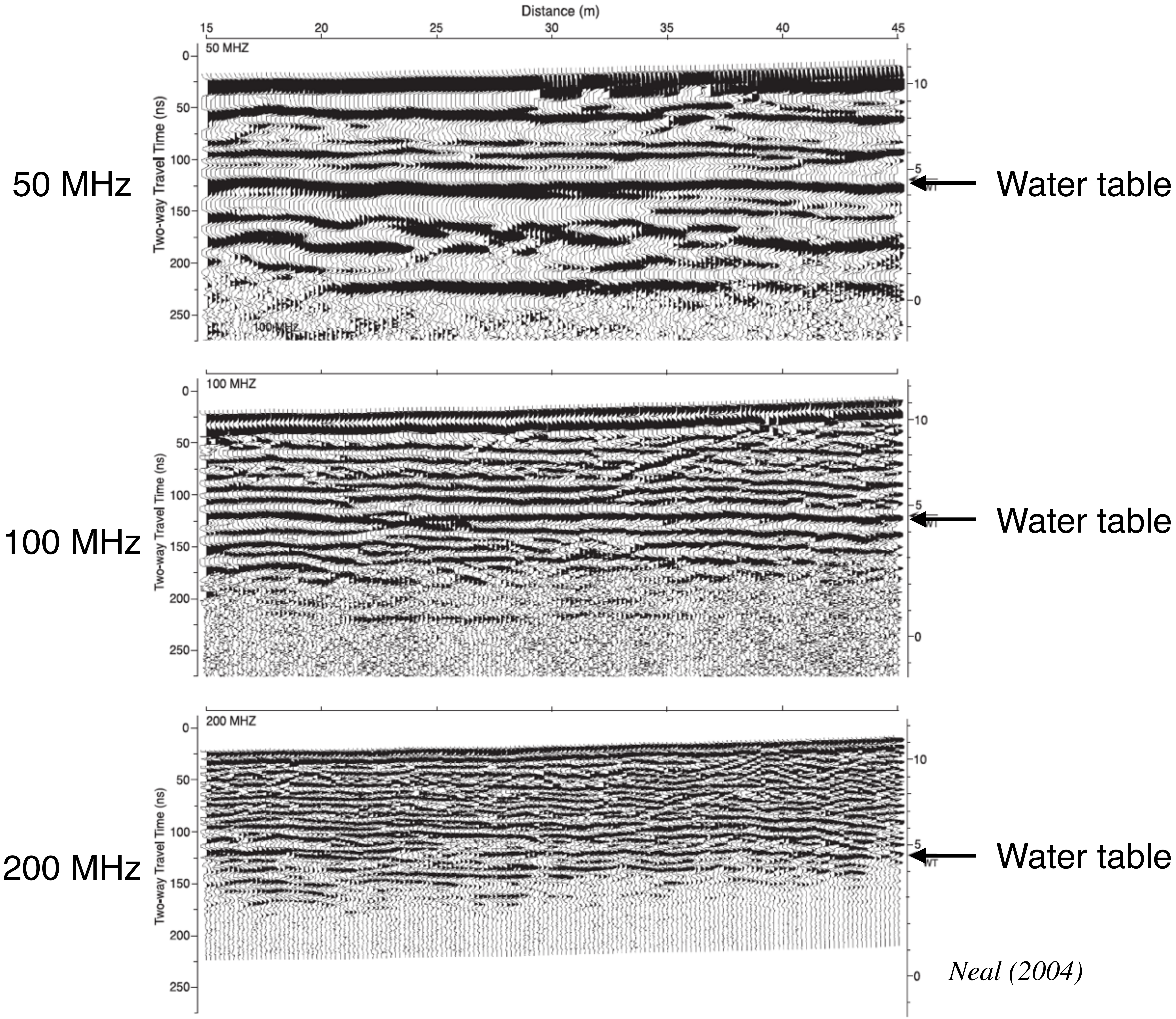
The CMP method uses the move-out from a single reflector beneath the mid-point to estimate the velocity as a function of depth. Here you can see that the airwave comes in at about 70 ns at 20 m spacing, or 0.3 m/ns.   The ground wave is 0.12 m/ns. Asymptotic move-out on the water table reflection is about the same.





Neal (2004)

Survey Design



Neal (2004)

Choose center frequency, f_c

Higher frequencies have shorter wavelengths and so higher resolution.

Higher frequencies also have higher attenuation, and will not propagate as deeply.

$$\alpha = \sqrt{\frac{\omega \mu_o \sigma}{2}}$$

Recording time window: $W = 1.3\,\dfrac{2 \times \text{max depth}}{\text{min velocity}}$ The 2 comes from two-way travel time.

Time sample rate: $\Delta t = \dfrac{1}{6 f_c}$ This is three times the Nyquist frequency.

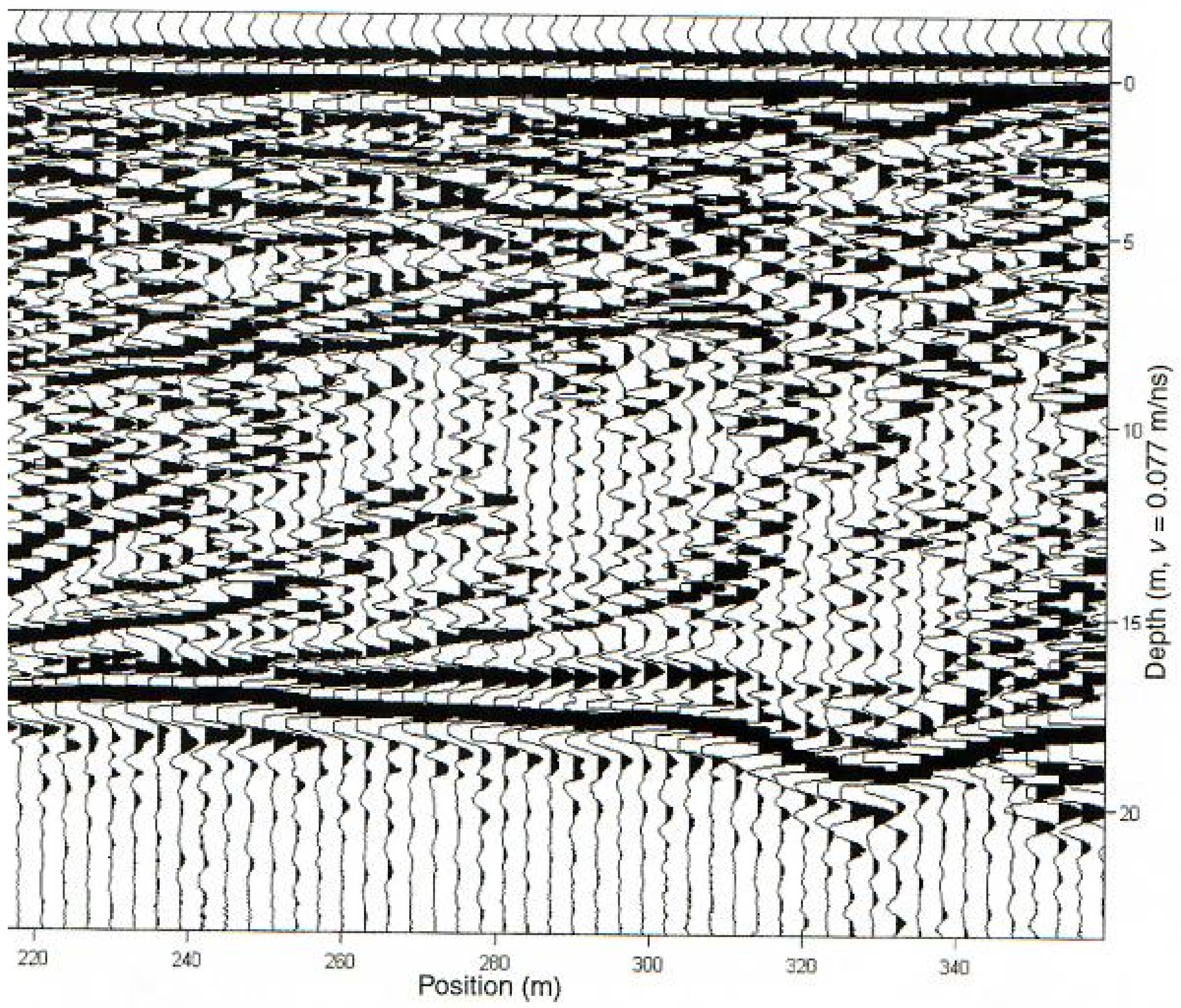
Antenna separation (bistatic only): At least half a wavelength to avoid overloading receiver (about 1.5 m for a 100 MHz system). Less than about half the target depth.

Spatial sample interval:

The spatial Nyquist is given by

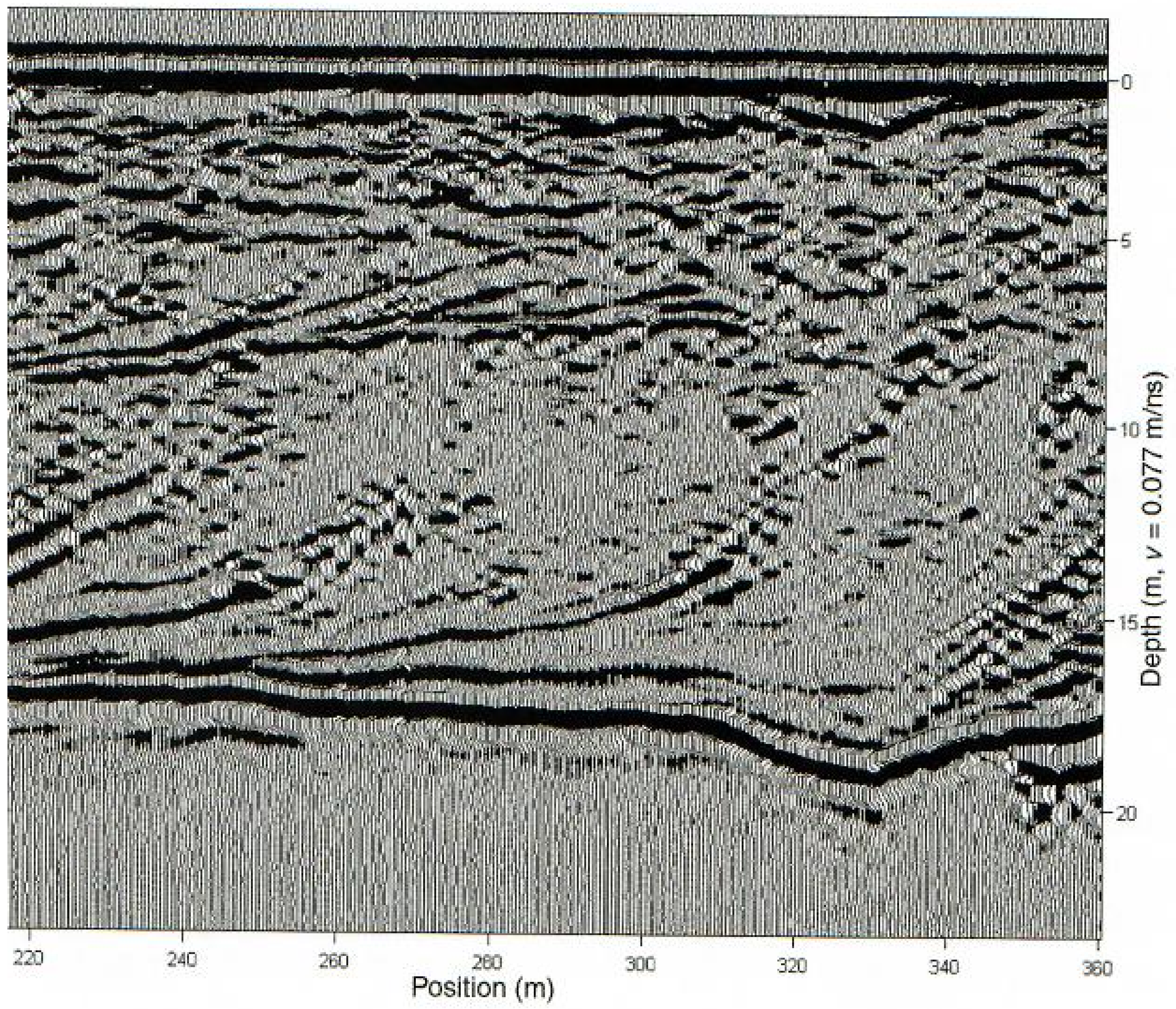
$$\Delta x = \frac{c}{4 f_c \sqrt{\epsilon_r}} \approx \frac{75}{f_{(MHz)} \sqrt{\epsilon_r}} \text{ m}$$

No real advantage in over-sampling, but under-sampling will not resolve steep interfaces.



