Magnetic Methods

The study of the Earth's magnetic field is one of the oldest disiplines in geophysics, and similarly represents one of the earliest geophysical prospecting methods. Maps of the magnetic field are used to estimate the distribution of magnetic minerals in crustal rocks, which is in turn used to infer the structural relationships of the rock units. The main use for petroleum geophysics of magnetic methods is in estimating the thickness of sedimentary sequences by estimating the distance to magnetic sources presumed to be in the basement rocks. In mining geophysics the magnetic minerals comprising, or asscociated with, ore bodies are used as a basis for direct detection of mineralization. Regional aeromagnetics maps are useful in the interpretation of regional geological structure, as the geological 'grain' is usually well portrayed by the distribution of magnetic minerals. An increasingly important use of magnetic methods lies in environmental geophysics. High resolution mapping of the magnetic field, or its gradient (measured directly by using two magnetometers simultaneously), is used in environmental and archaeological applications of geophysics to map and characterize waste dumps, unexploded ordinance, disturbed soil, foundations, and so on.

Although magnetic surveys may be, and are, conducted by land crews just as gravity surveys are, the instrumentation and nature of the magnetic field allow measurements to be made from a moving platform, and so much of today's magnetic surveying is done either from a ship or an aircraft. Although magnetic fields may be defined in terms of a potential, and therefore have much in common with gravity in terms of mathematical representation (both gravity and magnetics are implied when potential methods are referred to), there are some important differences between the two methods in exploration applications. Firstly, magnetic sources have both a magnitude (like gravity) and direction (unlike gravity), making interpretation more complicated. Earth's magnetic field depends on latitude, and so the magnetic effect of a body also depends on latitude. Secondly, unlike gravity, which is the response of a bulk effect of the rocks, magnetic fields are the response of traces of magnetic minerals in rock.

Basic Theory.

A magnetic field is produced when a charge moves through space. Thus there is a magnetic field associated with a wire carrying an electric current (Figure 1). The magnitude of the magnetic field $|\mathbf{B}|$ at a distance r from a long, straight wire carrying an electric current I is given by

$$B = \frac{\mu_o I}{2\pi r}$$

The direction of the field is given by the right hand rule. Note that the units of $\mathbf{B}$ are T, teslsa, or Ns/Cm or N/Am or kg $s^{-2}A^{-1}$. $\mathbf{B}$ is also called magnetic induction or magnetic flux density to distinguish it from $\mathbf{H}$, which we will introduce later and is also often called the magnetic field, but here I will call $\mathbf{H}$ the magnetizing field. The constant μ_o is the permeability of free space, once defined as $4\pi \times 10^{-7}$ H/m (or N/A^2) by defining the values of the Coulomb and Ampere appropriately. In the 2018/19 revision of the SI system, the kilogram became a defined unit, and μ_o became a measured unit. It is still only one part in a billion different from the old definition, which may be used for all practical purposes.

The magnetic field has no effect on a stationary charge, but will exert a velocity-dependent force called the Lorentz force on a moving charge:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

If $\mathbf{v}$ and $\mathbf{B}$ are orthogonal the force $\mathbf{F}$ is given by the right-hand rule. In a uniform magnetic field, moving charges will travel in circles or helical paths, with no work done on them.

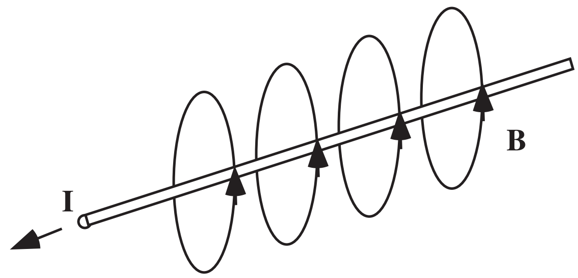


Fig.M.01: A current in a wire generates a magnetic field.

We can extend the Lorentz force in terms of current elements $d\mathbf{I}_1$ and $d\mathbf{I}_2$, having magnitude and direction. Then the force on $d\mathbf{I}_1$ due to $d\mathbf{I}_2$ in free space is given by

$$\mathbf{F} = \frac{\mu_o}{4\pi r^3} d\mathbf{I}_1 \times (d\mathbf{I}_2 \times \mathbf{r})$$

where $\mathbf{r}$ is the direction vector from $d\mathbf{I}_2$ to $d\mathbf{I}_1$ and $|\mathbf{r}| = r$. If you go through the cross products correctly you will see that parallel wires with currents flowing the same way will *attract* each other

We are going to be interested in magnetic minerals, rather than electric currents, and when discussing the magnetic method most texts on exploration consider magnetic fields as being generated by magnetic poles. The concept of a magnetic pole originated with the discovery of magnetism via the bar magnet, which was initially a piece of rock (magnetite, of course). In reality, the magnetization of a bar magnet is still due to moving charges, only at the atomic scale. (We will examine this later when we discuss magnetic materials.) The relation between bar magnets and moving charges can be clearly seen when its field is compared to the field of a solenoid, Fig M.02 below. The north pole of a magnet is defined as the pole which points towards the Earth's north pole when the magnet is free to pivot. The north pole of a solenoid is the one from which positive field lines emanate.

The magnetic field produced at the center of a single loop of wire radius r carrying current I is

$$B = \frac{\mu_o I}{2r}$$

so for a loop with n turns this will be

$$B = \frac{\mu_o n I}{2r}$$

In practice we can't place n turns all in the same place, and for a solenoid with n_l turns per unit length the field inside is uniform away from the ends and is given by

$$B = \mu_o n_l I \quad .$$

The dipole moment of a small loop of wire of area S and carrying a current I is simply IS, with a direction perpendicular to the plane of the loop and given by the right hand rule, and units of Am^2. For a bar magnet, if we have north (or positive) and south (or negative) poles of strength m separated by a distance l then the *dipole moment* is simply $ml\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector from the south pole towards the north pole. (m thus implicitly has units Am).

Magnetics, gravity, and electrostatics are all examples of **potential fields**. A common theme in potential theory is that if the force between two charges/masses/magnetic moments is divided by one of the

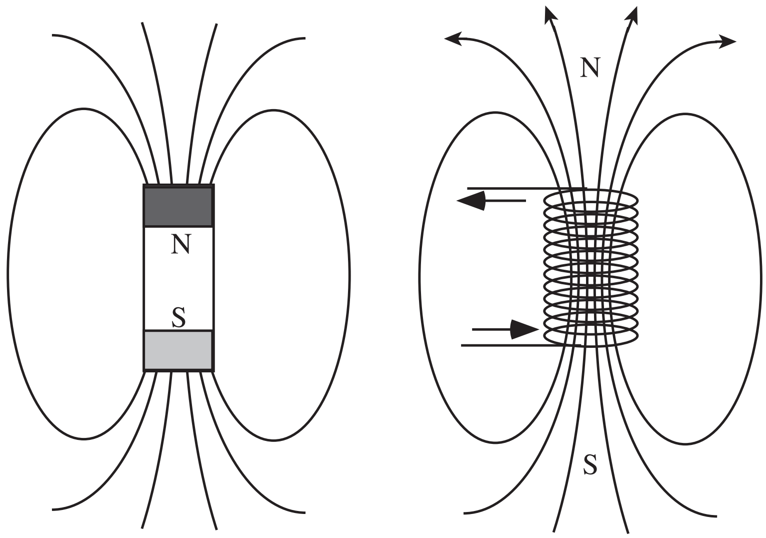


Fig.M.02: A permanent bar magnet and a current-carrying solenoid produce similar (dipolar) magnetic fields.

charges/masses/magnets, one gets the definition of a field. If one indulges in the fiction of an isolated magnetic pole (which is easier, and eventually more useful, than dealing directly with currents) this may be seen by considering the force on a test pole m due to a pole m_1 at distance R:

$$\mathbf{F} = \frac{\mu_o m_1 m}{4\pi R^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector from m_1 to m. Then magnetic field may be defined by dividing by the magnitude of the test pole:

$$\mathbf{B} = \frac{\mu_o}{4\pi}\frac{m_1}{R^2}\hat{\mathbf{r}} \quad .$$

Potential is defined as the work required to move a point charge/mass/magnetic pole from an infinite distance to the point P:

$$U_P = \int_{\infty}^{P} \mathbf{F}.d\mathbf{r} \quad .$$

Note that force is a **vector** (i.e. it has a magnitude and a direction) but that potential is a **scalar** (i.e. it has magnitude only). Both are functions of three dimensional space. Dealing with potential is often easier than dealing with the field directly. Any field **G** is obtainable from the potential U by differentiation:

$$\mathbf{G} = -\nabla U$$

and in particular the magnetic field can be obtained from the magnetic scalar potential V_m

$$\mathbf{B} = -\mu_o \nabla V_m \quad .$$

The potential is **conservative**, that is, it doesn't matter what path the integration takes to go from point to point, the total work done is the same. Potential fields are also additive, or linear, so the effect of an extended body or several bodies may be obtained by integrating the effect of all the constituent material.

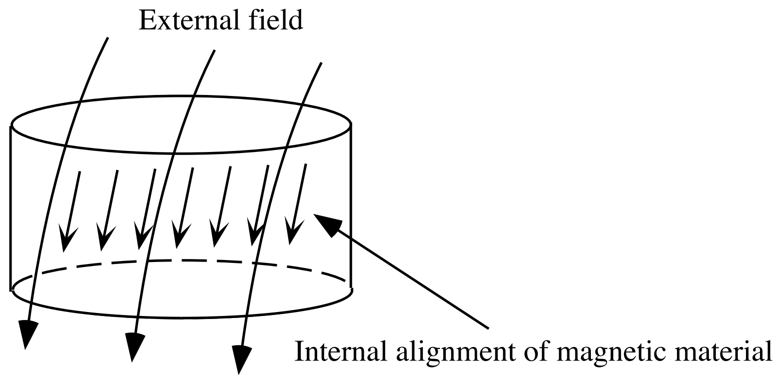


Fig.M.03: An external field aligns magnetic moments with a material to produce an internal field in the same direction. This phenomenon is called polarization and the size of the internal field is quantified by the magnetic susceptibility of the material.

The spinning of electrons within atoms produces magnetic dipoles, so all substances are intrinsically magnetic, that is, in the presence of an external field the small dipoles within the material will align and produce an additional, internal, magnetic field (if they are not already aligned to some extent, as they are in the bar magnet). This aligning of the internal magnetic dipoles is termed polarization or magnetic induction.

Inside magnetic materials, is it usual to introduce the magnetizing field **H** whose units are A/m. For most rock forming materials, the magnetization **M** is simply proportional to the magnetizing field **H**, so

$$\mathbf{M} = k\mathbf{H}$$

where the scalar k is termed the *magnetic susceptibility* of the material. Note that, like Ohm's law, this is a linear approximation to reality – we will see below that for large **H**, then **M** will saturate.

