Global Electrical Conductivity and Magnetic Satellite Induction Studies

Steven Constable

Catherine Constable

Scripps Institution of Oceanography

sconstable@ucsd.edu

http://mahi.ucsd.edu/Steve/MDAT/

Contents:

- Background: Earth conductivity
- Background: EM induction
- Some satellite results

Why mantle electrical conductivity?

- Highly senstitive to phase transitions
- Sensitive to mantle temperature
- Influenced by volatiles and trace materials

Conductivity of Earth materials:

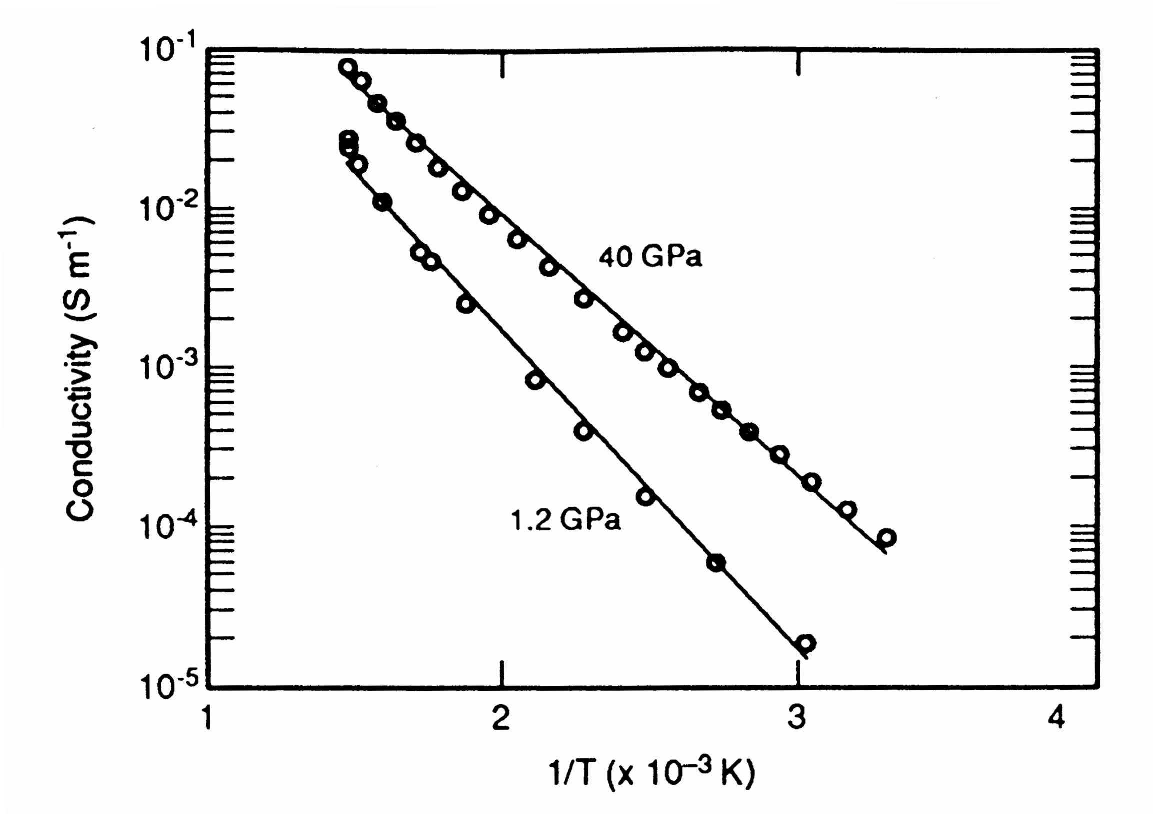
Conductivity σ has units of S/m.

Electrical resistivity ρ has units of Ωm.