50 Mz Antenna:

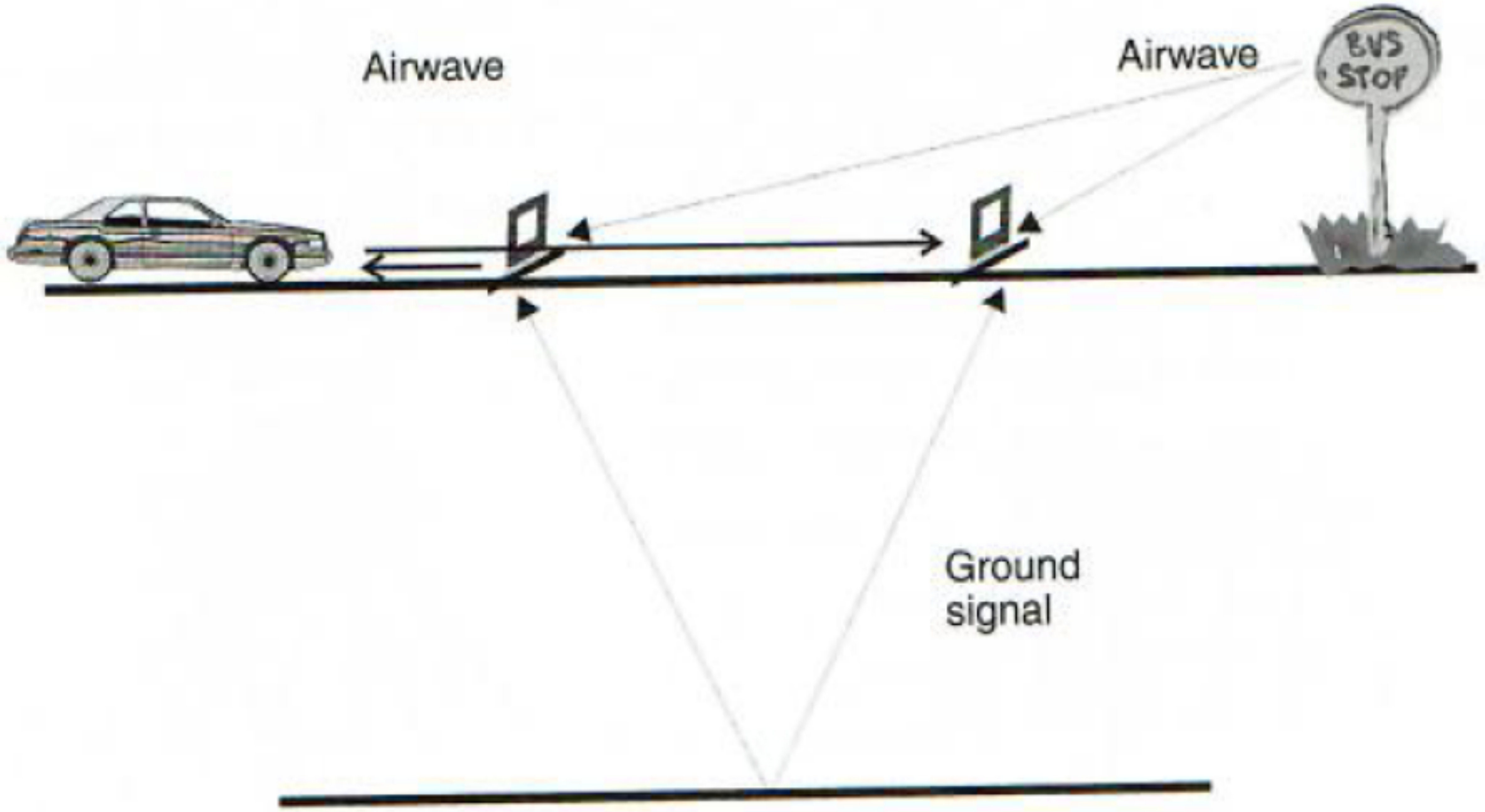
3 m station spacing



0.5 m station spacing (about equal to the Nyquist)

Neal (2004)

Be wary of surface reflectors. These are characterized by NMO velocities of air (0.30 m/ns).



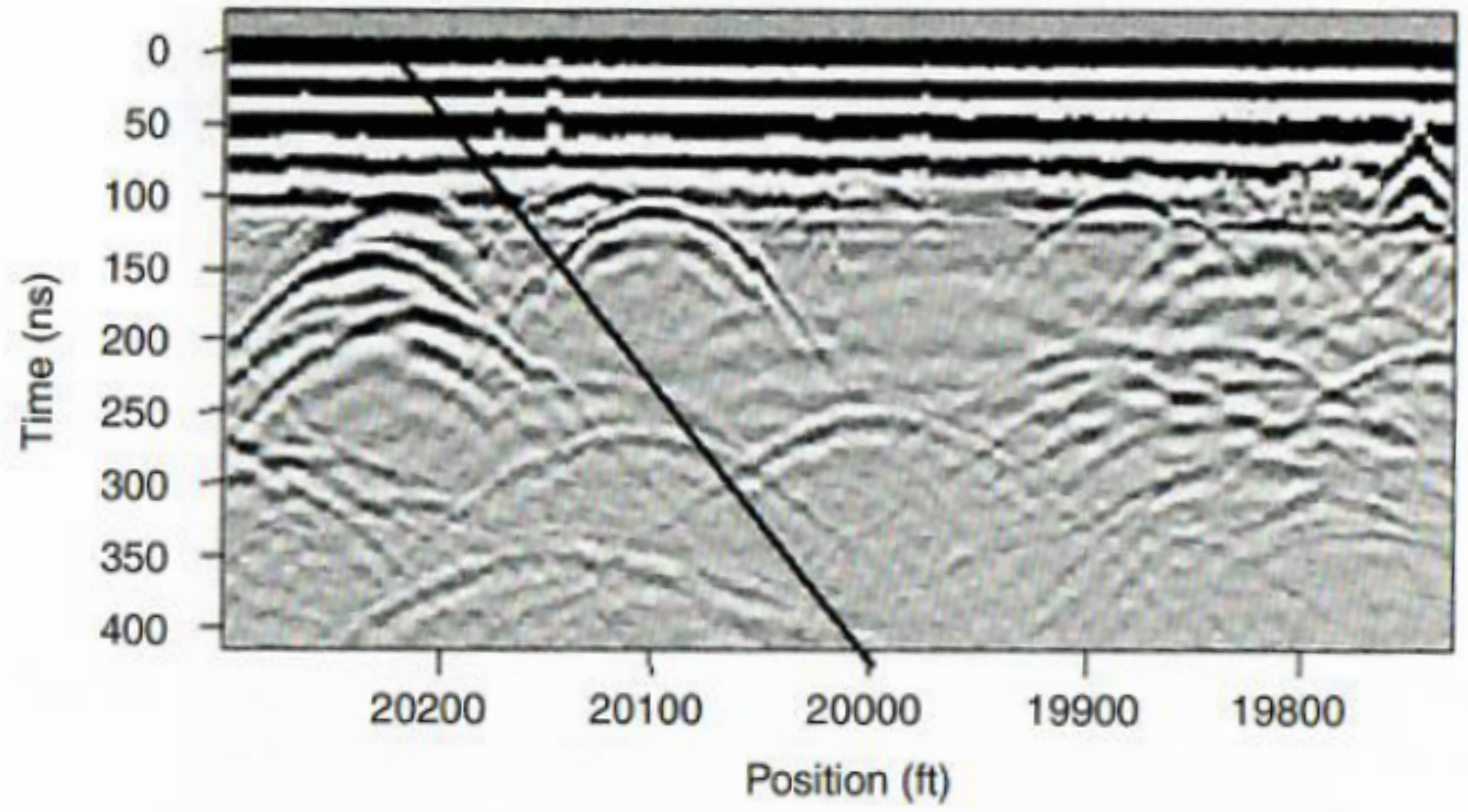
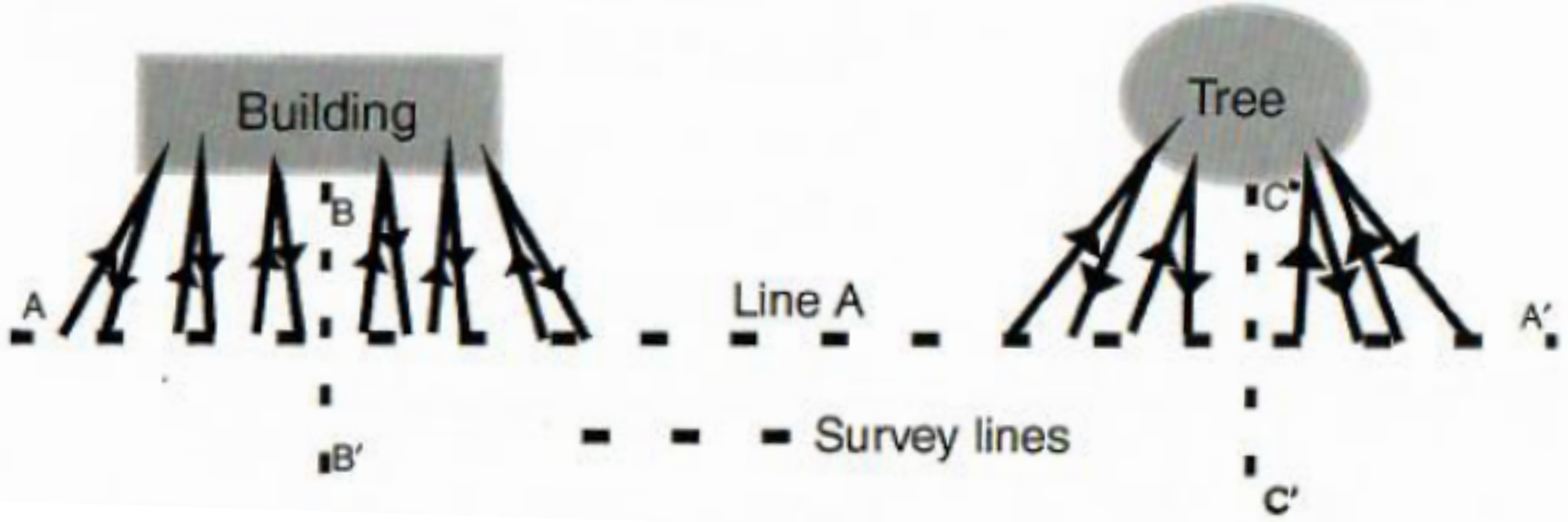


Figure 135. Airwave reflections from trees in the GPR survey area. All the reflectors below 100 ns in the data are caused by surface objects. The straight line has the airwave slope for reference.