Outside the material (in a vacuum or anywhere that $k = 0$) we have that the external magnetic field $\mathbf{B}_e$ is given by

$$\mathbf{B}_e = \mu_o \mathbf{H}$$

so if we add the internal magnetization **M** to the external field, we have that the internal field $\mathbf{B}_i$

$$\mathbf{B}_i = \mu_0(\mathbf{H} + \mathbf{M})$$

or

$$\mathbf{B}_i = \mu_0(\mathbf{H} + k\mathbf{H}) = \mu_0(1 + k)\mathbf{H} = \mu\mathbf{H}$$

where $\mu = \mu_0(1 + k)$ is the permeability of the material. The relative permeability, $\mu_r = \mu/\mu_o$ is sometimes given in which case $\mathbf{B}_i = \mu_r\mu_o\mathbf{H}$. The intensity of the induced magnetization per unit volume is the polarization and depends on the magnitude of **H**.

Note that

$$\mathbf{H} = -\nabla V_m \quad .$$

The motion of electrons within the atoms contributes to a material's magnetic properties in several ways.

The gradient operator, ∇ , equals

$$\mathbf{x}\frac{\partial}{\partial x} + \mathbf{y}\frac{\partial}{\partial y} + \mathbf{z}\frac{\partial}{\partial z}$$

where **x, y, z** are the cartesian unit vectors. Applied to a scalar, ∇ Ñ becomes gradient and results in a vector:

$$\nabla U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)$$

Dotted into a vector, ∇ becomes divergence and is a scalar:

$$\nabla \cdot \mathbf{V} = \frac{\partial \mathbf{V}_x}{\partial_x} + \frac{\partial \mathbf{V}_y}{\partial_y} + \frac{\partial \mathbf{V}_z}{\partial_z} \quad .$$

Crossed into a vector, ∇ becomes the curl or rotation and is another vector:

$$\nabla \times \mathbf{V} = \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \mathbf{x}\big(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\big) + \mathbf{y}\big(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\big) + \mathbf{z}\big(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\big)$$

Diamagnetism. The motion of electrons in their orbitals, which may be thought of simplistically as being circular orbits, produces tiny magnetic dipoles just as a circular electric current would. These dipoles are usually oriented randomly, but in the presence of an external magnetic field the orbitals align slightly in such a way as to **oppose** the external field, weakening it slightly. This is a very small effect, but since all materials are made of atoms, all materials are diamagnetic. Susceptibility is small and negative for diamagnetism, which is the dominant effect in minerals such as quartz, halite, feldspar.

Paramagnetism. The spin of electrons also produces a magnetic dipole. Electrons tend to be in spin ‘up’ and spin ‘down’ pairs and so the magnetic effect cancels, but for odd electrons there is a net magnetic moment on the atom. This effect is greatest for transition elements, which have many unpaired electrons in the d orbitals. The normally random magnetic dipoles due to electron spin will align along an external field, and so k is positive, but again paramagnetism is a relatively small effect.

Ferromagnetism. In some elements, the crystal bonding is such that the spin dipoles of the atoms are aligned over volumes which are large compared to atomic scales, so that a net magnetic moment is obtained over macroscopic distances. These volumes are called ‘domains’, and there magnetization is not due to external fields, although an external field can move the boundaries of the domains (the domain walls) if sufficiently strong, resulting in very large, positive, values for k. At sufficiently high temperatures, thermal agitation destroys the alignment and the magnetization is destroyed. The temperature at which this occurs, dependent on the material, is called the Curie temperature.

If within a domain, the spin dipoles of atoms all align one way the material is truely ferromagnetic, as in iron, nickel and cobalt. If the dipoles are alternately aligned one way and another such that there is no net magnetic effect then the material is anti-ferromagnetic. If the dipoles are alternately aligned one way and another, but the dipoles in one direction are larger, then the material is ferrimagnetic. Magnetite and ilmentite are ferrimagnetic and have large susceptibilities. If the alternately aligned dipoles are of similar size but slightly canted in places due to defects in the crystal lattice, the material, such as haematite, is anti-ferromagnetic but weakly magnetic.

(Ferromagnetism and anti-ferromagnetism are actually a result of quantum exchange interactions related to Pauli exclusion. If the exchange constant J_{ab} is positive, parallel spins (ferromagnetism) are energetically favorable, while if J_{ab} is negative antiparallel spins (anti-ferromagnetism) are energetically favorable.)

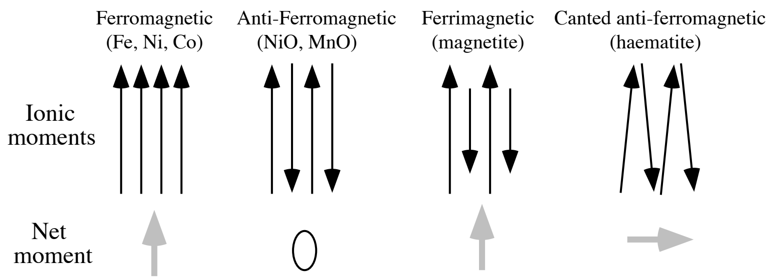


Fig.M.04: Four types of permanent magnetization in materials.

The susceptibility of a rock containing magnetic material is a complicated function of the size and shape of the magnetic grains. A grain of magnetite or haematite may consist of one or more magnetic domains; it is either a single domain grain or a multidomain grain. This variability makes it difficult to relate the susceptibility of a rock directly to its magnetite content.

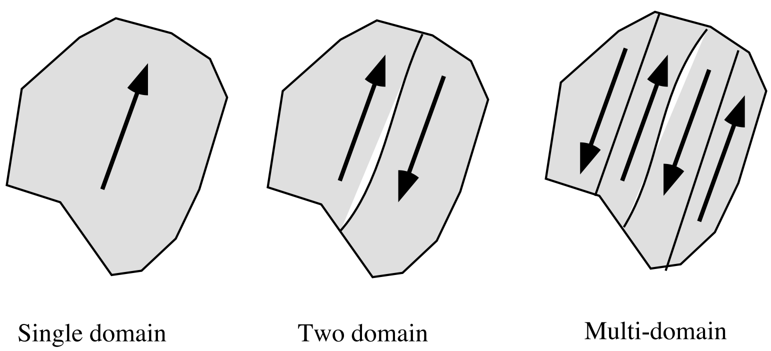


Fig.M.05: Single and multi-domain materials.

Furthermore, the susceptibility of a ferromagnetic substance is a function of the external field and the history of the field. A plot of **M** as a function of **H** is called a *hysteresis curve*. If k where simply a constant, independent of **H**, then the curve would be a straight line through the origin. The slope of the curve would give the magnitude of k. However, there are two aspects of ferromagnetic domains which affect k. Once the magnetic field reaches a certain strength all the domains have lined up and the internal magnetic field, and hence magnetization, cannot increase. This level of internal field is called saturation magnetization. Secondly, it requires effort to move the domain walls and magnetization, so if the material is magnetized and the external field is then reduced to zero, some of the domains remain aligned, leaving an induced magnetization even though the external field is zero. To remove the residual field, an external field of some

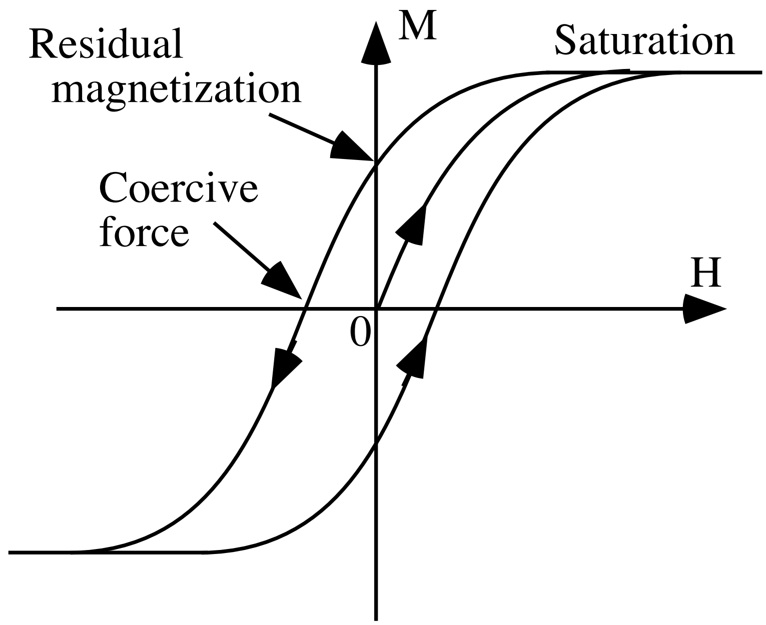


Fig.M.06: A hysteresis loop. If the material starts with no permanent internal magnetization, then a rising external field **H** will produce an internal magnetization **B** that is limited by the saturation of the material. Reducing the external field to zero leaves a permanent magnetization in the material (residual magnetization) and it takes a negative external field equal to the coercive force of the material to remove the residual magnetization. A larger negative field will saturate the material in the opposite direction, and reversing the field again takes one around the opposite side of the hysteresis loop.

magnitude and opposite to the internal magnetization must be applied. The size of this external field is a measure of the *coercive force* required to demagnetize the material.

For now, we have to warn you that the units used in magnetism are very confusing. The cgs system is set up so that μ_o = 1, so **B** and **H** are the same magnitude in free space. Field intensity has units of oersted ($4\pi \times 10^{-3}$ Oe = 1 A/m) and magnetic induction has units of gauss (10^{-4} T). These units are large compared with the variations measured in exploration, and the gamma (= 10^{-5} gauss) is very commonly used for mapping field variations, so 1 gamma = 1 nT (although in older textbooks the gamma is defined as = 10^{-5} Oe, making it a unit of H, not B). Unlike gravity, the nT is widely accepted today and is the preferable unit, and you won't come across gammas unless you are looking at pretty old work. (Note that in cgs **B** = $(1 + 4\pi k)$**H**.)

Magnetic properties of rocks.

There are extensive and sophisticated techniques and instrumentation for the measurement of remanent magnetization for the purpose of discovering the past magnetic field. These are only of peripheral interest to the explorationist, and interested students are referred to D.W. Collinson, Methods and Techniques in Palaeomagnetism, Elsevier, 1983 and D. J. Dunlop & O. Özdemir, Rock Magnetism: Fundamentals and Frontiers, CUP 1997. Since induced magnetization is the property of interest in exploration, the measurement of susceptiblility is of more direct importance. It is generally undertaken using a susceptibility bridge, a balanced, air cored pair of coils whose resonance is dependent on the susceptibility of material placed between the them. Note that $k(SI) = 4\pi k(cgs)$.

Magnetite, with a susceptibility of about 1, is by far the most important magnetic material in prospecting. Quartz and silicate minerals have a susceptibility of about 10^{-6}. Thus

$$
\begin{array}{ccccccc}
k_{sediments} & < & k_{metamorphics} & < & k_{acid\ igneous\ rocks} & < & k_{basic\ igneous\ rocks} \\
5 \times 10^{-4} & & 5 \times 10^{-3} & & 1 \times 10^{-2} & & 5 \times 10^{-2}
\end{array}
$$

Telford *et al.* give a table of typical susceptibilities on page 74. Figure 7 gives an idea of the ranges in susceptibility of rocks and also iron and steel. Note that for iron and steel objects, remanent magnetization can be up to ten times as large as induced magnetization.