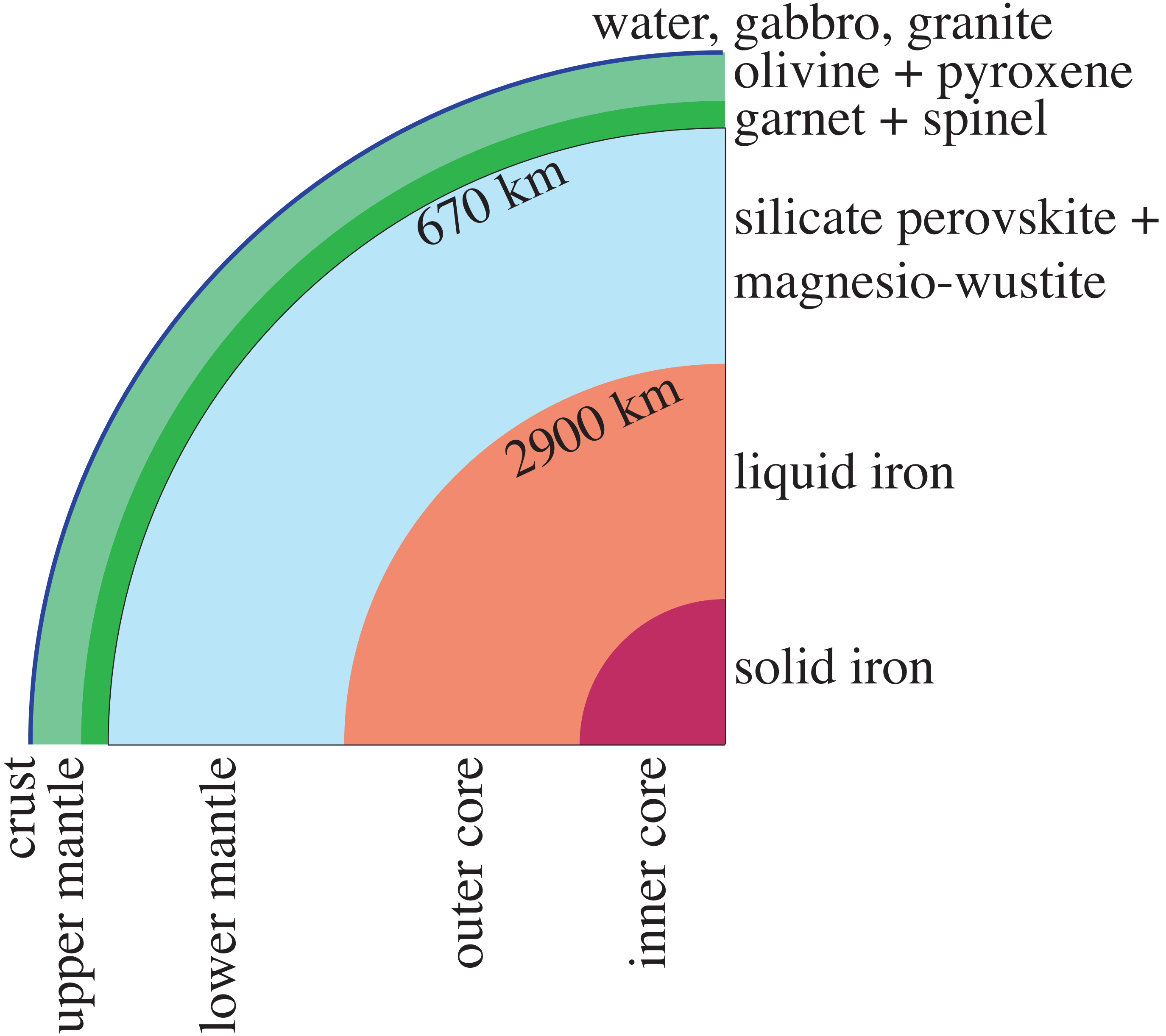
Silicate minerals are semiconductors $\sigma = \sigma_o e^{-\frac{A}{kT}}$