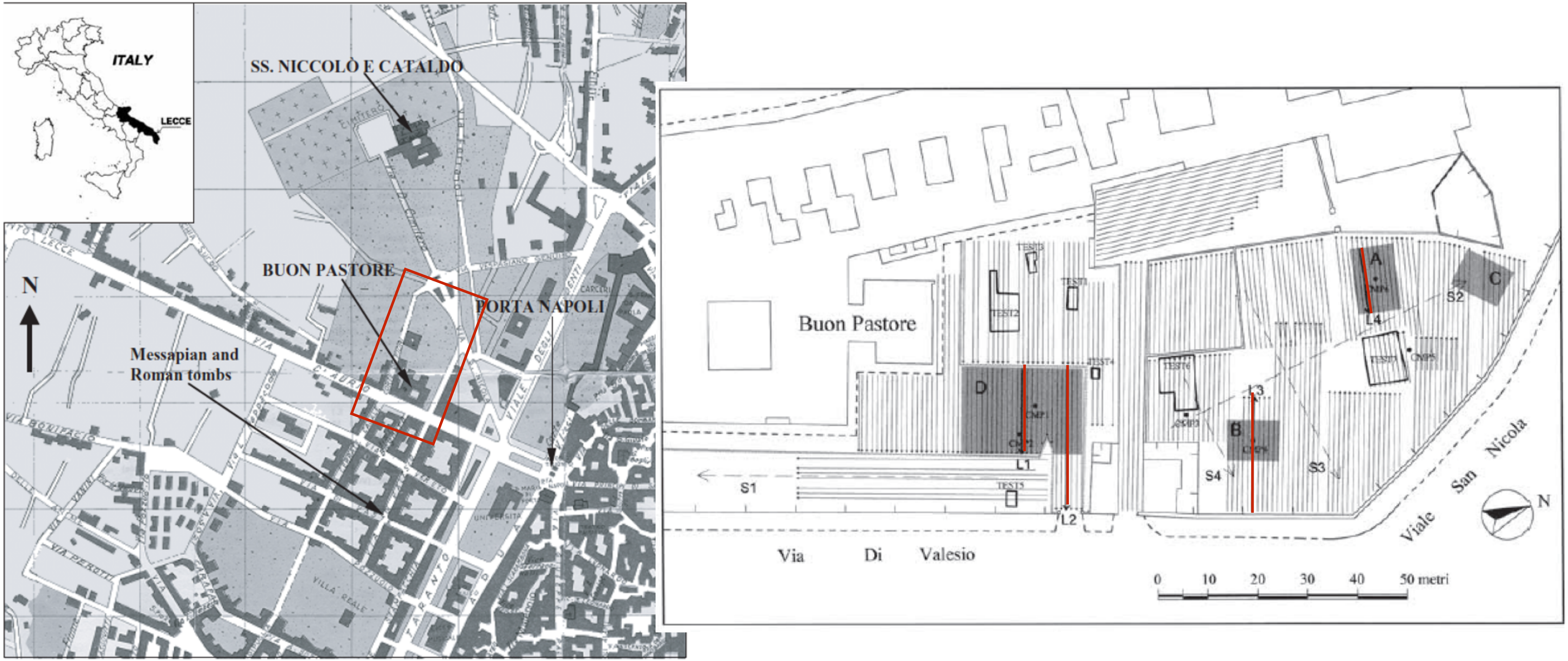


Annan (2005)

Examples

Luigia Nuzzo, Giovanni Leucci, Sergio Negri, Maria Teresa Carrozzo and Tatiana Quarta
Osservatorio di Fisica e Chimica della Terra e dell'Ambiente, Dipartimento di Scienza dei Materiali, Università di Lecce, Italy

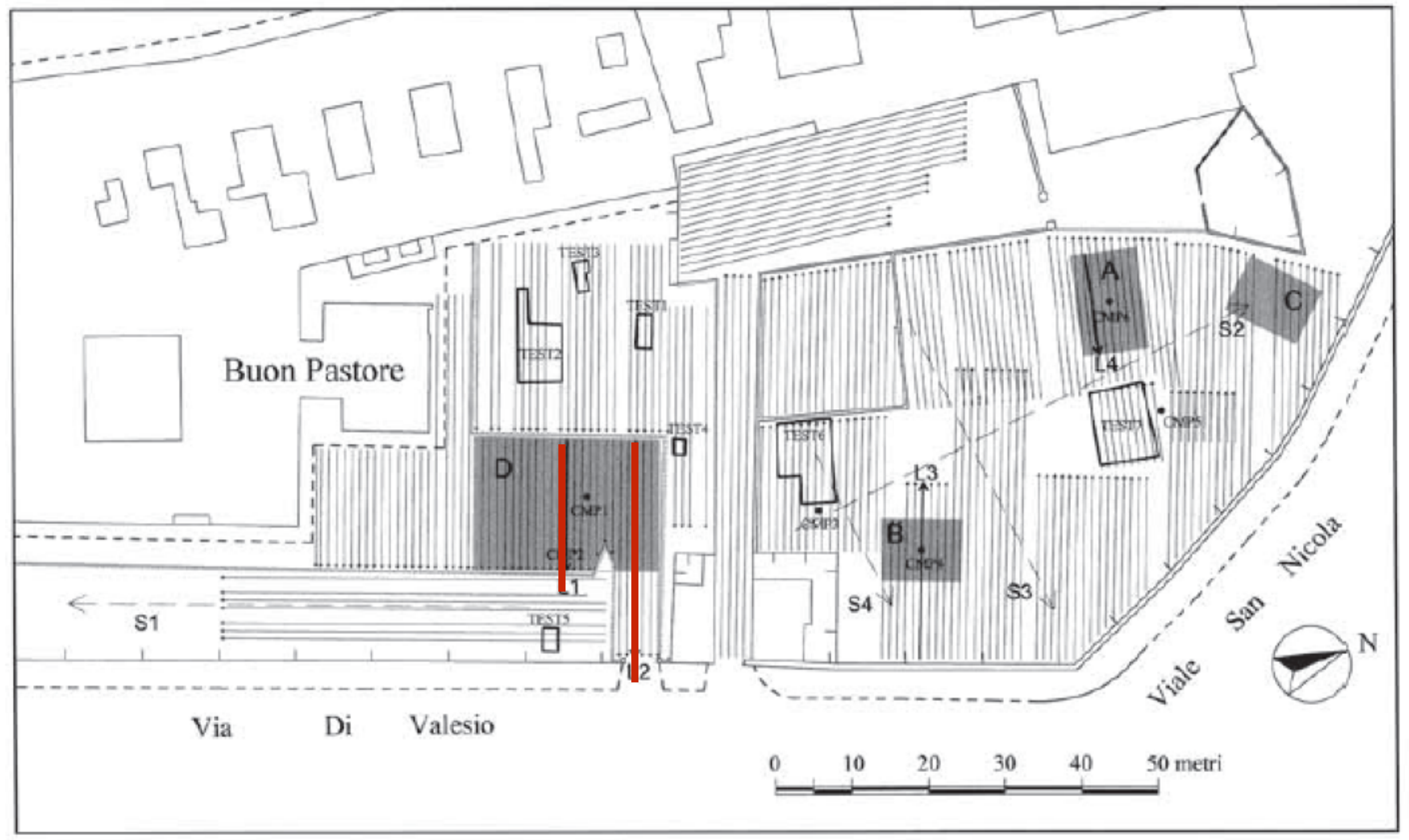
ANNALS OF GEOPHYSICS, VOL. 45, N. 2, April 2002