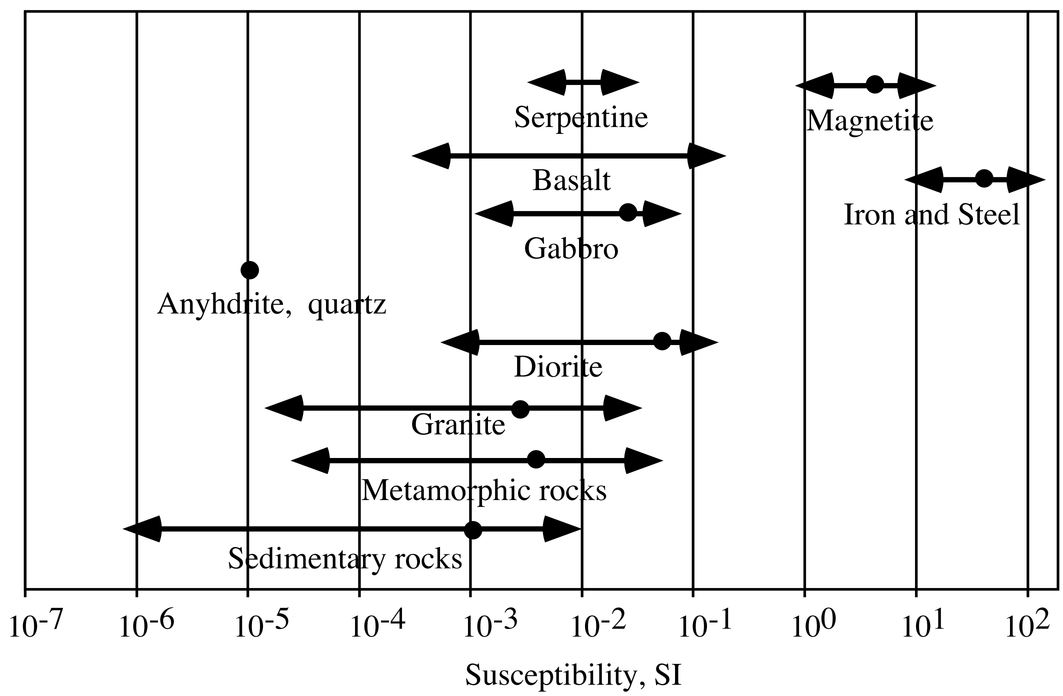


Fig.M.07: Graphical presentation of susceptibilities given in Telford et al.

Induced and remanent magnetization.

Magnetization induced in rocks causes them to behave like magnets, and so making measurements of the magnetic field on Earth's surface, from ships, aircraft, or even satellites can allow us to infer something about the distribution of magnetic material inside the crust.

While for exploration purposes we are mostly interested in magnetization induced by Earth's field, it is important to remember that rocks can have a remanent magnetization caused by a variety of processes. One is the coercive force discussed above, either from exposure to Earth's magnetic field over a long period of time or more intense magnetic fields associated with lightning strikes (viscous remanent magnetization). Another is thermal remanent magnetization created as the magnetic mineral in an igneous or volcanic rock cool from above their Curie point in Earth's field. Sediments can acquire a depositional remanence as mineral grains settle out of suspension in water. Chemical remanence can be created during diagenesis. Sometimes separating an induced magnetization from a remanent magnetization can be tricky.

Since the induced magnetization **M** depends on the magnitude and direction of **H**, we need to know something about Earth's magnetic field before we can fully interpret magnetic anomalies.

Magnetism of the Earth.

The elements used to measure and record the Earth's magnetic field are either the cartesian coordinates (X,Y,Z) where X is true north and Y is east, or spherical coordinates (F,D,I) where F is the magnitude of the field, D is declination and I is inclination. Declination is the angle between the horizontal projection of the field vector **F** and true north, and inclination is the angle between the field vector and the horizontal. These coordinates originate because accurate measurements of (F,D,I) are easier to make than (X,Y,Z), by measuring the total field (which does not require a magnetometer to be oriented) and the two angles (which use a sensor mounted on the theodolite and then oriented perpendicular to the field, thus recording zero, which can be done very precisely).

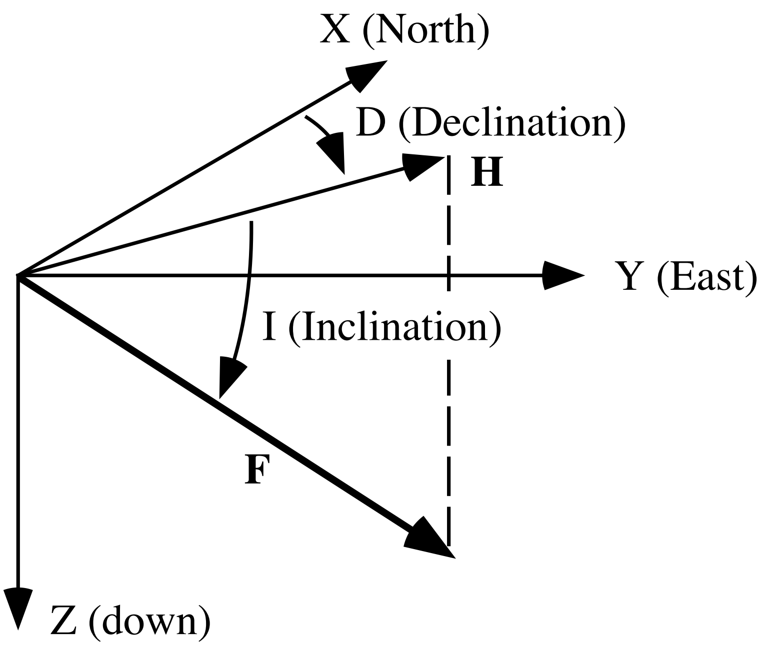


Fig.M.08: The elements used to describe Earth's magnetic field.

The main field. Most of Earth's magnetic field is generated in the outer core, where motion of electrically conductive liquid iron forms the *geodynamo* or geomagnetic dynamo. The energy source is convection fueled by latent heat of crystallization as the inner core grows in size, gravitational energy as the denser inner core grows, and liberation of an incompatible light element at the inner core which drives convection as is rises. The motion is complicated and changes with time, but Earth's rotation results in a system that is dominated by an *axial dipole* most of the time. Thus, the largest part of the Earth's magnetic field, about 90%, can be described by a dipole presently near the centre of the Earth, inclined at 11° and offset from the rotation axis about 490 km. (Remember; this dipole is a fiction and is the result of a best fit to a more complicated field.) As we shall see, at a given distance from a dipole, the field off the end (polar field) is twice as strong as the field to the side (equatorial field). For Earth the total field (F) is about $60\mu T$ at the poles and $30\mu T$ at the equator. The geomagnetic poles are the points where the axis of this best fitting dipole intersects the Earth's surface, and the North pole was at $78.8^\circ N, 70.9^\circ W$ in 1980 but changing quite rapidly since then. We see that inclination is 90° at the geomagnetic poles and 0° at the geomagnetic equator. The relation between geomagnetic latitude, θ, and inclination is $\tan(I) = 2\tan(\theta)$. This simple relation is at the heart of the palaeomagnetist's ability to reconstruct positions of continents given the inclination of the Earth's fields in times past recorded by rock magnetism. Note that the geomagnetic poles and equator do not correspond to their geographic counterparts.

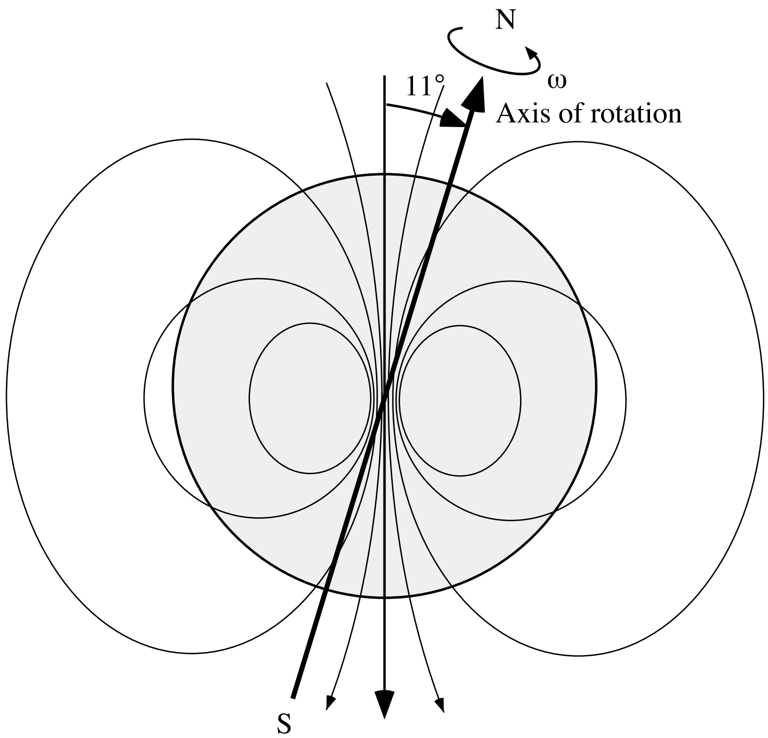


Fig.M.09: Most of Earth's field can be described by a dipole inclined at an 11° angle to the rotation axis.

<u>The non-dipole field</u>. A significant proportion of the Earth's field is does not fit the simple dipole model. The following diagrams, taken from Parkinson's text on geomagnetism, illustrate the morphology of the long period components of the main field. Imagine what these diagrams would look like for a simple dipole. The structure of the main magnetic field is codified in the *International Geomagnetic Reference Field*, a mathematical description of Earth's magnetic field that is updated every few years.

<u>The time-varying magnetic field</u>. Earth's magnetic field is a combination of internal and external sources. The field generated in the core varies slowly from year to year (called *secular variation*). Another part of the internal field is due to remanence in crustal rocks below their Curie temperatures, along with the field induced in surface rocks by the main field (the part of the field of interest to exploration). This part of the field is essentially static. An external field is caused by movement of currents in the ionosphere and movement of the magnetosphere driven by the solar wind and the Earth's rotation, producing time variations in the magnetic field at periods of seconds to the 11-year sunspot cycle (these will be of interest when we consider electromagnetic induction as an exploration method). Intense increases in the solar wind caused by coronal mass ejections can create quite large, and rapid, changes in Earth's magnetic field called magnetic storms. Lightning produces electromagnetic energy trapped in the waveguide formed by the resistive atmosphere which separates the electrically conductive Earth and its ionosphere. This cavity resonates at 8Hz, the *Schumann resonance*. Taken together, these effects produce variations in the magnetic field on all time

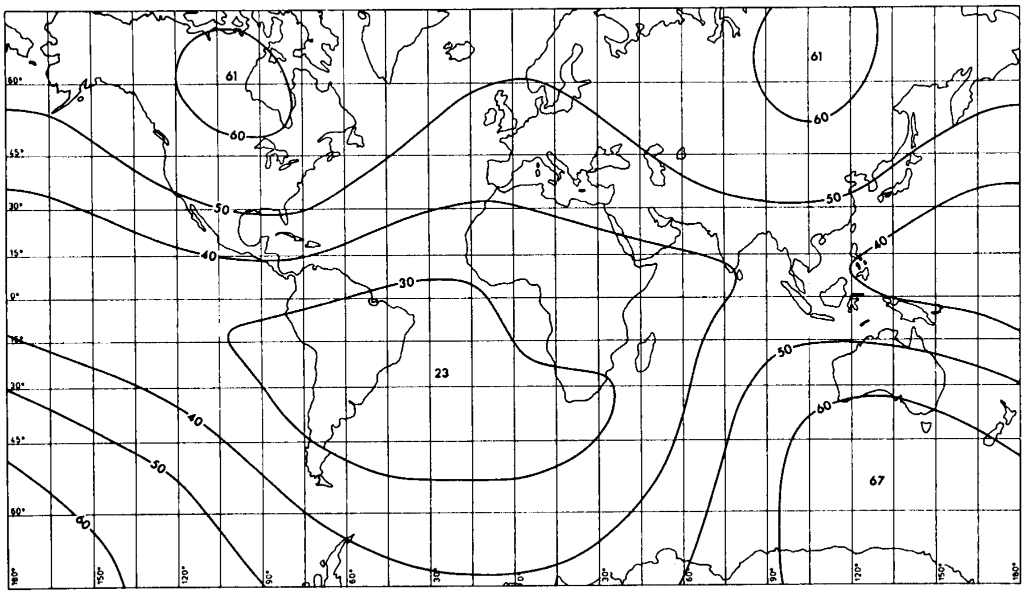


FIG. 27. Total intensity (F), in μT, based on MAGSAT data (Langel et al., 1980)

Fig.M.10: Total field intensity (F) in μT based on MAGSAT data.