Shankland, Peyronneau

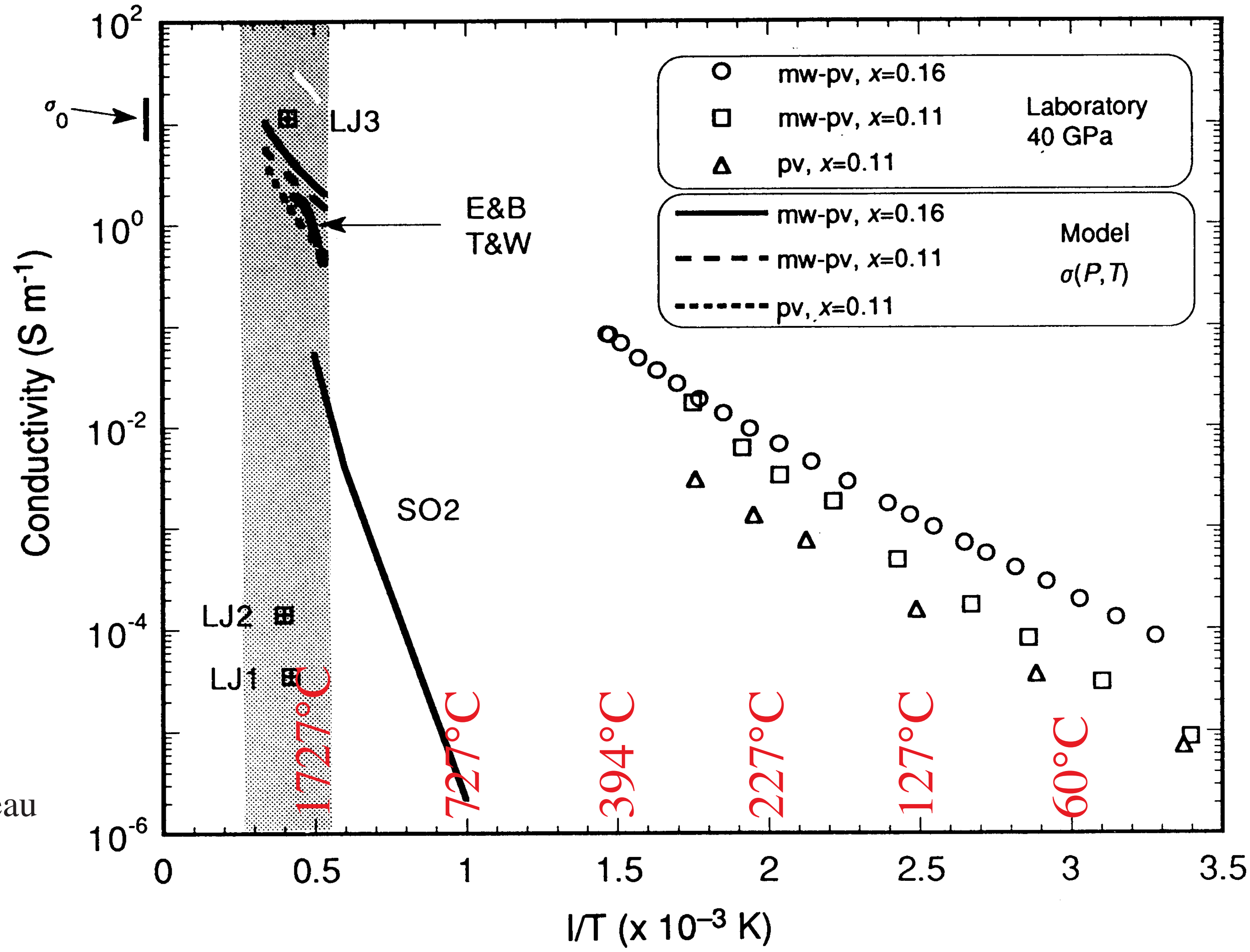
& Poirier (1993)



A simplified Earth:



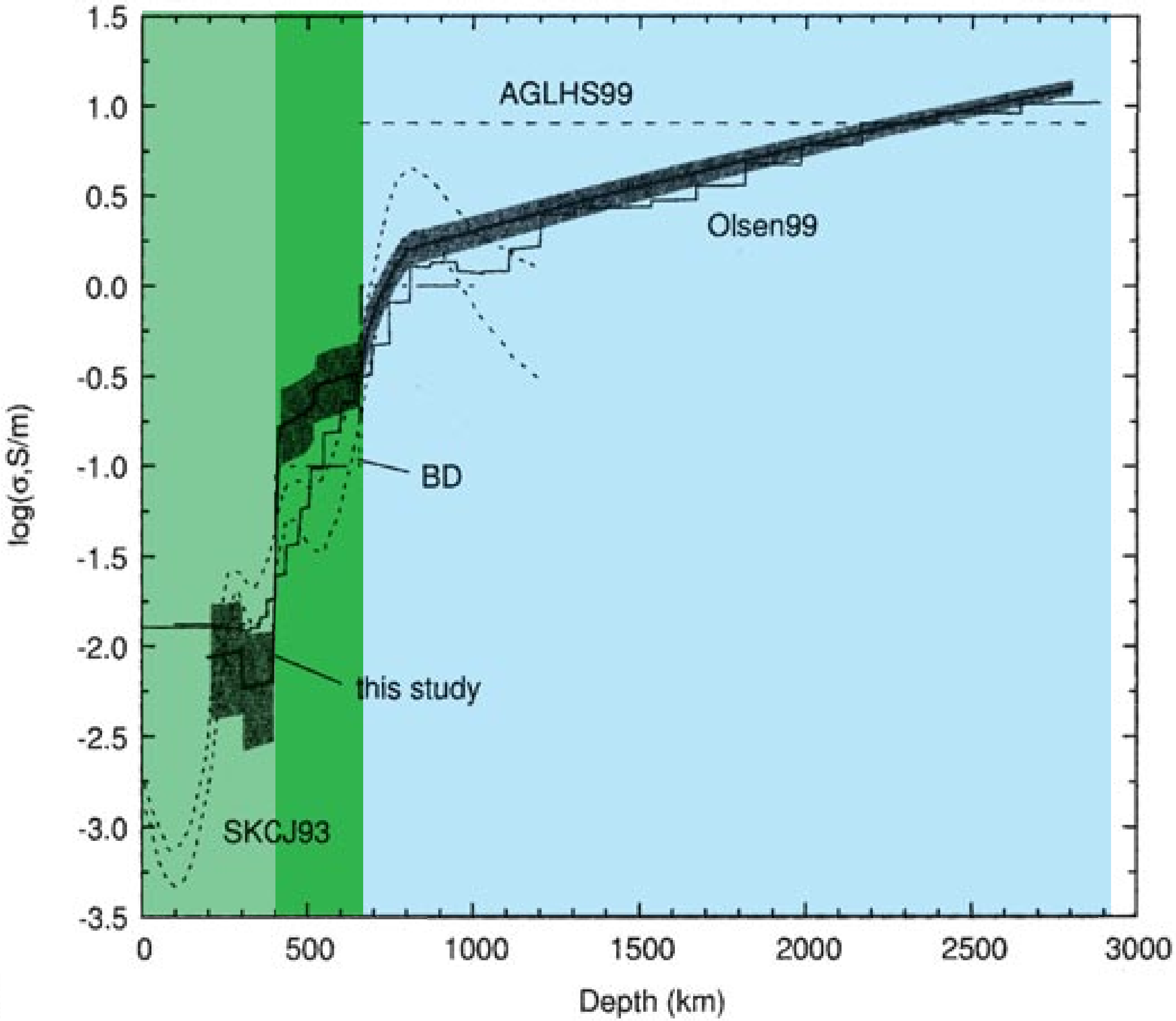
Silicate perovskite and olivine