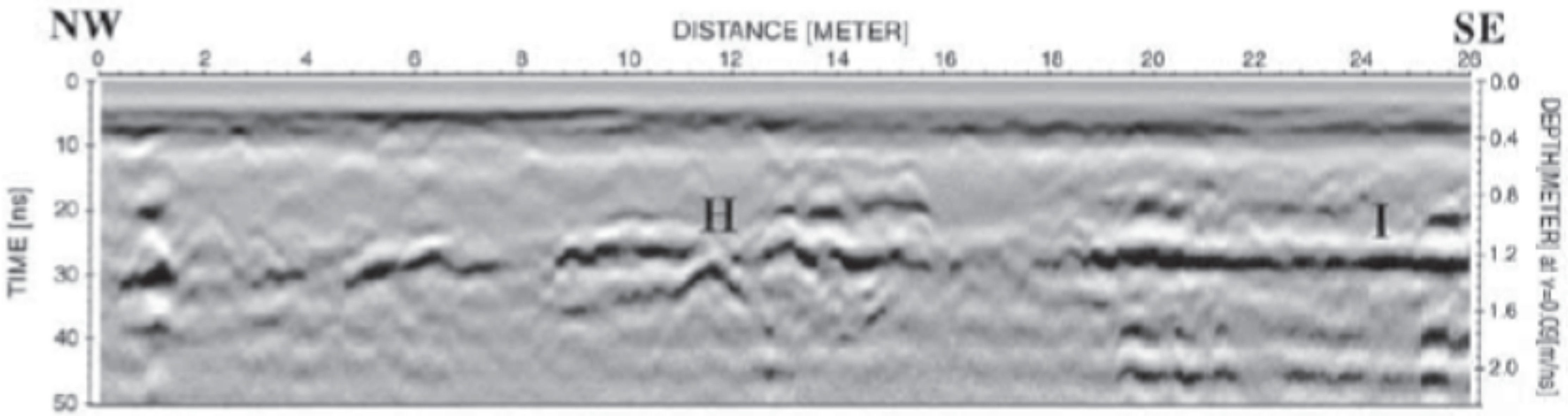


I = bedrock

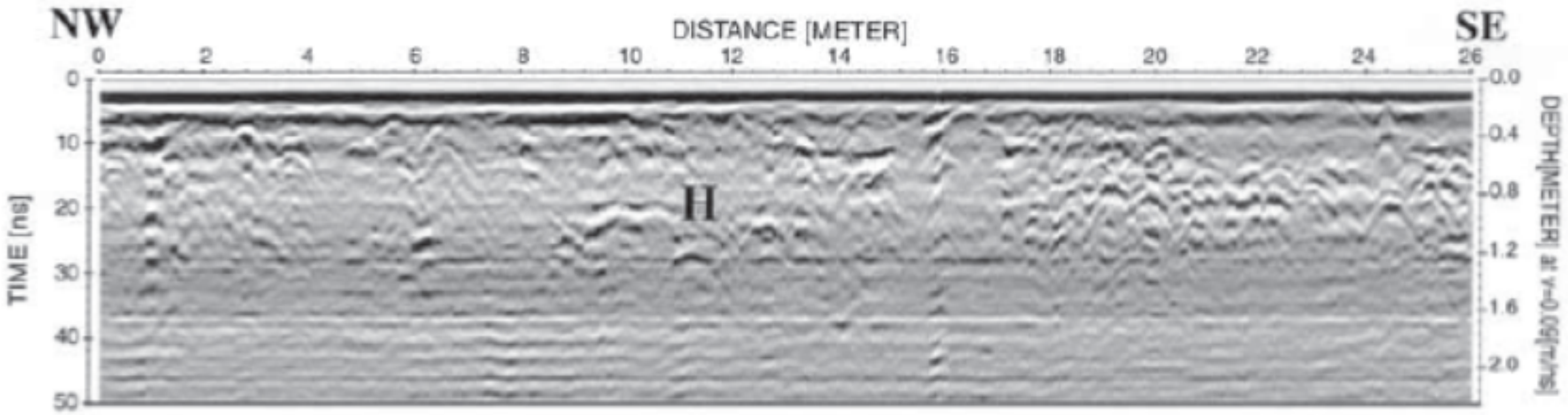
H = channel in bedrock

C = cistern

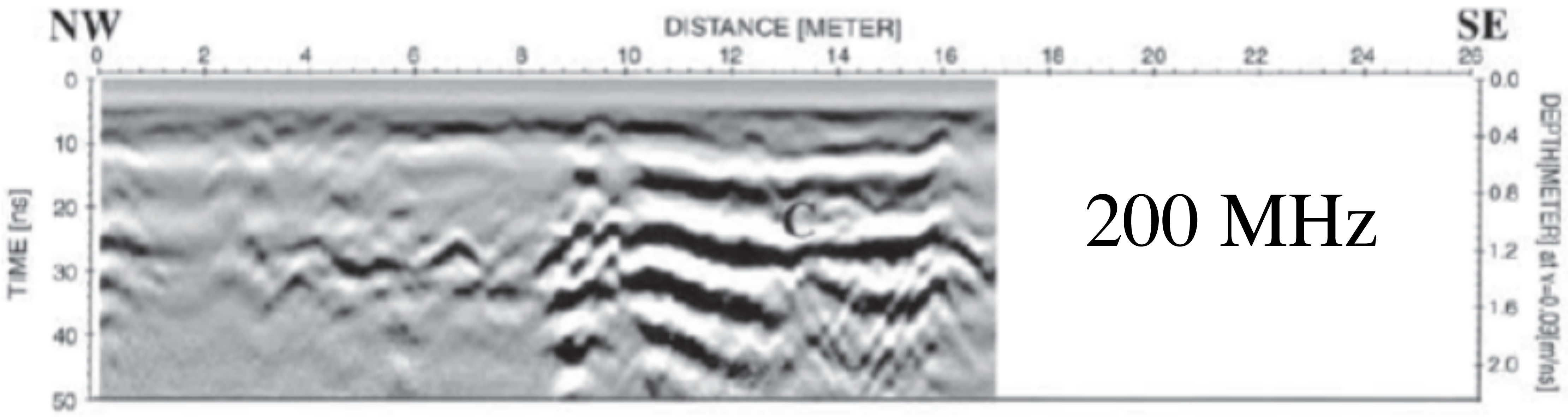


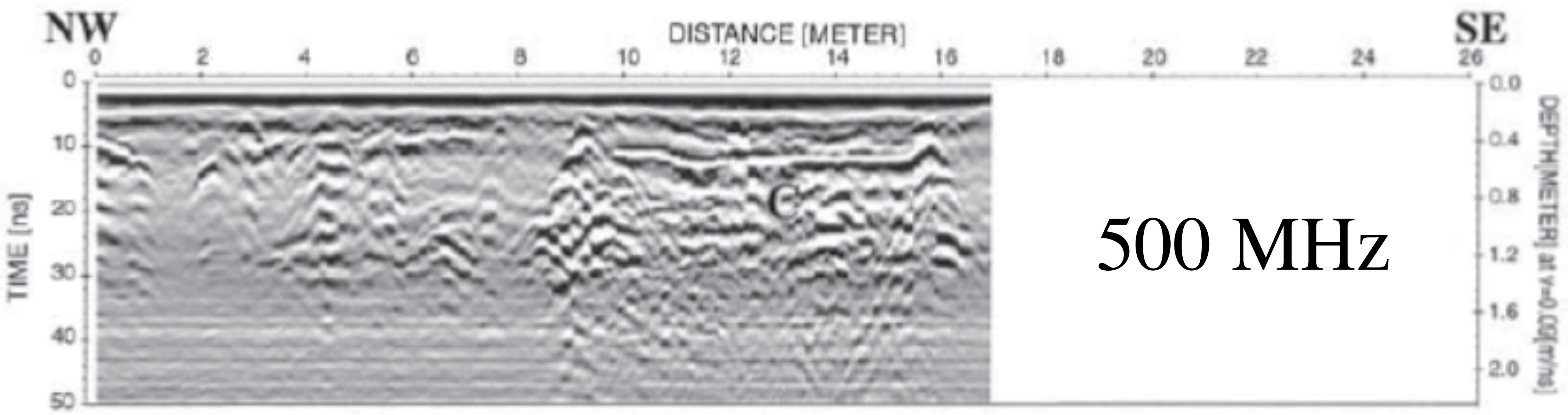


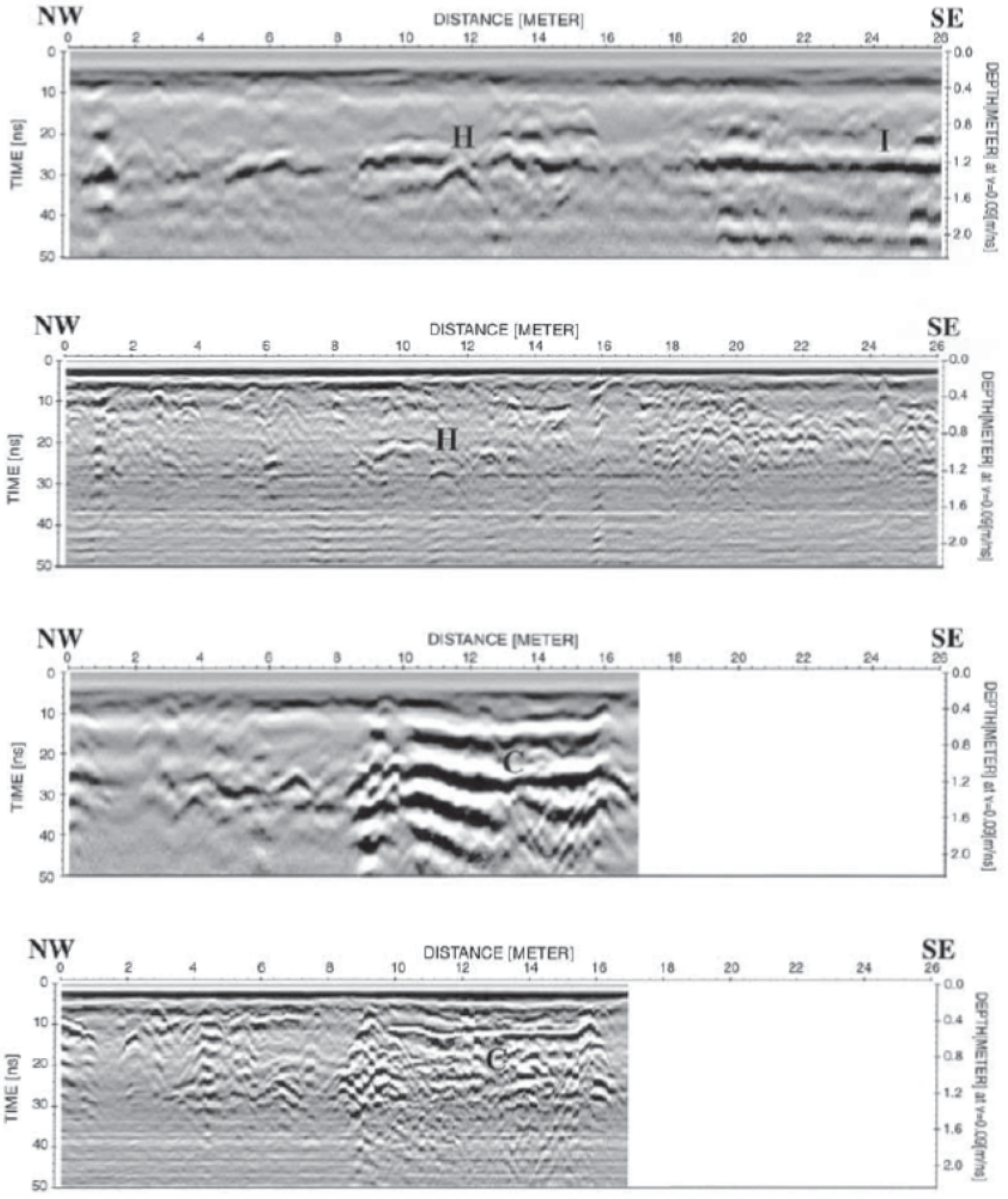
200 MHz

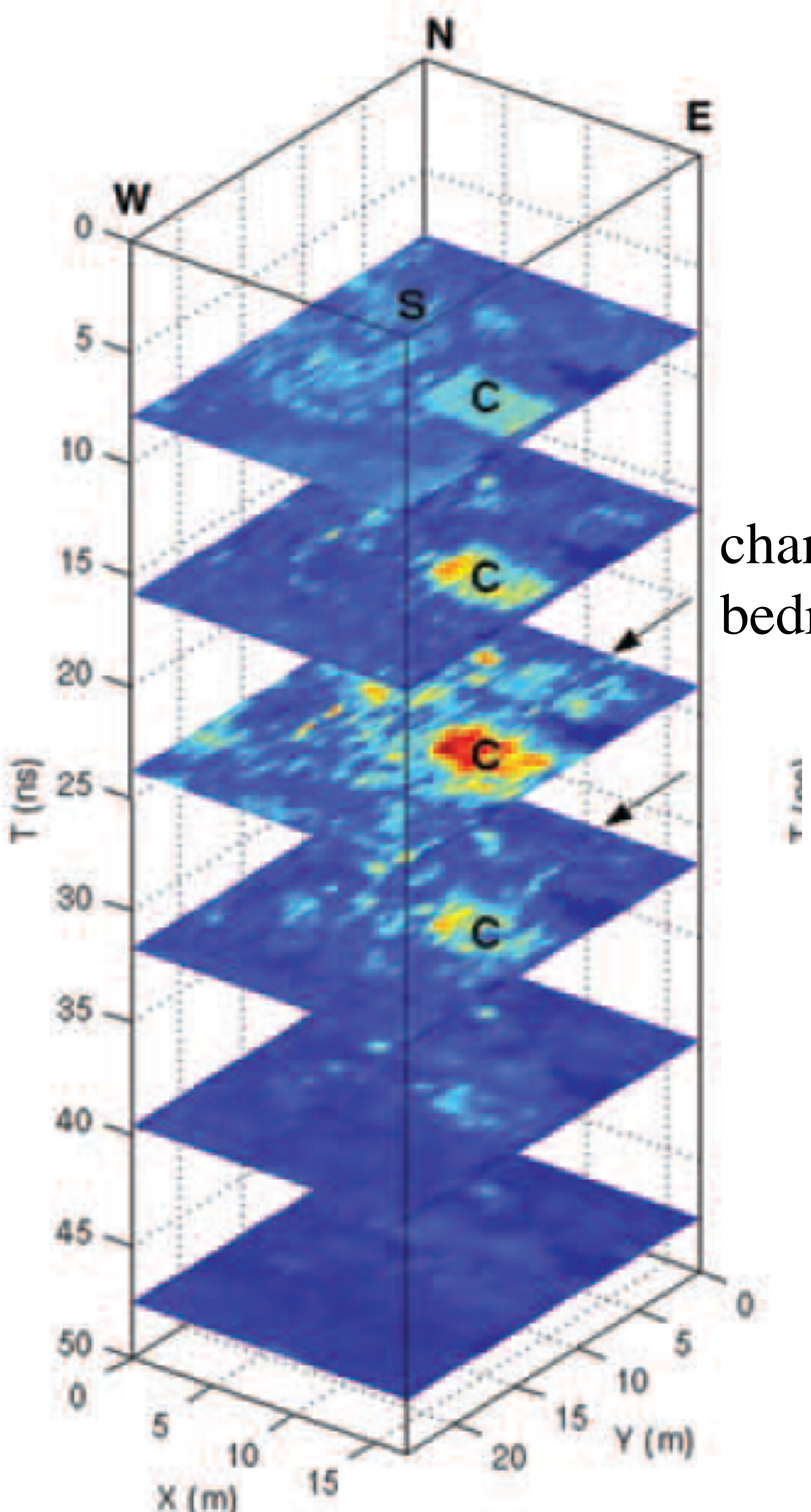


500 MHz

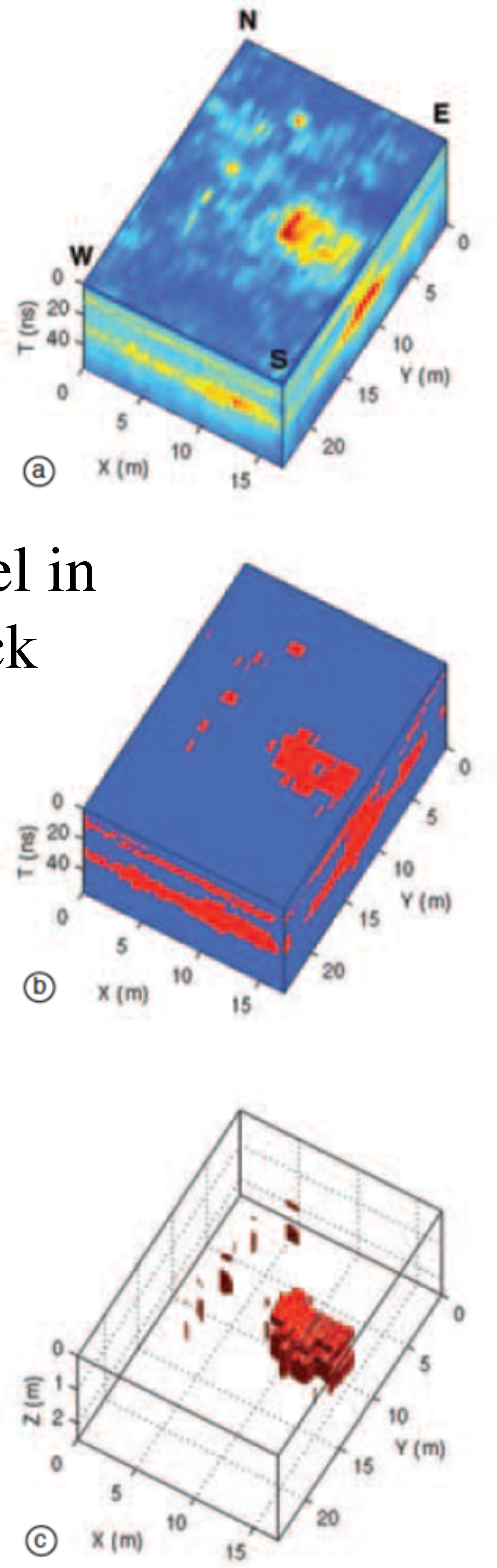