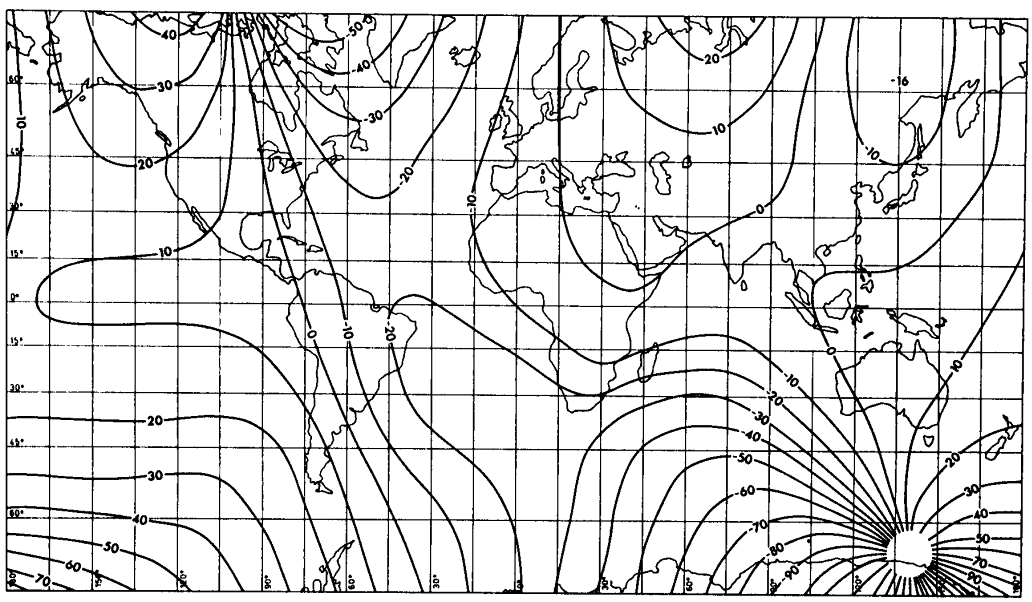


FIG. 23. Declination (D), in degrees, based on MAGSAT data (Langel et al., 1980)

Fig.M.11: Declination (D) in degrees based on MAGSAT data.

scales from MHz to millions of years.

The main magnetic field changes very slowly, at periods on the order of a year and longer. The only impact of this secular variation on the explorationist is that maps of inclination and declination need to be corrected for the yearly changes in these indices. Declination varies about -0.1 to -0.2° per year across the U.S.A. The magnetic field due to external electric currents changes on time scales of a few seconds to more than

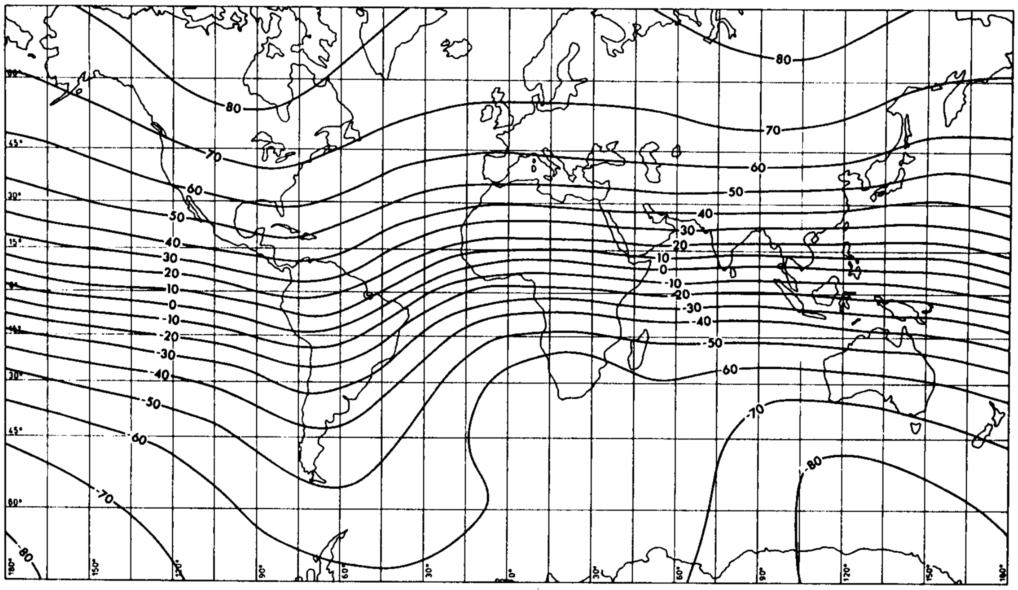


FIG. 24. Inclination (I), in degrees, based on MAGSAT data (Langel et al., 1980)

Fig.M.12: Inclination (I) in degrees based on MAGSAT data.

one day. The daily component, due to modulation of the ionosphere as the Earth rotates within the solar wind, is a 50 - 200 nT variation in total intensity. Since the daily variation is smooth with time, it is easily removed by a base station type drift correction if better than 50 nT accuracy is required. Magnetic storms, caused by variations in the magnetosphere induced by solar flares, are more serious, because they may reach amplitudes of several hundreds of nT over a few tens of minutes. It is usual for magnetic prospecting to be suspended during times of severe storm activity.

You will want to know the magnetic inclination when you are carrying out a geomagnetic survey. The most convenient way to do this is to consult the IGRF (International Geomagnetic Reference Field), a set of spherical harmonic coefficients that include not only the description of the main field at given times (epoch), but also an estimate of the secular variation (year to year changes). By running a simple program these coefficients can be combined and interpolated or extrapolated to give an estimate of the magnitude and direction of Earth's magnetic field at any time and location. (There are many calculators available on the web, one is at https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml but others may be found by searching on "IGRF calculator".)

More Basic Theory - From Monopole to Dipole.

As is usual, potential is defined as the work required to move a point charge/mass/magnetic pole from an infinite distance to the point P:

$$U_P = \int_{\infty}^{P} \mathbf{F}.d\mathbf{r}$$

and that any field **V** is obtainable from the potential U by differentiation:

$$\mathbf{V} = -\nabla U \quad .$$

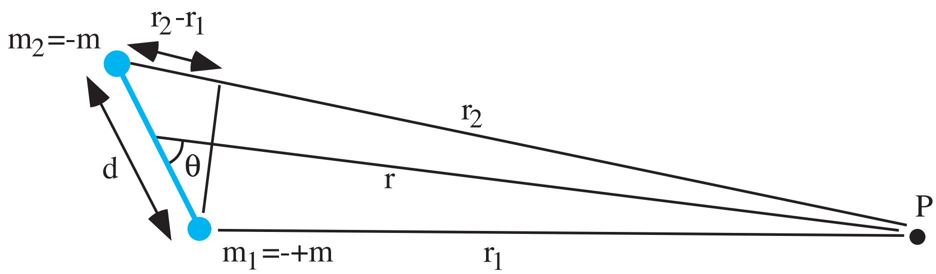


Fig.M.13: A pair of 'monopoles' forming a dipole. The field is calculated at point P.

Going back to our fiction of an isolated magnetic pole and we consider Figure 13, we have formed a dipole from two monopoles separated by a distance d. We can calculate the field at point P by summing the potential due to the two monopoles and differentiating to recover the field. The magnetic potential at P due to m_1 is found by integrating to work required to move our test pole from infinity to P:

$$\mathbf{F} = \frac{\mu_o m_1 m}{4\pi R^2}\hat{\mathbf{r}}$$

$$U_P^{(m_1=+m)} = \frac{\mu_o}{4\pi}\int_\infty^{r_1} \frac{(+m)}{R^2} dR$$

$$= \frac{\mu_o}{4\pi}\int_\infty^{r_1} \frac{(+m)}{R^2} dR$$

$$= -\frac{(+m)\mu_o}{4\pi R}\Bigg|_\infty^{r_1} = \frac{\mu_o}{4\pi}\frac{(+m)}{r_1} \quad .$$

(Note that the proper expression for the force on a unit pole for the $\mathbf{B}$ field is $\mathbf{F} = m_2\mathbf{B}$, but if we had defined an $\mathbf{H}$ field above we would have had to use $\mathbf{F} = \mu_o m_2 \mathbf{H}$, to achieve the same expression for the potential.) In a similar fashion we find that

$$U_P^{(m_2=+m)} = \frac{\mu_o}{4\pi}\frac{(-m)}{r_2}$$

Because the magnetic potential is a linear, scalar, field the total potential due to the dipole is

$$U_P = U_P^{(-m)} + U_P^{(+m)} = \frac{\mu_o}{4\pi}\left(\frac{(+m)}{r_1} + \frac{(-m)}{r_2}\right) = \frac{\mu_o m}{4\pi}\frac{(r_2 - r_1)}{r_1 r_2}.$$

But if $r >> d$, then $r_2 - r_1 \approx$ d cosθ and $r_1 r_2 \approx r^2$ (this approximation is safe because for dipoles, by definition, d is negligibly small). So now

$$U_P = \frac{\mu_o}{4\pi}\frac{md\cos\theta}{r^2} = \frac{\mu_o}{4\pi}\frac{A\cos\theta}{r^2}$$

where $A = md$ is the dipole moment and θ is the azimuth (the angle between the dipole and the direction to the observer).

We know that the magnetic field is the negative of the gradient of the potential, just as in gravity, or

$$\mathbf{B} = -\nabla U_P$$

Let us look at our dipole again:

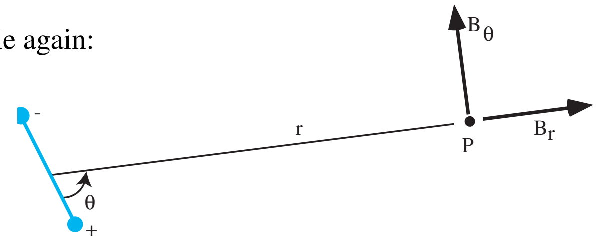


Fig.M.14a: Azimuthal ($\mathbf{B}_\theta$) and radial ($\mathbf{B}_r$) components of a dipole field.