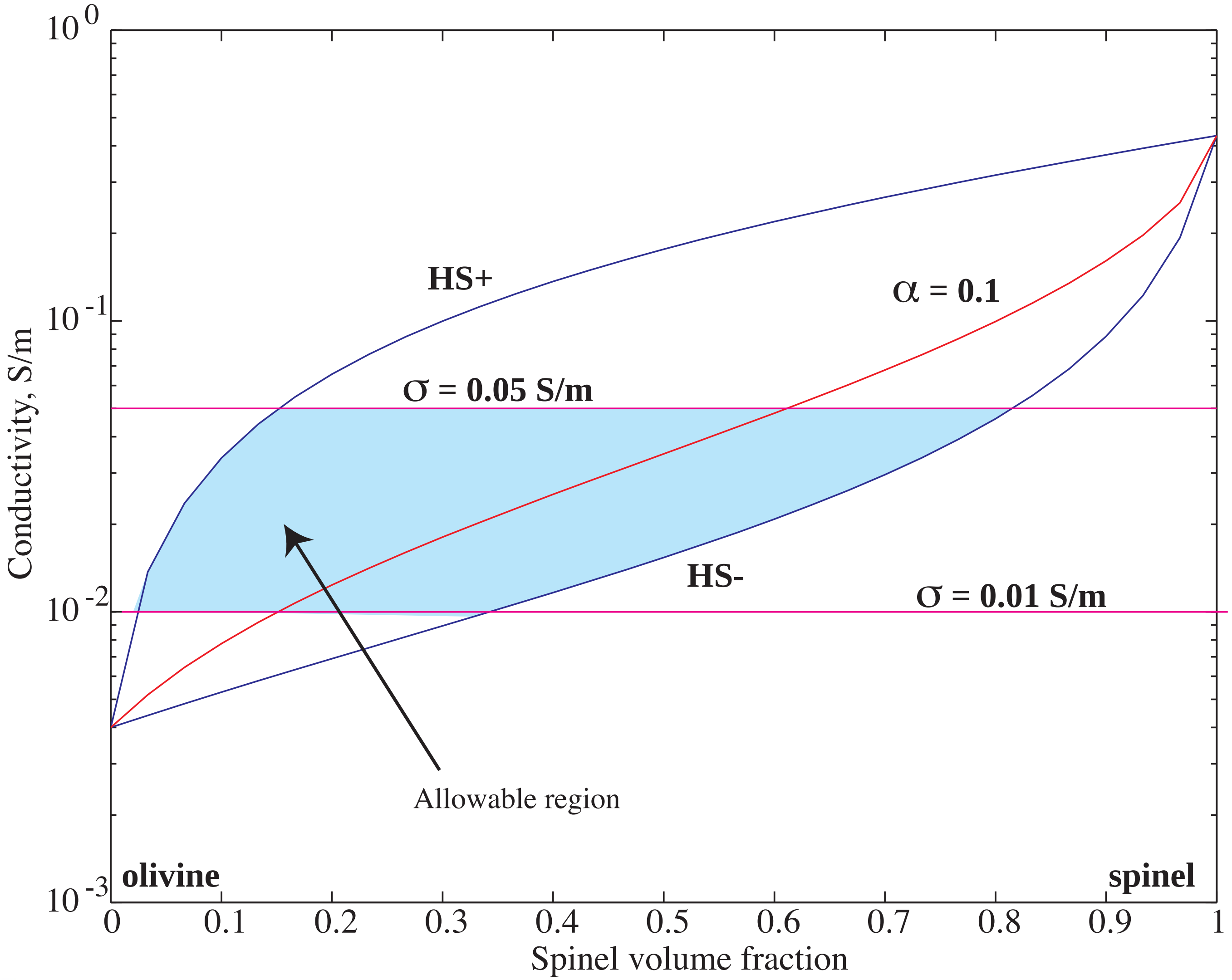
Shankland, Peyronneau & Poirier (1993)

Olivine, spinel, and perovskite:

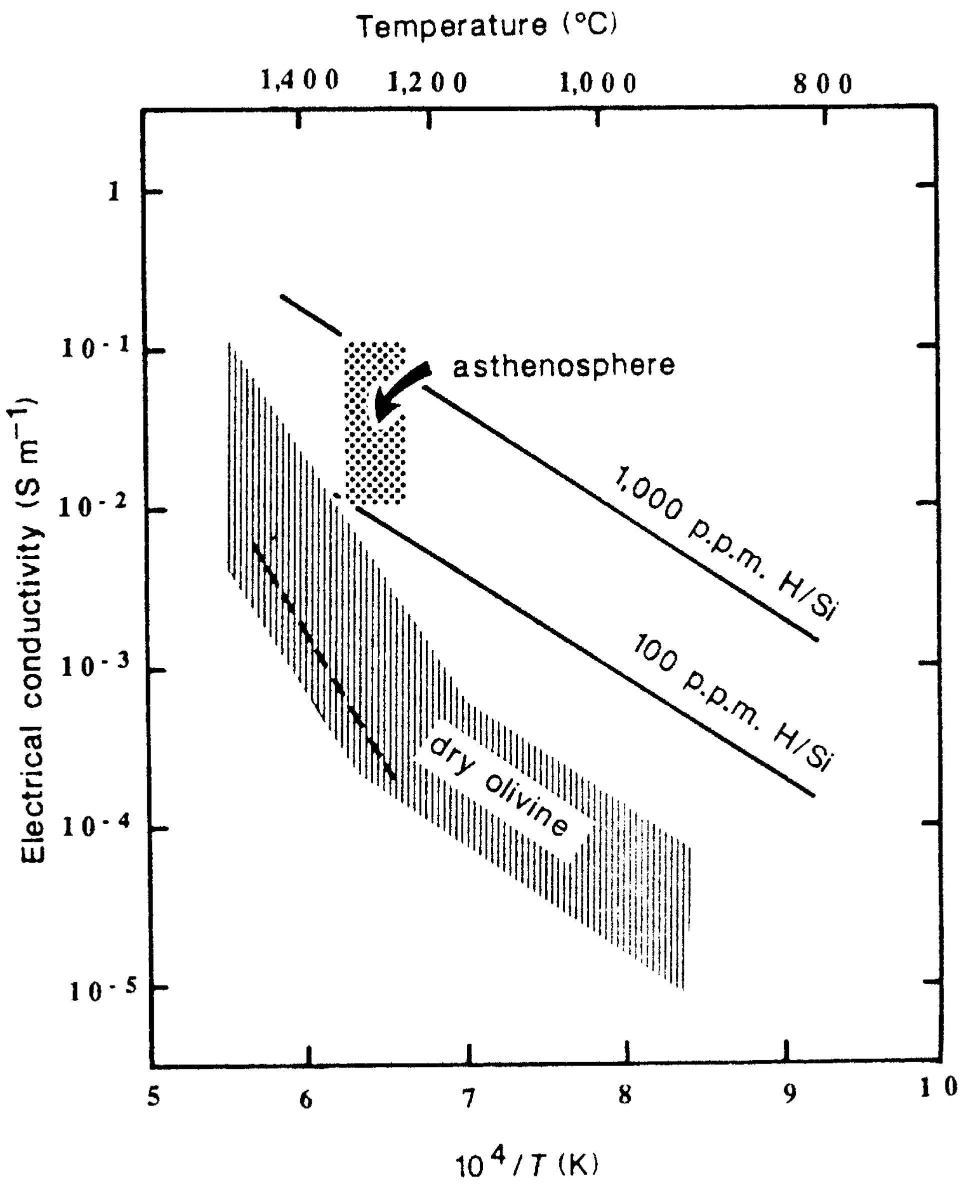
Xu et al (2003)



Mixing olivine and spinel:



Volatiles in the mantle?



Karato

Nature (1990)

Electromagnetic Induction:

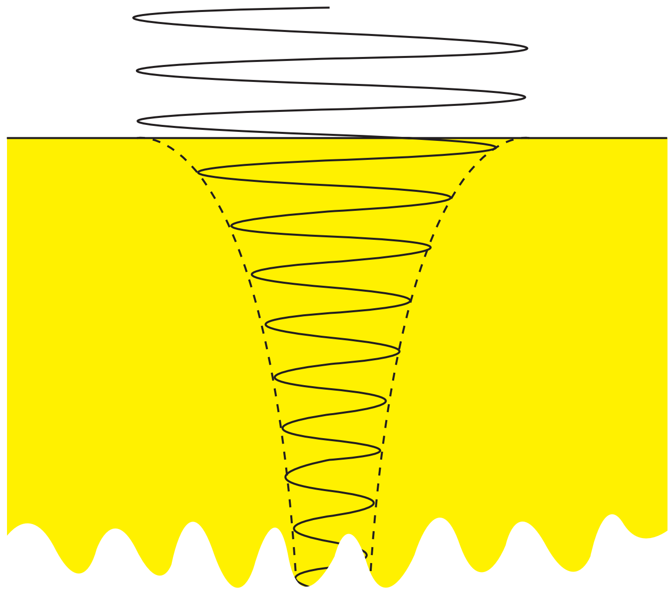
Faraday's Law states that a time-varying magnetic field induces electric currents in conductors.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Secondary magnetic fields created by these currents appose the primary magnetic field.