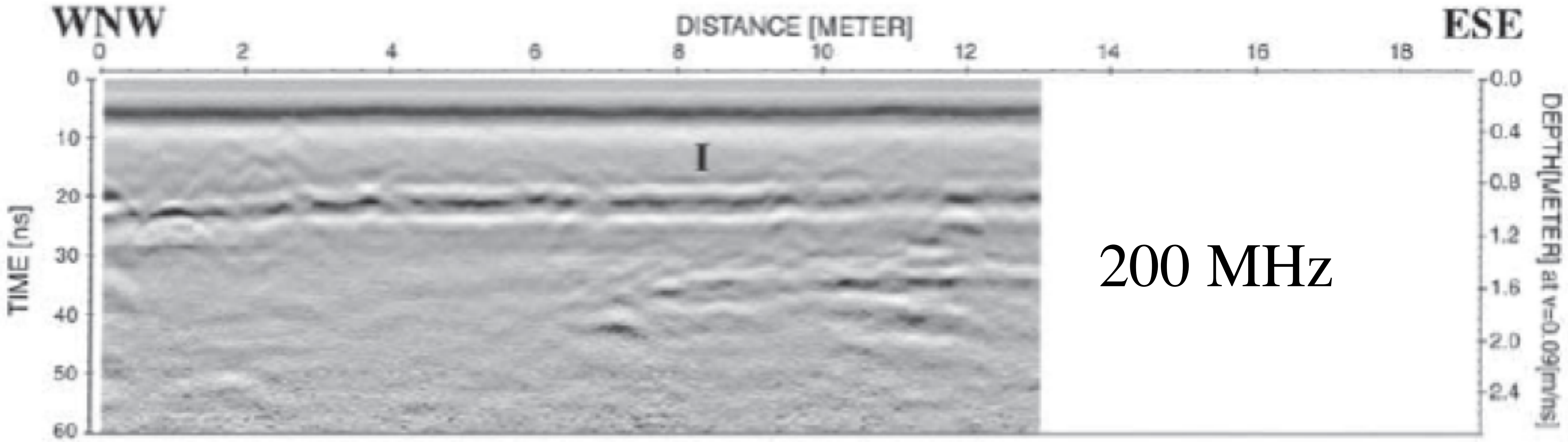


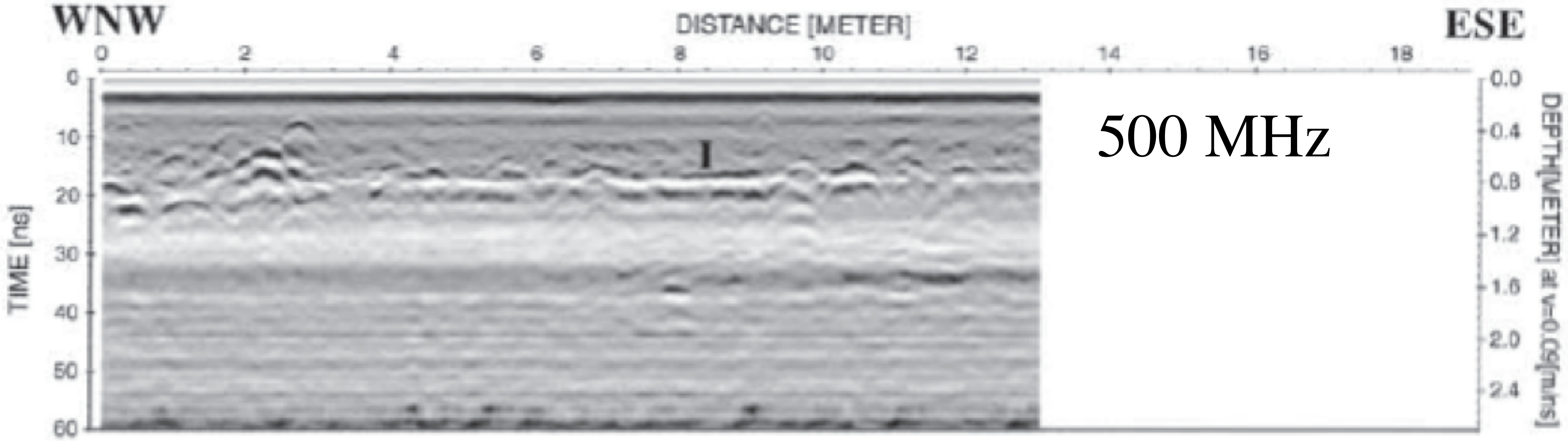


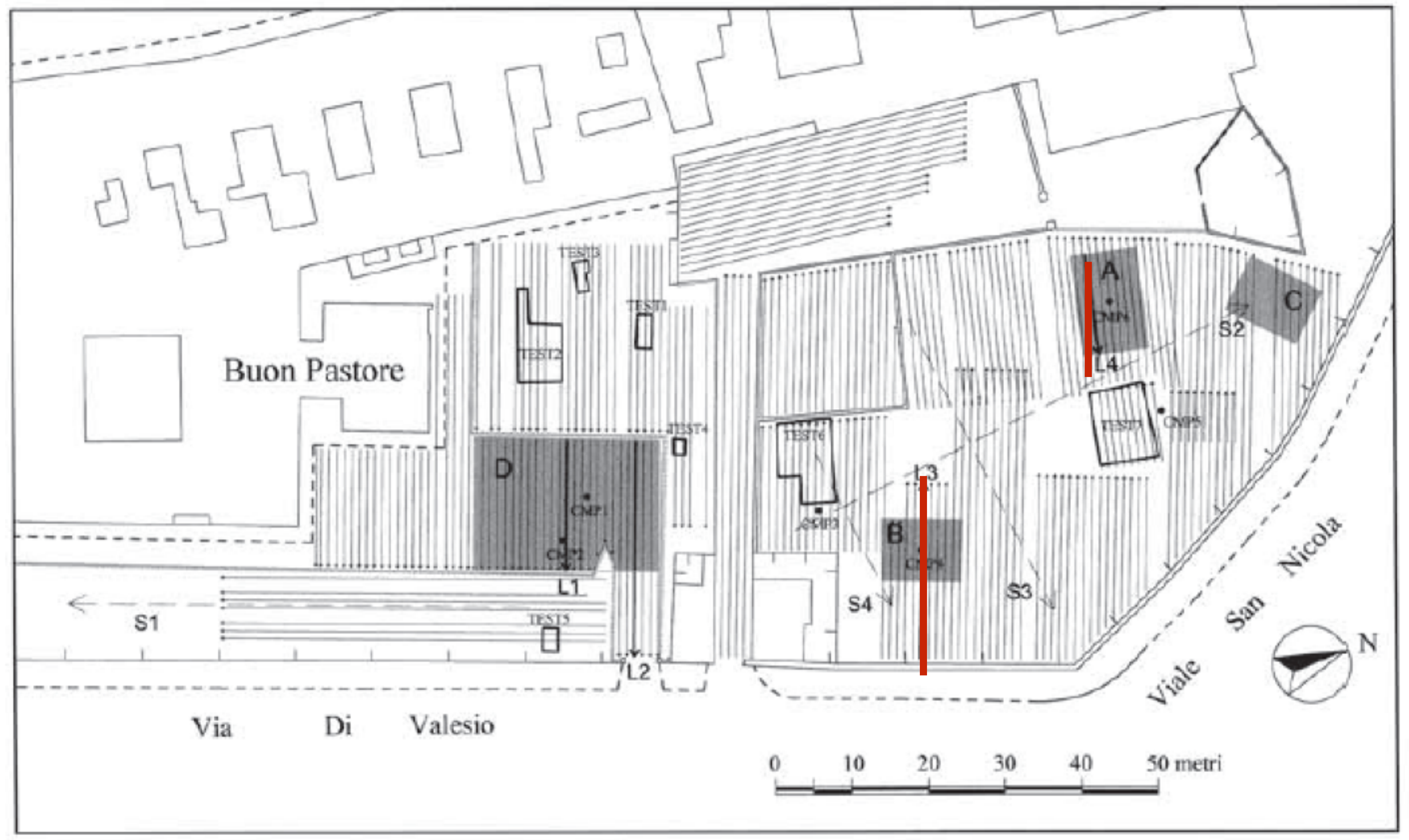
channel in bedrock

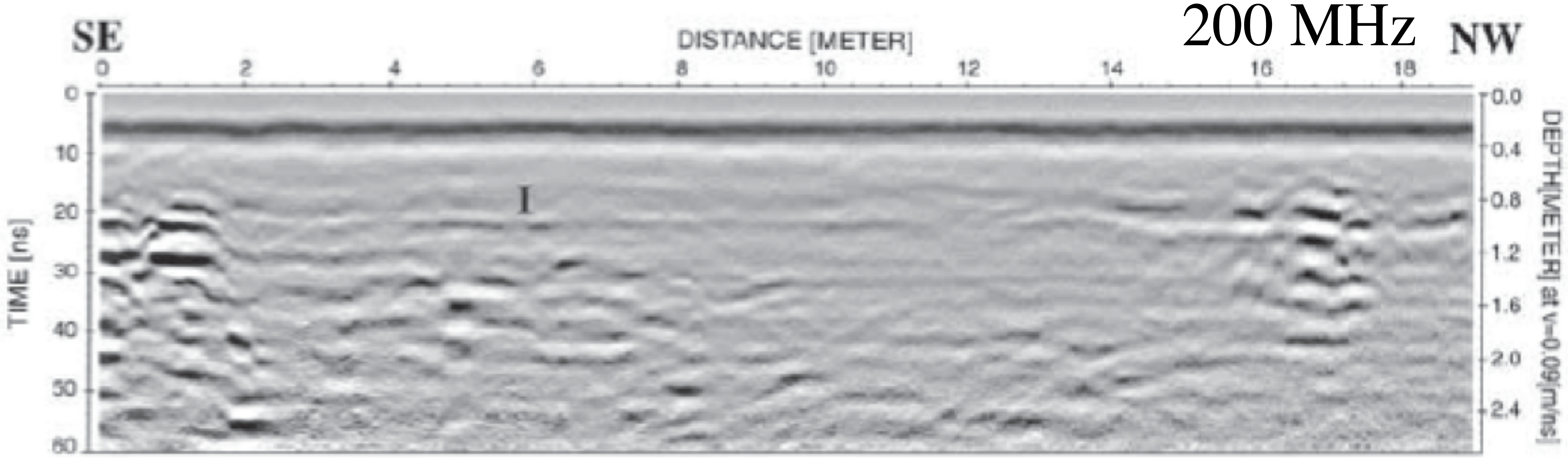


I = bedrock

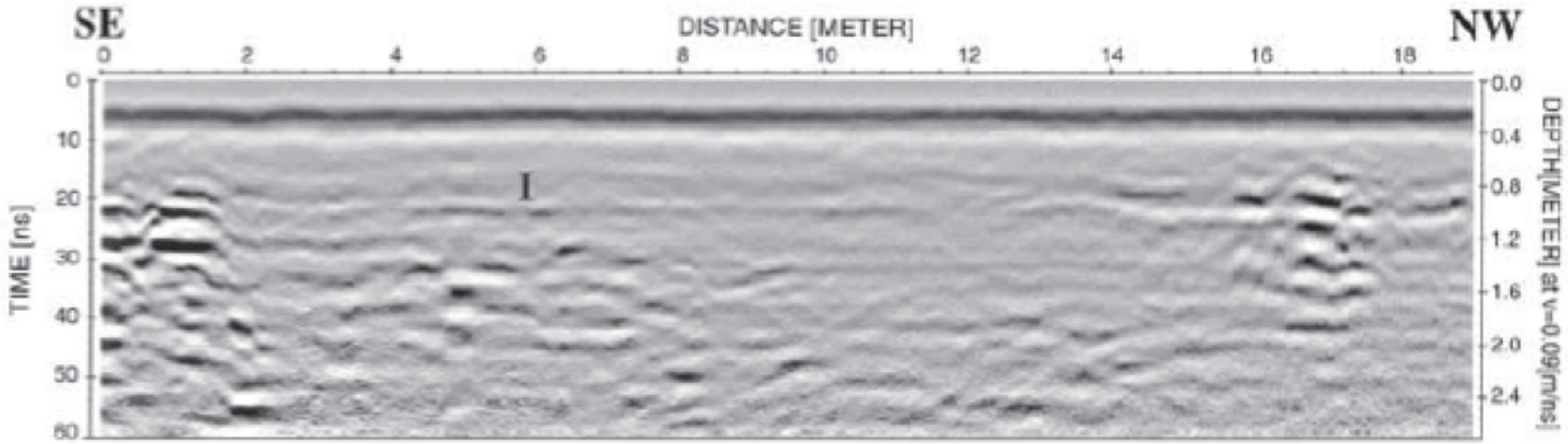

WNW
ESE
DISTANCE [METER]
TIME [ns]
DEPTH[METER] at v=0.09[m/ns]
I
200 MHz

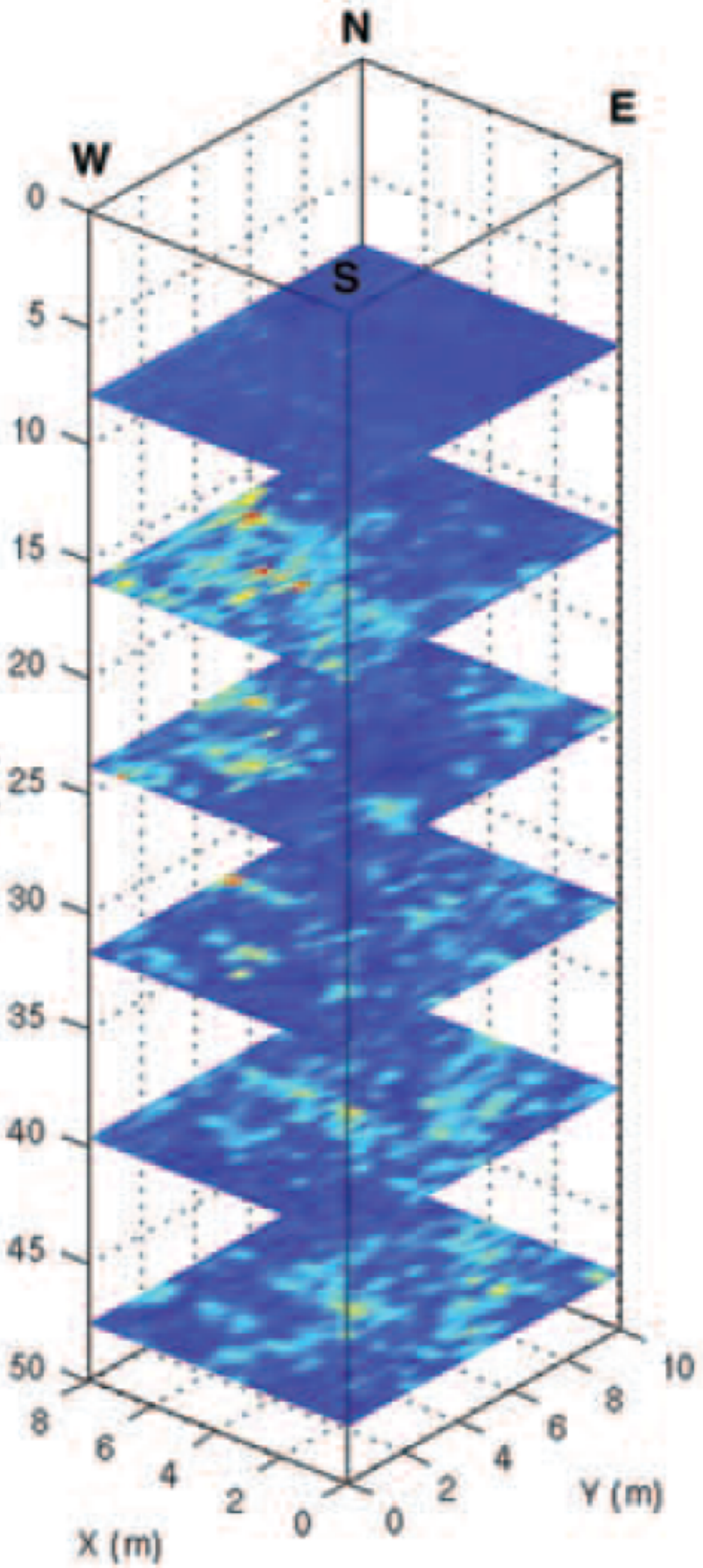