We can define radial, B_r, and azimuthal, B_θ, components of the magnetic field $\mathbf{B}$ in spherical coordinates. Spherical coordinates are tricky, so let's go through them again. In plane polar coordinates, θ is positive anti-clockwise from the x axis towards the y axis. In spherical coordinates (Figure 14b), that is the job of ϕ, and θ is now the angle between the z axis and the actual vector. In the figure, the $\hat{e}$ are the unit vector directions. For a magnetic dipole, there is no ϕ dependence for the dipole geometry (i.e. as you rotate around the dipole axis, nothing changes except ϕ). If we take the dipole to be aligned along the z axis, with the positive pole in the positive z direction, then you can see where we get Figure 14 of the notes (below). Defined this way, the azimuth is positive anti-clockwise from the positive pole.

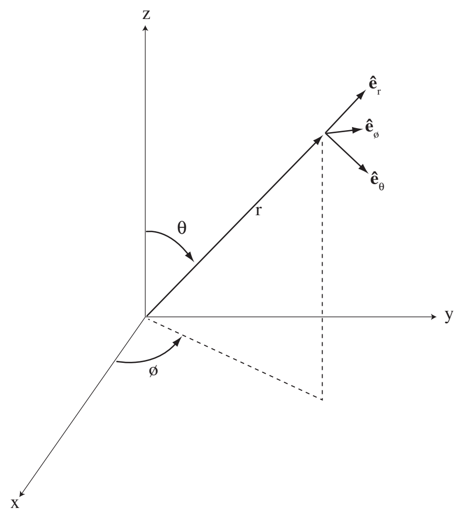
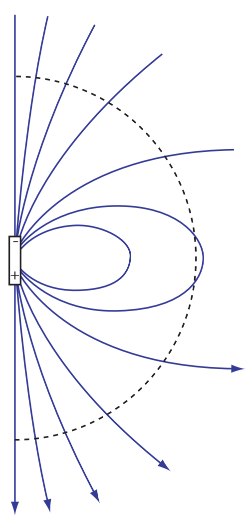


Figure 14b. Spherical coordinate system.



On Earth, B_r =Z and B_θ =H (but notice that θ is co-latitude rather than latitude). Conventionally, flux lines go from the positive pole to the negative, so if you think about this everything is right with the math; for $\theta = 0°$, along the axis in the positive direction, $\mathbf{B}_r$ is positive (outward directed field), for $\theta = 90°$, the purely $\mathbf{B}_\theta$ field is positive (but always pointing anti-parallel to the z-axis), and for $\theta = 180°$ the field is again purely radial, but since cos 180° is -1, $\mathbf{B}_r$ is now negative (pointing inward). For other θ, there are both radial and azimuthal components to the field and it looks a bit like Figure 14c. I have plotted the positive pole downwards to make the field look like Earth's field if north were upwards. That is, Earth's magnetic field dips into the earth in the northern hemisphere (i.e. the radial component is negative). North poles on compasses and bar magnets are the positive poles, and are named north because they are "north-seeking poles" (i.e. the compass needle north points north). Thus, the north geographic pole of Earth corresponds to the south magnetic pole, but nobody ever calls it that (except, I'm told, the French, who call the positive pole of a magnet the south pole).

Figure 14c. A dipole field and an imaginary Earth surface.

We can obtain expressions for B_r and B_θ by differentiating in spherical coordinates. We recall from our consideration of volume elements in spherical coordianates that the distance elements in spherical coords are $dr,\ r.d\theta,\ r.\sin\theta.d\phi$, so our differential operator, which in cartesian coords is

$$\nabla = \mathbf{x}\frac{\partial}{\partial x} + \mathbf{y}\frac{\partial}{\partial z} + \mathbf{z}\frac{\partial}{\partial z}$$

in spherical coords becomes

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \boldsymbol{\theta}\frac{\partial}{r\partial\theta} + \boldsymbol{\phi}\frac{\partial}{r.\sin\theta.\partial\phi}$$

so

$$\begin{aligned}\mathbf{B} &= -\frac{\mu_o A}{4\pi}\left[\frac{-2\cos\theta}{r^3}\hat{\mathbf{r}} - \frac{\sin\theta}{r.r^2}\boldsymbol{\theta} + 0\boldsymbol{\phi}\right] \\ &= \frac{\mu_o A}{4\pi}\frac{2\cos\theta}{r^3}\hat{\mathbf{r}} + \frac{\mu_o A}{4\pi}\frac{\sin\theta}{r^3}\boldsymbol{\theta}\end{aligned}$$

Consider the extreme cases of $\theta = 0°$ (polar field, in the radial direction only) and $\theta = 90°$ (equatorial field, in the azimuthal direction only):

$$\begin{aligned}\theta = 0° \qquad &\mathbf{B} = \frac{\mu_o}{4\pi}\frac{2A}{r^3}\hat{\mathbf{r}} \quad + \quad 0\boldsymbol{\theta} \\ \theta = 90° \qquad &\mathbf{B} = \quad 0\hat{\mathbf{r}} \quad + \frac{\mu_o}{4\pi}\frac{A}{r^3}\boldsymbol{\theta}\end{aligned}$$

We see that there is an r^3 dependence for the field due to a dipole in all directions, that the polar fields is twice that of the equatorial field, that the polar field is purely radial and the equatorial field is purely horizontal. This is a general result that holds for all dipoles, as well as Earth's magnetic field.

The dipole moment A has units of Am^2, that is field (A/m) times volume V. Thus, if one knows the susceptibility k of the material, the dipole moment of a small body of volume V is $A = kHV$ where H is the inducing (Earth's) field, or $A = kBV/\mu_o$ in the more familiar induction units. However, it is important to remember that the pattern of a magnetic dipole anomaly depends on the geomagnetic latitude, as the diagrams of total field anomalies in Figure 15a, showing both profiles and contours, indicate.

The field in the northern hemisphere points down (and the inclination is defined as positive down), the induced field at mid-latitudes will look something like Figure 15b (this is representative of the beach anomaly in the field work).

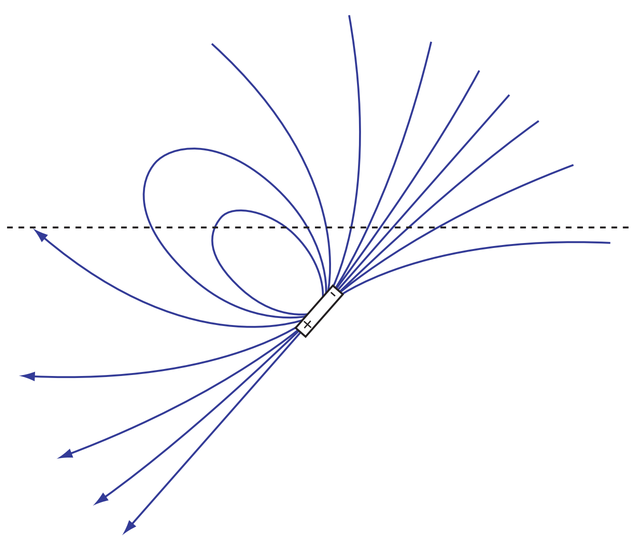


Figure 15b. Induced field in northern hemisphere mid-latitudes, with imaginary earth surface.

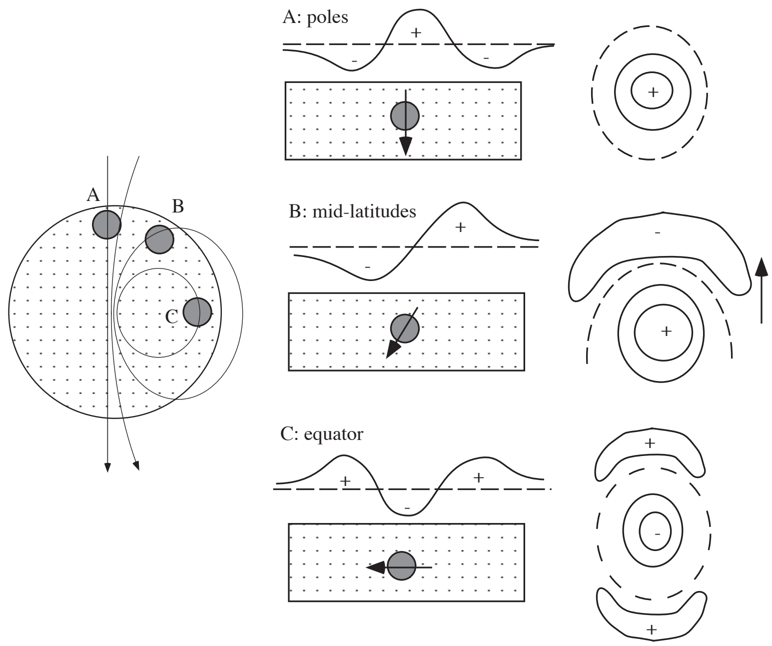


Fig.M.15a: The effect of a susceptible body on Earth's magnetic field depends on latitude. These diagrams illustrate the behavior of the total field anomaly.

When you come to write the Matlab script to analyze the beach anomaly, you need to compute r and the direction of r for all x. The diagram in Figure 15c might help you. Note that ϕ here is not necessarily the azimuth. Think about it. Once you have computed the anomaly field, it needs to be added vectorially to the main field (which does not change direction over this small distance) by computing the components of the main field in the radial and azimuthal directions. When writing matlab scripts, work up slowly. Compute r first and plot it - make sure it looks right. Compute your angles and plot them – does it look right? Think about where they should be zero, ninety°, 180°.

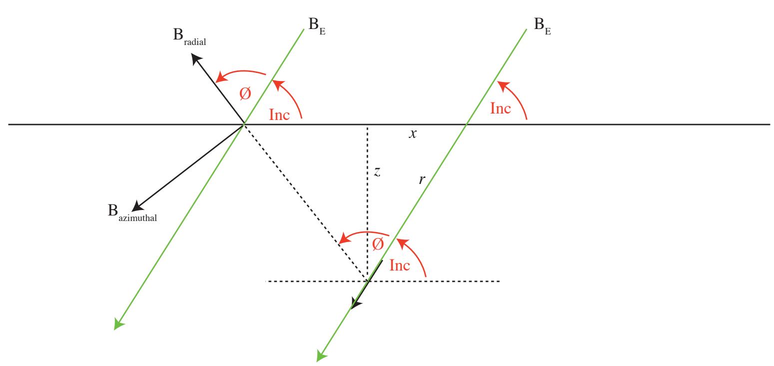


Figure 15c. Computing r and the direction of r for all x.

Reduction to the pole.

Because magnetic anomalies at mid-latitudes are somewhat complicated, and the maximum does not necessarily lie above the body creating the anomaly, people have developed (complicated) ways to transform magnetic anomaly maps into what they would look like if they were measured at the poles. The mathematics involve Fourier transformations and filtering, and is very complicated, but there are computer codes available that will do this sort of thing. The method is called *Reduction to The Pole,* or RTP.

Magnetometers.

Magnetic field sensors fall into two broad categories; total field magnetometers and vector magnetometers. Total field proton and nuclear precession devices are used for magnetic mapping, and are used extensively to calibrate measurements at observatories and in satellites. For EM induction vector measurements are needed, either using induction coils or fluxgate sensors. Superconducting (SQUID) magnetometers have been used for induction studies, but so rarely they will not be discussed here.

Torsion magnetometers

Prior to WW2 most magnetic measurements were made with *variometers*, devices which are essentially small magnets suspended on torsion fibers and free to rotate (Figure 16). A mirrored magnet is freely suspended by a quartz fiber and is allowed to rotate in Earth's field. A magnet of moment **M** in a field **H** will experience a torque **L** such that

$$\mathbf{L} = \mathbf{M} \times \mathbf{H}$$

and so a freely suspended magnet will align with the field component perpendicular to its axis of suspension, or at least to the point where the restoring torque of the suspension is balanced by **L**. If we consider a D component magnetometer (where we can ignore the mass of the magnet), the scale value S_D, defined as the angular change in the declination (in minutes of arc) which produces a deflection of one millimeter on the photographic recording drum is given by

$$S_D = \frac{G}{M}\frac{3438}{2R}$$

where R is the baseline of the optical lever in millimeters, G is the torsion coefficient, and M is the magnet moment. It took me a while to work out where 3438 came from – it is the number of arc-seconds in a radian; $180 \times 60/\pi$).

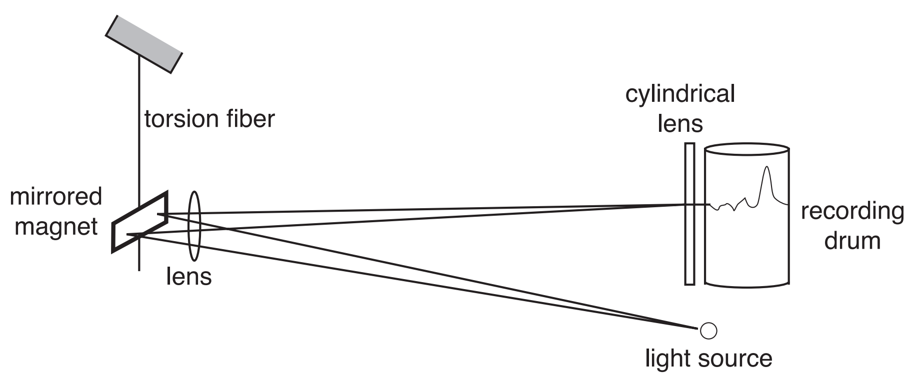


Fig.M.16: Schematic of a torsion fiber magnetometer.

To make R large (and S_D small), these instruments take up whole (darkened) rooms at an observatory. There are complications caused by misalignment and stray magnetic fields which we have ignored, and the whole thing needs to be calibrated using Helmholtz coils (pairs of large diameter, identical coils of radius r spaced r apart, which maximizes the uniformity of the enclosed field by making d^2B/dx^2 equal to zero at the center). Both G and M will depend on temperature, and so the rooms are thermostatically held to within about a degree and perhaps including more complicated systems of external magnets on bimetallic strips etc. Timing is carried out by briefly lighting an external timing lamp or deflecting the magnet with a coil.

Freely suspended magnets cannot be used in field equipment, but by running a tensioned fiber between two supports one can make a field instrument based on this principle. The most famous land version of this was the *Gough-Reitzel* magnetometer, which contained three axes of suspension in a 2 m by 0.25 m cylinder, which could be buried entirely to maintain a constant temperature (think about it...). 'Data' were recorded on 35 mm film. They could run for about 2 months off one car battery.

Most people stopped using torsion fiber variometers in the late 1990's. However, some third world countries continued to use them for several more years and some of the more remote Chinese observatories may have been using torsion fiber variometers up until a few years ago. "I would not be surprised if there are a few countries, such as India or China, that might still be operating one as a back up." (Bill Worthington, personal communication.)

Total field nuclear precession magnetometers

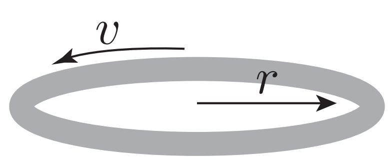
The *proton precession magnetometer* is based on the *Larmor precession* of a proton in an external magnetic field. The protons of a bottle of water, kerosene, benzene, or other suitable fluid are polarized by strong magnetic field provided by a coil wound about the bottle, aligning all the protons at a high angle to Earth's magnetic field. When the polarizing field is removed, the protons are pulled back towards the direction of Earth's field, but because of their spin angular momentum precess around the direction of $\mathbf{B}_{\text{Earth}}$ as they relax over the period of a few seconds. The precession frequency, known as the *Larmor frequency*, is given by

$$\omega = \gamma_p B \quad \text{or} \quad f = \frac{\gamma_p}{2\pi} B$$

or the product of the *gyromagnetic ratio* of the proton, γ_p, and the magnitude of a magnetic field $B = |\mathbf{B}|$. Thus, by measuring the precession frequency with a pickup coil also wound around the bottle (which may be, in fact, the excitation coil gated by a relay) B_{Earth} can be estimated from

$$B_{\text{Earth}} = 2\pi f / \gamma_p$$

The gyromagnetic ratio of a particle, γ, is the ratio of its magnetic moment to spin angular momentum. For the proton, $\gamma_p = 2.675 \times 10^8$ radians $\mathrm{T}^{-1}\mathrm{s}^{-1}$. We can derive a back of the envelope calculation for γ_p by considering a spinning proton to be a small (fictional) circuit of electric current with velocity v with radius r:



Circuit current is just a flow of one Coulomb per second, so we need to distribute the charge on the proton q along the length of the circuit $2\pi r$ and multiply this by velocity v to get

$$I = \frac{qv}{2\pi r}$$

We know that the magnetic dipole moment of a single loop is current times area A, or

$$DM = IA = \frac{qv}{2\pi r}\pi r^2$$

If we cancel the πr and multiply by m/m where m is the mass of the proton, we have

$$DM = \frac{q}{2m} mvr$$

Now the second term mvr is just the angular momentum, so γ, the ratio of dipole moment to spin angular momentum, is

$$\gamma = \frac{DM}{mvr} = \frac{q}{2m}$$

The charge on the proton is 1.602×10^{-19} C and the mass is 1.672×10^{-27} kg, so $\gamma_p = 4.79 \times 10^7$ C/kg (a C/kg is the same as $T^{-1}s^{-1}$). The difference between this estimate and the actual value, 5.58, is called the *g-factor* . For the electron, the g-factor is -2.002. Dirac predicted it to be -2 – the difference (and the value for the proton) is due to quantum electrodynamic effects which I don't understand.

Getting back to magnetometers, the sensitivity of a proton precession instrument is $dB/df = 2\pi/\gamma_p$ = 23.4859 nT/Hz. For a 45,000 nT field the frequency is around 2 kHz. It is accurate to about 1 nT, implying precession frequencies are measured to about 1/20 Hz. This is done by timing a certain number of oscillations using a highly accurate crystal oscillator running at considerably higher frequency than the Lamor precession, or by using a phase lock loop to multiply the precession frequency and count that directly.

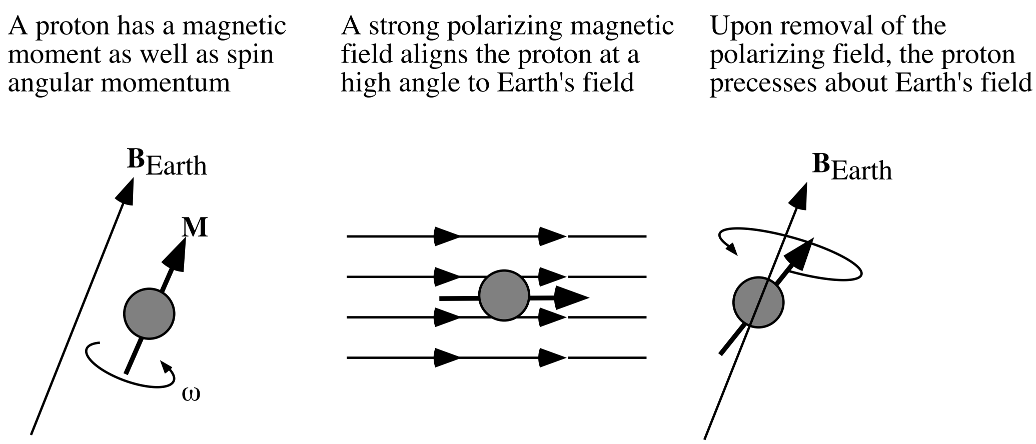


Fig.M.17: The operation of a proton precession magnetometer.

Note that as long as the polarizing field is at a reasonably high angle to the Earth's field (greater than about 10°), the orientation of the sensor is not important and the instrument measures the total field without regard to direction. The disadvantages of the proton precession instrument are that it takes several seconds to make a measurement and the instrument does not operate properly in regions of high magnetic gradients (because the protons will not only precess, but also translate, in a gradient, and the bottle of water will not be oscillating at a pure frequency). Because the proton precession magnetometer is only able to measure total field, it is convenient for airborne and marine use, but this aspect of its operation can be a disadvantage on land. It is, however, a very convenient, stable and accurate instrument to use, and for this reason have been employed extensively in observatories and in satellites.

Optically pumped magnetometers (or alkali vapour magnetometers) obtain higher sampling rates and better precision than the PPM by using the gyromagnetic ratio of the electron $\gamma_e = 1.761 \times 10^{11}$ radians $T^{-1}s^{-1}$, which is three orders of magnitude bigger than proton, mainly as a result of the much smaller mass. The ground-state energy level of the unpaired valence electron in alkali elements (sodium, rubidium, cesium, etc.) splits into two energy levels in an external magnetic field, B (Figure 18). This *Zeeman* effect occurs because the magnetic moment of the electron, and hence its spin axis, can either be parallel or anti-parallel to the applied magnetic field. The electrons in the lower energy level have spin +1/2 and dipole aligned with the field, while the upper level has spin -1/2 electrons. The difference in energy levels, ΔE, is

$$\Delta E = \hbar \gamma_e B$$

where as before γ is the gyromagnetic ratio and $\hbar = h/2\pi$, where h is Planck's constant. The spin -1/2 energy level can be depopulated by passing polarized light filtered to just the right frequency through alkali gas in a cell; electrons elevated to a high energy level fall back into either ground state but those falling into the -1/2 state are immediately elevated into the higher level again. (The polarization means that only one spin state is selected.) While the electrons are absorbing light energy the gas appears opaque; once the -1/2 state is empty the light passes through the vapour without being absorbed. This difference in transparency can be monitored electronically using a photocell. Light of the correct frequency is produced by using a lamp made from the same alkali as in the gas cell, made from the same alkali as the gas chamber plus appropriate filters.