WNW
ESE
DISTANCE [METER]
TIME [ns]
DEPTH[METER] at v=0.09[m/ns]
I
500 MHz

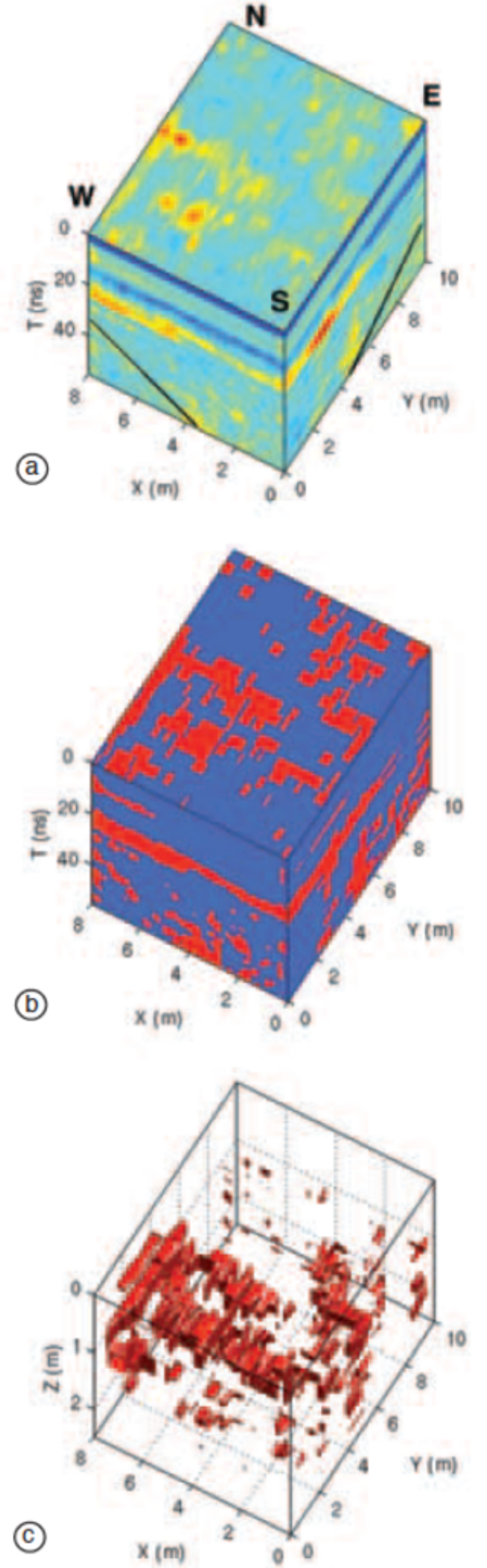

Buon Pastore
A
B
C
D
S1
S2
S3
S4
L1
L2
L3
L4
Via Di Valesio
Viale San Nicola
N
0 10 20 30 40 50 metri


200 MHz
SE
NW
DISTANCE [METER]
TIME [ns]
DEPTH[METER] at v=0.09[m/ns]
I

Rectangular patterns in bedrock thought to be a quarry.