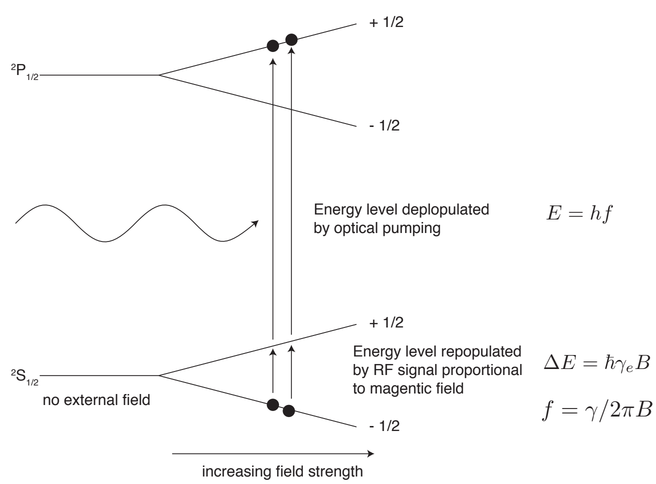


Fig.M.18: Zeeman splitting of the energy level of the valence electron & optical pumping.

The -1/2 state can be repopulated by applying just the right radio frequency (RF) signal, f to a coil wrapped around the gas cell. The photons of the RF signal have energy $E = \hbar\omega = hf$, thus

$$f = \gamma_e/2\pi B \quad .$$

This is, once again, the Larmor frequency, of the system. If the radio frequency is swept through Larmor frequency, the light will go opaque at just the right frequency. This would be an effective, but cumbersome, magnetometer.

Instead, most modern magnetometers are operated as self-oscillating systems using a feedback circuit (Figure 19). For a narrow band of frequencies about the Larmor frequency the phase of the electron precession locks onto that of the radio frequency excitation frequency, and the light level fluctuates at the Larmor frequency. It is this property allows the instrument to be operated as a feed-back device. The fluctuating light level is measured by a photo cell, amplified, and fed-back to set the RF frequency of the coil (with a 90 degree

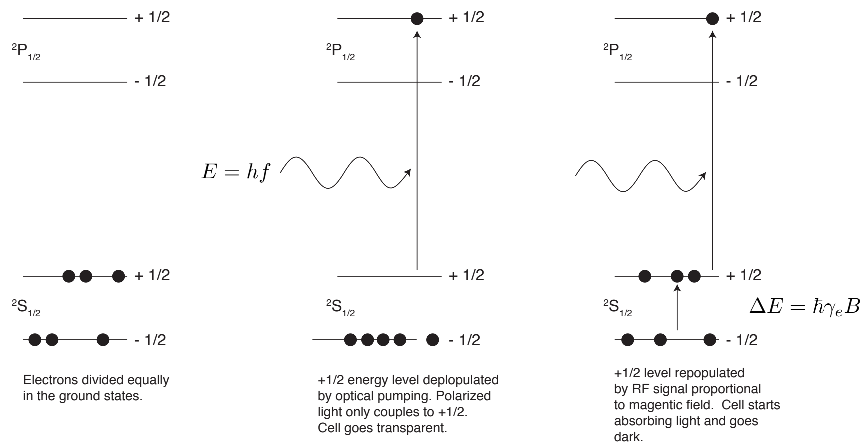


Fig.M.18: Zeeman splitting of the energy level of the valence electron & optical pumping.

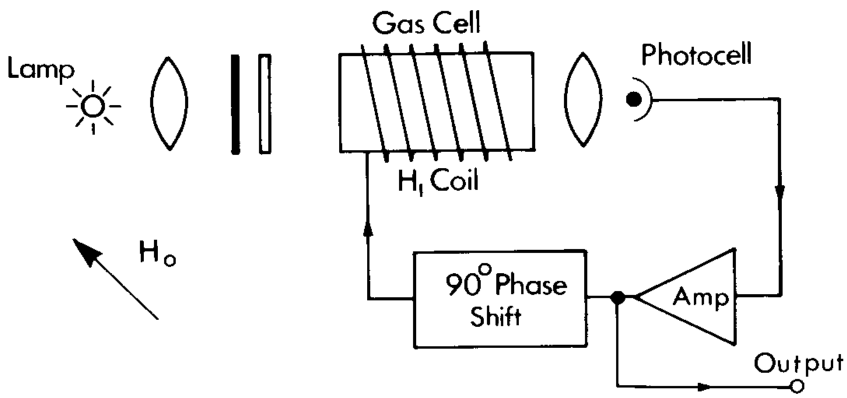


Fig.M.19: Schematic of a self-oscillating alkali vapour magnetometer (from Forbes, 1987).

phase shift to keep it synchronized). In this way the operating frequency is made to track the changes in the Earth's field, and an alkali vapor magnetometer is capable of making many measurements a second.

The sensitivity of an alkali vapor magnet, $2\pi/\gamma$, depends on the alkali. However since they all depend on the precession of electrons rather than the heavier proton, the sensitivity is much better than for a proton precession magnetometer. For a cesium vapor magnetometer, the sensitivity is 0.29 nT/Hz, approximately 100 times better than the proton precession magnetometer. The accuracy is similarly improved from $\sim$1 nT to $\sim$0.01 nT. For the G-858 cesium vapor magnetometer, Geometrics states an accuracy of 0.05 nT at 10 samples/sec, improving to 0.01 nT for 1 sample/sec (http://www.geometrics.com/858-d.html).

Induction coil magnetometers.

A simple loop of wire can be used as a magnetic sensor based on Faraday's Law, in which a time-varying magnetic flux of density B through area A will induce a voltage V in a loop of wire with N turns:

$$V = -NA\frac{dB}{dt} = -NA\mu\frac{dH}{dt}$$

where H is the magnetic field perpendicular to the loop and μ is permeability, which for an air-cored loop

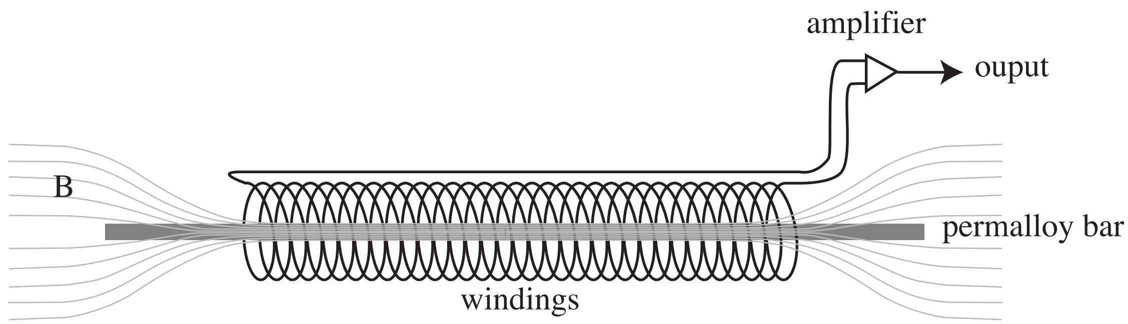


Fig.M.20: Induction coil magnetometers.

is the free-space value of $4\pi \times 10^{-7}$ H/m.

We see that V increases with frequency, but since time variations in B are in pico- to nano-teslas at the frequencies where induction coils work, unless A is huge this does not produce much of a voltage. However, large coils with multiple turns and integrating amplifiers have been used to measure micropulsation activity in Earth's field.

One can boost V by inserting a material of high relative permeability μ_r, such as mumetal or permalloy (alloys of Ni, Fe, Mo, and Mn), and thus increasing the flux through the coil. The core captures the magnetic field and multiplies the flux by an effective permeability μ_e, determined by a combination of geometry and relative permeability, producing a voltage

$$V = -NA\mu\mu_e \frac{dH}{dt}.$$

The relative permeability of mumetal alloys is of order 10^4–10^5, and it turns out that for high μ_r and long, thin cores, μ_e depends only on geometry, rather than μ_r, thus removing any temperature dependence in μ_r from the sensor (Figure 9). In effect, the core becomes a gatherer of flux rather than an amplifier of μ (Figure 10). For long, cylindrical, highly permeable cores of length L and diameter d the effective permeability depends only on geometry and is given approximately by

$$\mu_e = \frac{\left(L/d\right)^2}{(\ln(2L/d) - 1)}$$

(Tumanski, 2007).

We see that increasing length improves sensitivity, and typical broadband MT coils are of order 1 m long. For a harmonic variation in B of frequency f and amplitude B_o, the frequency response of a coil is

$$V(f) = 2\pi f a N A B_o \quad .$$

Since most of the flux is trapped in the core material, A becomes the cross sectional area of the core, rather than the area of each turn of wire. For large N, achieved by making the windings out of very fine wire, the dominant source of noise becomes the thermal resistance noise, or Johnson noise, of the wire. So for example, a 6000 Ω winding has a Johnson noise of 10^{-16} V^2/Hz or something like 10 microvolts at 1 Hz. Here we see the second reason for a long, thin, magnetometer. Piling windings upon windings increases the diameter of the turns, and so R starts to increase more rapidly than V.

A good induction coil magnetometer has a noise level of about 10^{-8} nT^2/Hz at around 1 Hz (i.e. less than a picotesla), with a red noise spectrum at lower frequencies because of the dB/dt loss of sensitivity. Long period response is limited to about 5,000 s, while high frequency response is limited by capacitive and inductive losses in the core material. This is addressed by laminating the core material and, ultimately, going to air-cored coils for the highest (low radio) frequencies. Because Earth's main field is around 40,000 nT, minute rotation of an induction coil magnetometer couples this field into the sensor. For a sub-picotesla noise floor, this corresponds to a rotation of only a nanoradian, or 1 mm in 1,000 km. For these reasons coils are buried 10 cm or so to avoid wind motion, and even so MT sites near trees or coastlines will be observed to have high noise levels.

Fluxgate magnetometer

Although integrating amplifiers or feedback can be used to flatten the frequency response of induction coils, the one-pole low-cut response of the dH/dt term is a fairly good match to the red spectrum of natural magnetic field variations. However, reasonable signal to noise performance for induction coils is still limited to periods shorter than several thousand seconds. For longer period MT and GDS studies, fluxgate magnetometers are preferred. Fluxgates use the principle that variations in magnetic flux may be generated for a fixed magnetic field by varying the permeability of the core material:

$$V = -NA\frac{dB}{dt} = -NA\mu\frac{d\mu_e}{dt}H.$$

This is achieved by winding the permeable core with an excitation coil which saturates the core in alternate directions using a sinusoidal current of around 1 kHz. A saturated core has an effective permeability of zero, since it is no longer influenced by the external field H, and so the excitation current generates the time-varying permeability required to get an inductive response.

The fluxgate magnetometer measures the total magnetic field, and so is useful for long period MT studies as well as magnetic observatory recording. On the other hand, since the main field is so large, dynamic range issues limit the total sensitivity to about 0.1 nT, which still makes it a better sensor than an induction coil for periods longer than about 10 s.