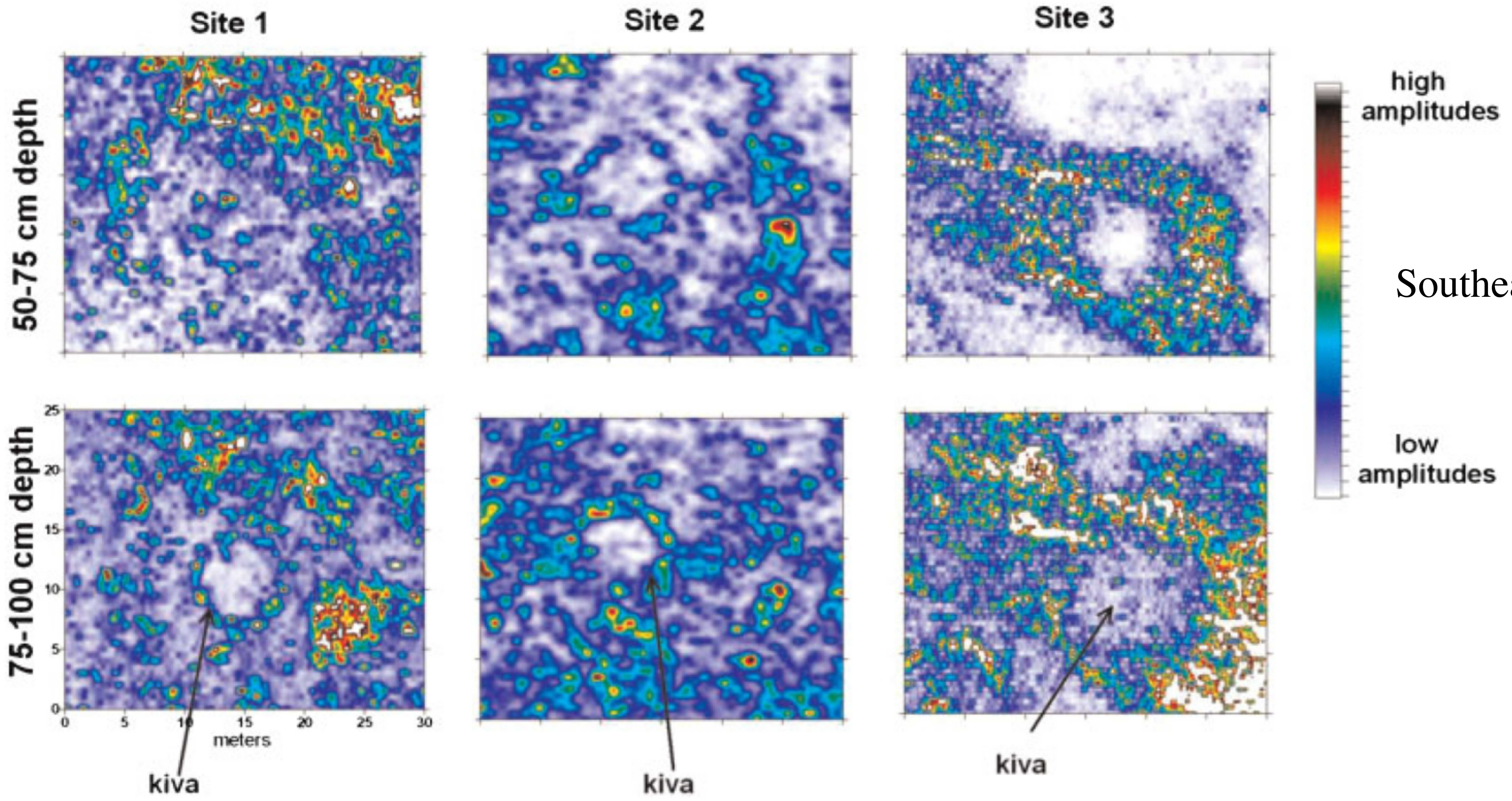




Geophysical Archaeology Research Agendas for the Future: Some Ground-penetrating Radar Examples

LAWRENCE B. CONYERS[1] AND JUERG LECKEBUSCH[2]*

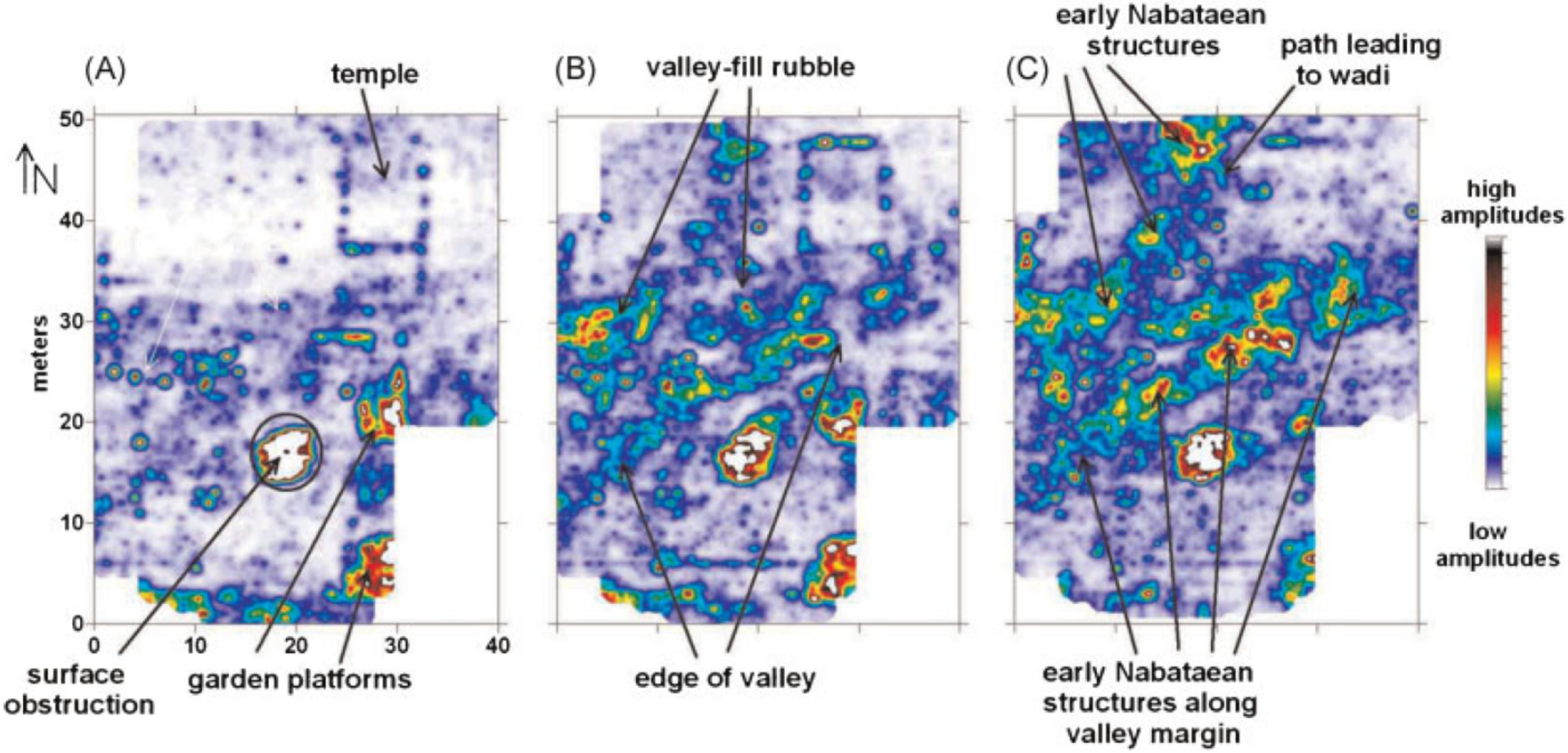
Archaeological Prospection
Archaeol. Prospect. **17**, 117–123 (2010)
Published online 11 May 2010 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/arp.379



Southeastern Utah

Petra, Jordan:

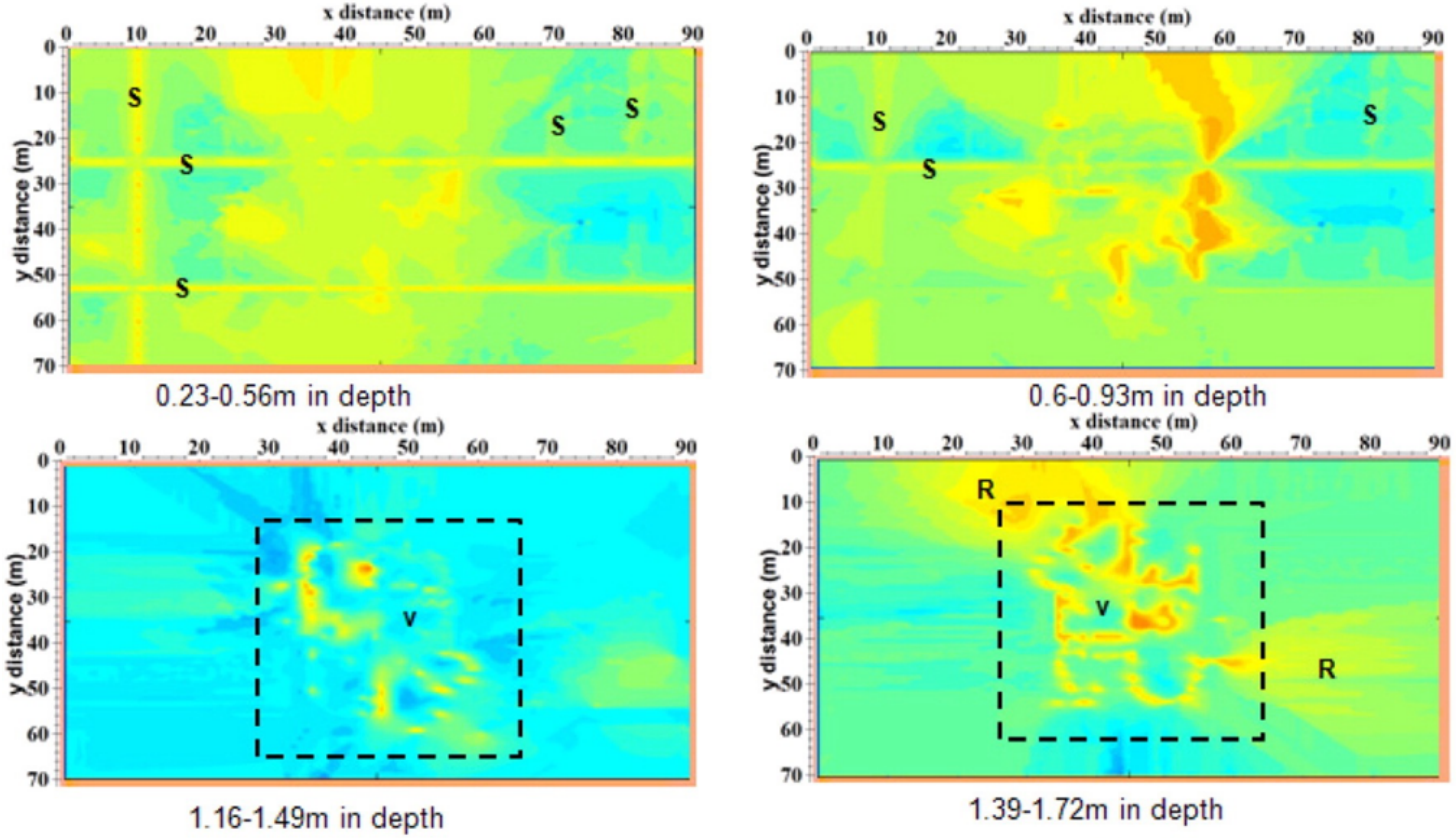
75—100 cm deeper deeper still

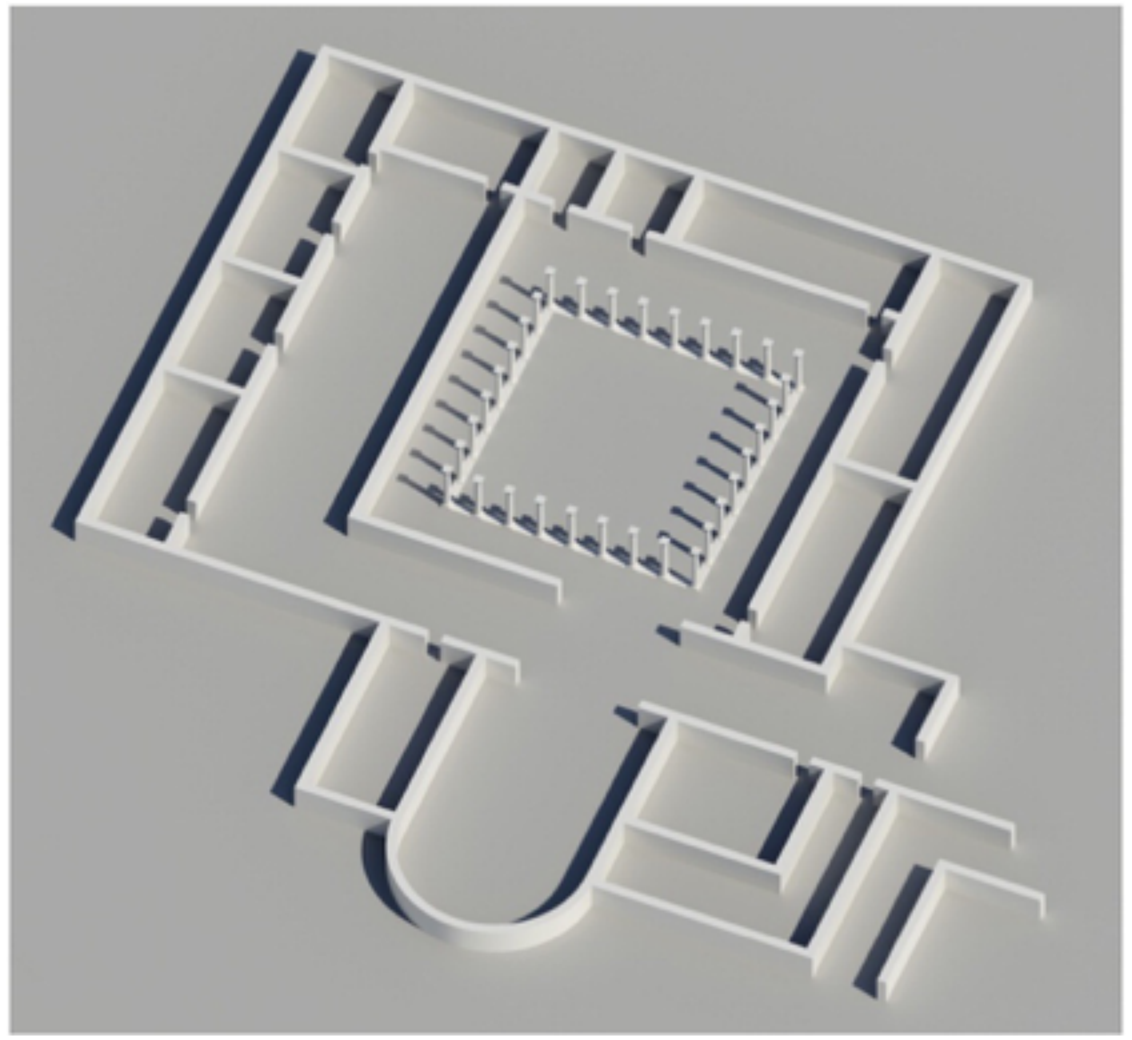


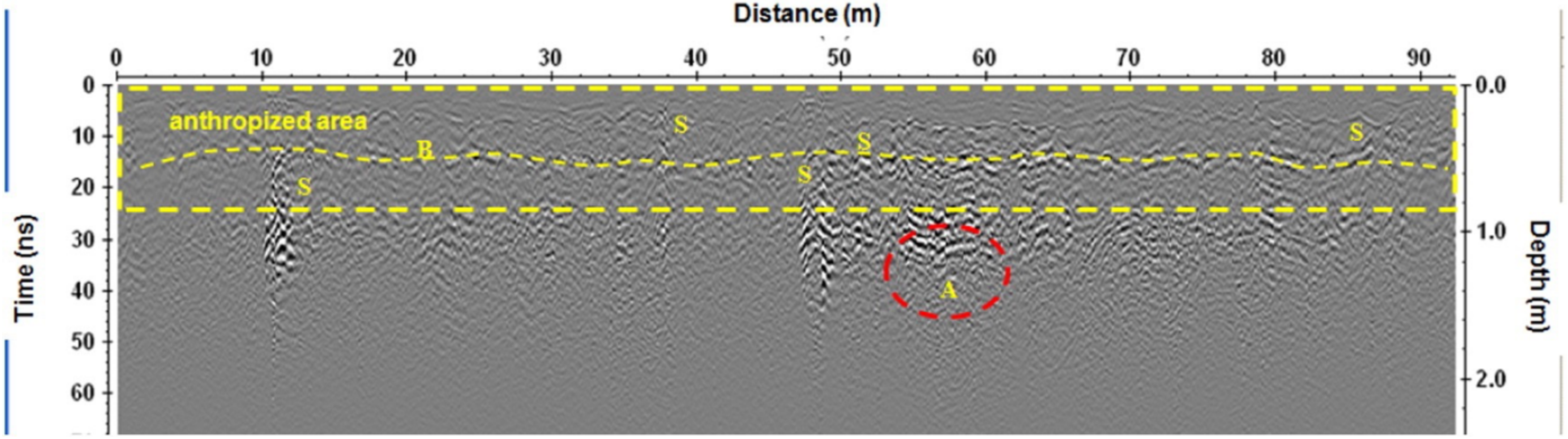
Journal of Archaeological Science: Reports
Volume 4, December 2015, Pages 276-284

The potential of integrated GPR survey and aerial photographic analysis of historic urban areas: A case study and digital reconstruction of a Late Roman *villa* in Durrës (Albania)

Daniele Malfitana [a], Giovanni Leucci [a], Giovanni Fragalà [a], Nicola Masini [a], Giuseppe Scardozzi [a], Giuseppe Cacciaguerra [a], Cettina Santagati [b], Eduard Shehi [c]



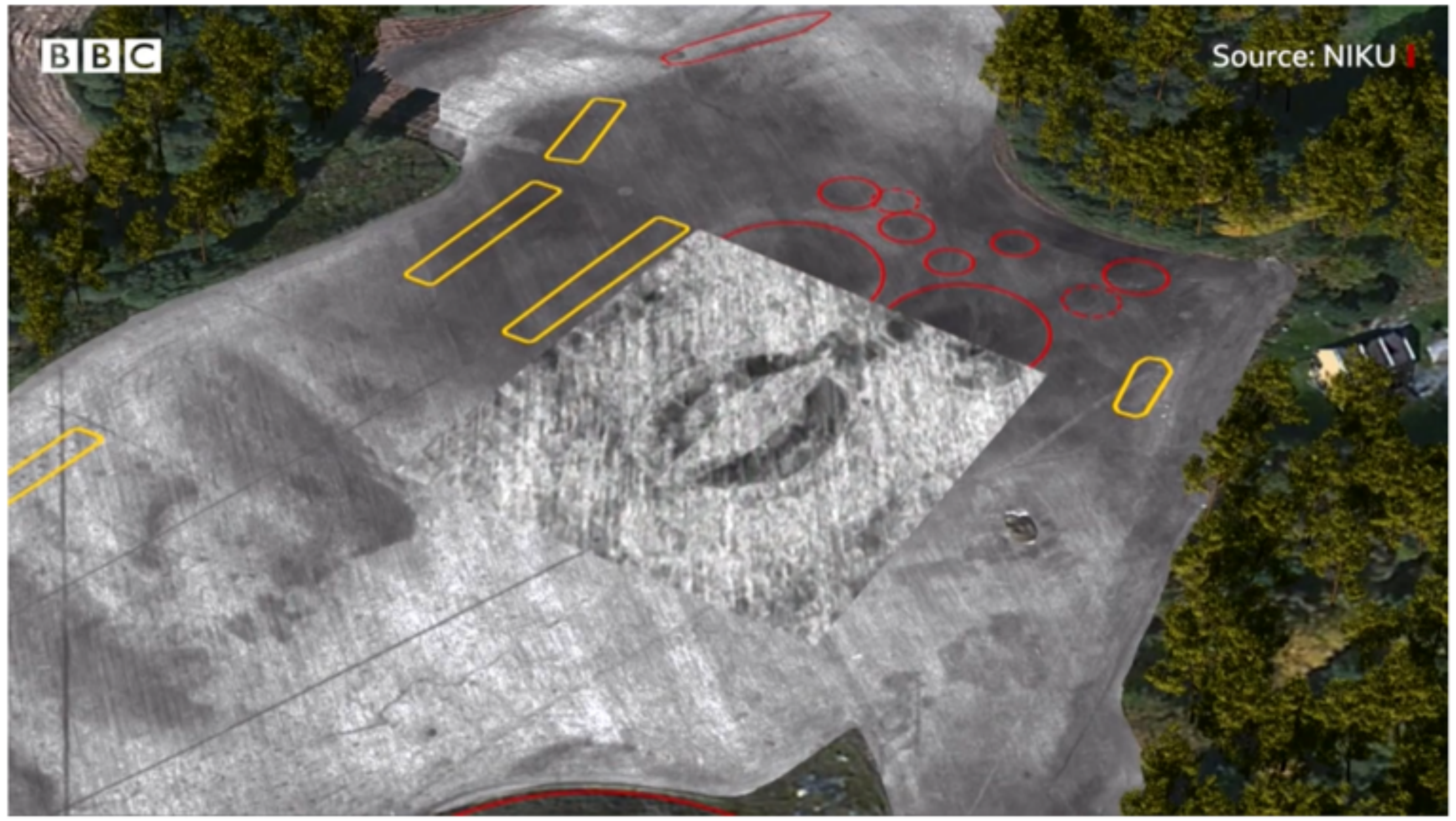




GPR in the news.

Police use GPR to investigate possible Jimmy Hoffa burial under Detroit driveway

November 14, 2012 · rockman · Geoarchaeology · Comments Off



Radar reveals buried Viking longship in Norway

Ground-penetrating radar added to ValuJet search

May 19, 1996
Web posted at: 3:00 p.m. EDT

DADE COUNTY, Florida (CNN) -- Trying to search under the mud where a ValuJet plane nose-dived eight days ago, investigators Sunday began using a ground-penetrating radar system.