Various core geometries can be used for fluxgates. In a double-rod fluxgate magnetometer (Figure 21), two primary coils are wound around two identical cores of permeable material, and connected in series so that a current passed through the primary circuit generates magnetic fields in the two cores which oppose each other. A 50-1000 Hz primary current is of sufficient strength to saturate the cores. In the absence of an external field the magnetization in the cores is equal and opposite, so there is no net magnetic field for the two-core system. In the presence of an external field, the core being magnetized in the direction of the external field saturates sooner than the opposite core. Thus, an asymmetry develops in the magnetization of the cores, leading to a time varying net magnetization. A secondary coil wound around the entire system measures the rate of change of this magnetization as a series of induced voltage spikes. These spikes are rectified and amplified to produce a voltage signal which is proportional to the magnetic field *along the axis of the sensor cores*. Note that the fundamental harmonic of the sensing voltage is twice the fundamental harmonic of the primary current.

The most common type of geophysical sensor uses a ring-core geometry (Figure 22). The excitation coil saturates the two sides of the core symmetrically in opposite directions; the side of the coil excited in the same direction as the external magnetic field saturates sooner, while the opposite side saturates later. This asymmetry produces a time varying net flux at twice the excitation frequency, which is detected by a pickup coil wound around the core; the orientation of the detection coil determines the direction of sensitivity. A lock-in amplifier tuned to twice the excitation frequency will thus have an output proportional to the external

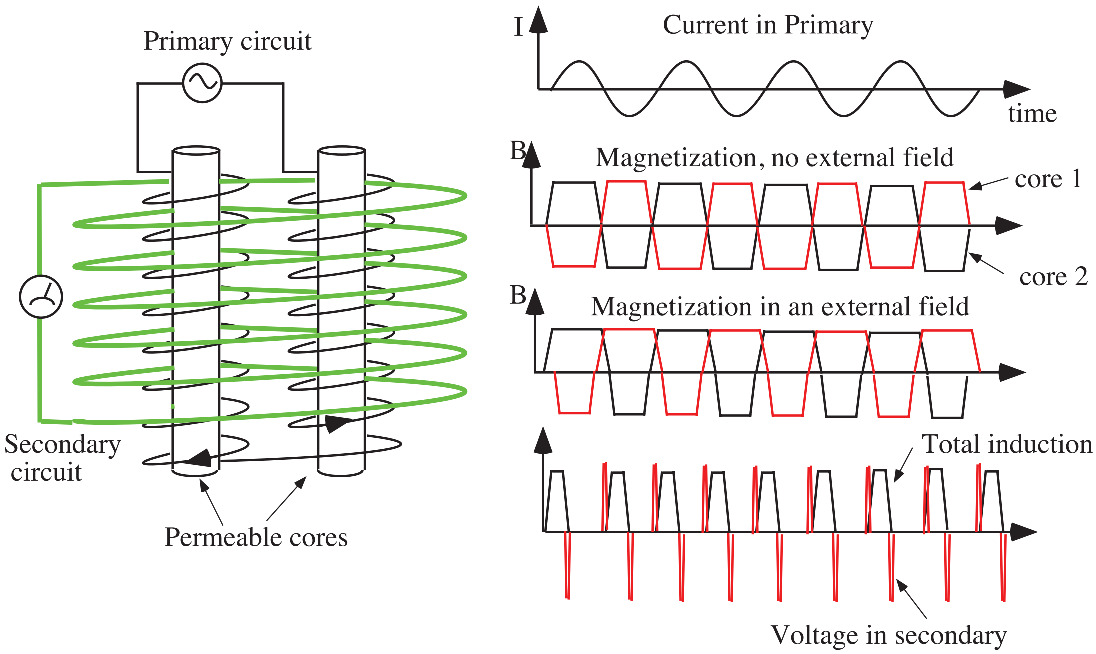


Fig.M.21: Fluxgate magnetometer.

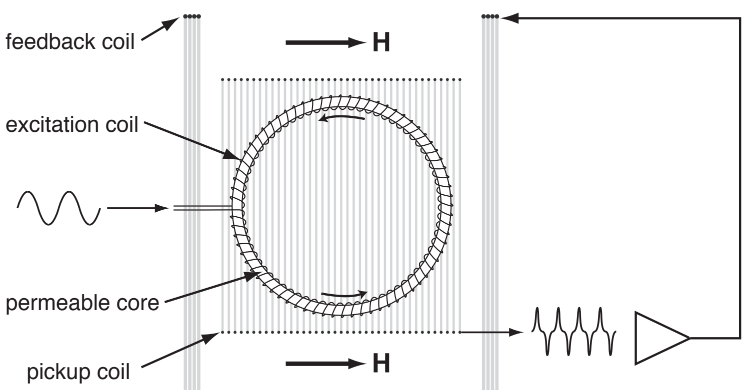


Fig.M.22: Ringcore fluxgate magnetometer. Measurement direction is indicated by the arrow.

field. However, this output is only linear over a range of several hundred nT, and also will be sensitive to variations in μ_e caused by temperature, so most observatory quality instruments are operated as null sensors. The total magnetic field (usually the component of Earth's field aligned with the sensor) is cancelled by a solenoid or Helmholtz coils. The output of the fluxgate is fed back to control the current in the nulling coils, and this current then becomes the measurement of low frequency changes in the magnetic field.

The effective permeability of ring-core of diameter L and thickness d is approximately

$$\mu_e = \frac{L}{d}$$

for large L/d; ring-core sensors are usually about 2 cm in diameter and a millimeter or so thick, with quoted noise levels of order 10^{-4} nT^2/Hz at 0.1 Hz (Primdahl, 1979). By design, fluxgates sense the static magnetic field component in a given direction, which is typically 40 μT on Earth (possibly as large as 60

μT), and so the low-frequency response of fluxgates extends to DC. Excitation frequencies are high enough that variations up to a few tens of hertz can be recorded, but long period MT instruments typically sample only at about 1 Hz, since the noise in fluxgates becomes higher than induction coils at a frequency of about 0.01 Hz. Also, making measurements of field variations in the presence of Earth's total field requires a large dynamic range, and even with 24-bit analog to digital conversion (ADC), the least count will be of order 0.01 nT, and resolutions of 0.1 nT are more typical. Least count for an induction coil sensors are of order 0.01 pT.

<u>SQUIDs</u>. One last magnetometer deserves a mention. Its principle of operation depends on the behavior of a superconducting Josephson junction, the details of which are difficult to understand and will not be explained here. It is called a SQUID, for *Superconducting Quantum Interference Device*, and is one of the most sensitive of modern instruments, accurate to about 0.00001 nT.

Because they are large, expensive, and require a supply of liquid helium for operation, they are not commonly used in exploration, but portable devices of this type are available and sometimes used for magnetotelluric prospecting.

Field operation.

<u>Aeromagnetics</u>. An aircraft equipped with a total field magnetometer can rapidly survey an area, regardless of ground acessability. Although the cost of the aicraft if relatively great, the speed of operation results in a cost per mile which is less than that for ground operation. The most critical aspect of airborne magnetometer operation is the navigation of the aircraft, whose position and height must be constantly monitored.

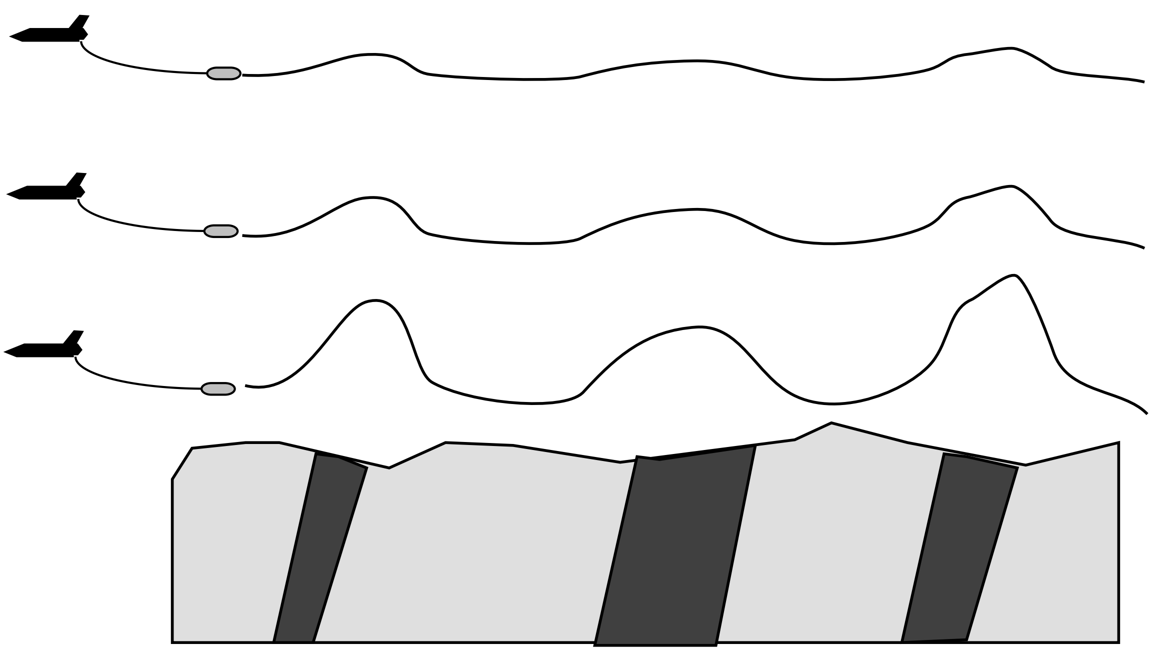


Fig.M.23: Increasing the height of an aeromagnetic survey removes high spatial frequencies from the magnetic field.

Increasing the flight altitude acts to filter the high spatial frequency components of the magnetic field (see the diagram below). The magnetometer sensor must be housed in a *bird* towed behind the aircraft or housed in a *stinger* protruding from the tail of the plane, in order to eliminate the magnetic effects of the aircraft from the measurements.

Ground surveys. When a small area must be surveyed and the cost of mobilizing a plane is prohibitive, or when a very precise survey is required, a ground magnetic survey may be conducted. Like gravity, this is a matter of taking an instrument, in this case either a fluxgate or, more likely, a proton precession device, and then occupying surveyed stations to take readings. A base station is returned to every few hours to remove the diurnal signal. It is usual with the proton precession magnetometer to mount the sensor on a rod to maintain a fixed distance of a few meters from the ground, reducing any large variations and gradients which might result from magentized soil very near the surface.

Sea surveys. It is now common practice to collect gravity and magnetic data on most geophysical ships, even if the main purpose of the operation is to collect seismic data. A proton precession magnetometer is towed behind the ship, again to reduce the magnetic effects of the towing vessel, and data collected every few seconds.

Much of the time magnetic data are interpreted qualitatively. Geological fabric and differing geological units show up nicely in large-scale aeromagnetic surveys, and for environmental work one often just wants to know where stuff is located. For small, often anthropogenic, objects, simple dipole modeling is very effective at getting location, size, and depth.

Depth rules in magnetics.

The most important quantitative interpretation is the estimation of the depth to the magnetized body. This allows the depths of sedimentary basins to be mapped by mapping the depth to igneous basement, which will usually have a much greater susceptibility than the sediments. The depth estimation problem is much more complicated for magnetics than for gravity – the simple formulas assume a vertical magnetization, which is only appropriate near the poles. However, as a reconnaissance tool in unmapped sedimentary basins the depth to magnetic sources is a powerful method. These methods are usually based on a vertical, prismatic body extending to infinite depth. This clearly is a good model for dykes intruding the basement rocks but not the sediments.