So, conductors attenuate magnetic fields.

Skin depth:



$$z_s = \sqrt{2/(\omega\mu\sigma)}$$

In practical units

$$z_s \approx 500 \text{ m}\sqrt{1/(\sigma f)} \approx 500 \text{ m}\sqrt{\rho T}$$

Thus 100 s period signal over a 100 Ω m earth has a skin depth of 50 km.

Skin depth can be a very useful indicator of energy penetration, but be careful!

Density, acoustic velocity, magnetization, and conductivity are the few physical properties are available for remote sensing, but resolution differs:

Radar: wave equation (same as seismics)

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

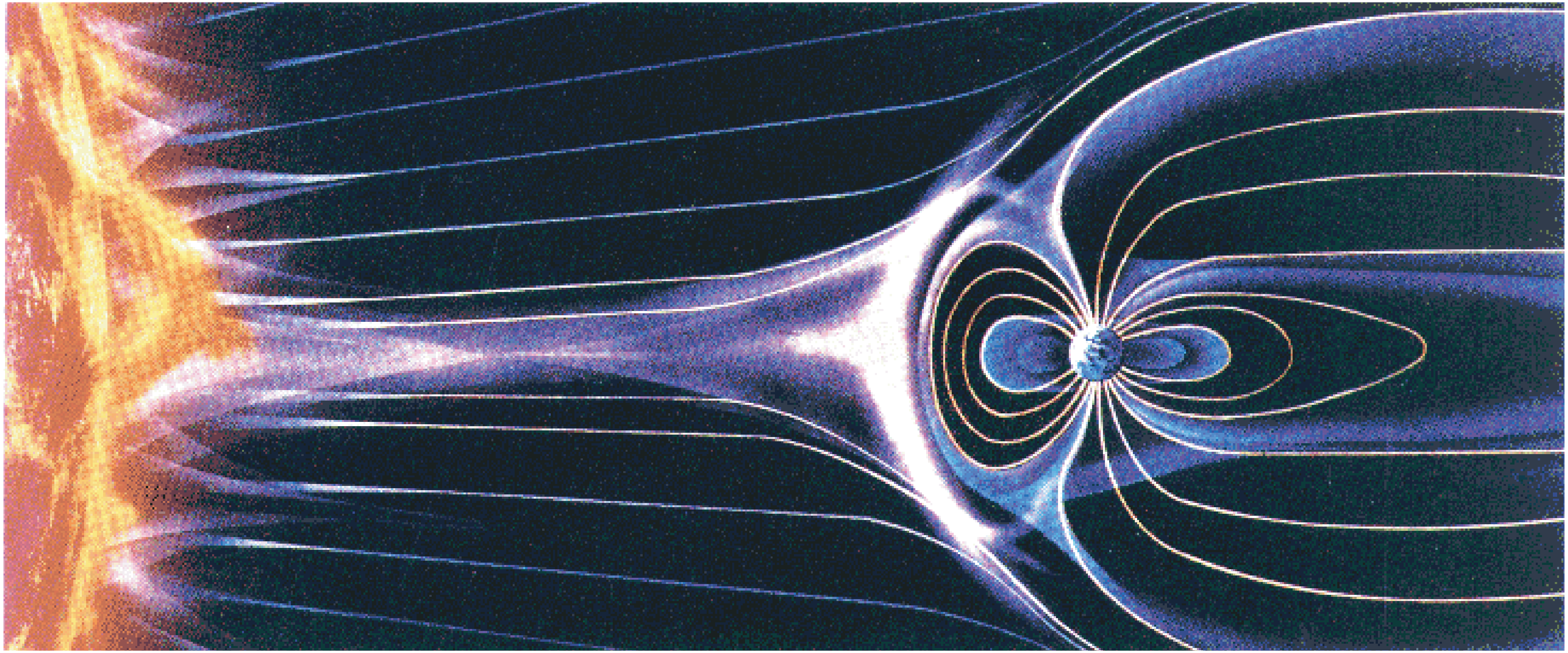
Inductive EM: diffusion equation

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t}$$

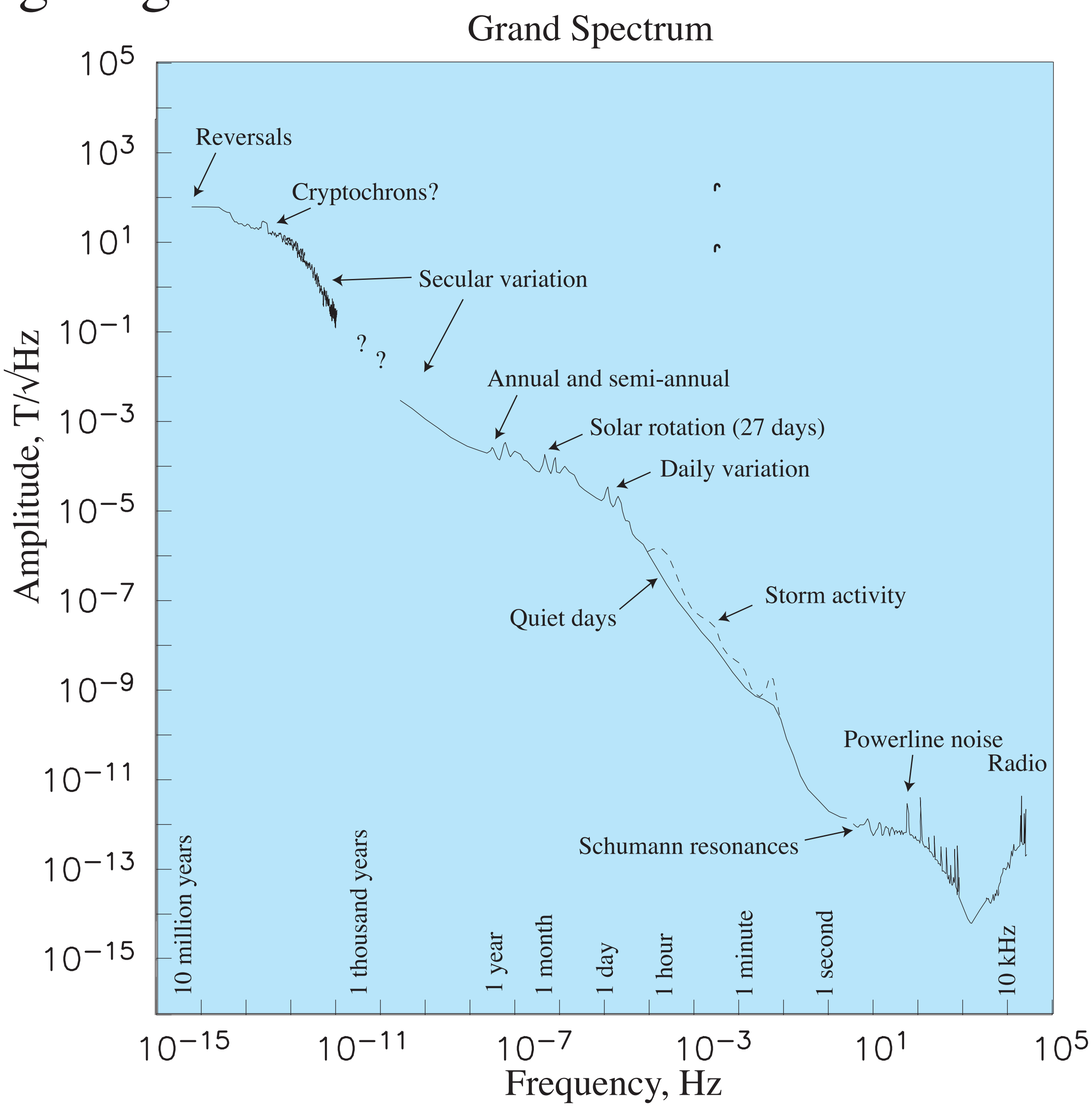
Gravity, Magnetics, Resistivity: potential fields

$$\nabla^2 \mathbf{E} = 0$$

A time-varying magnetic field:



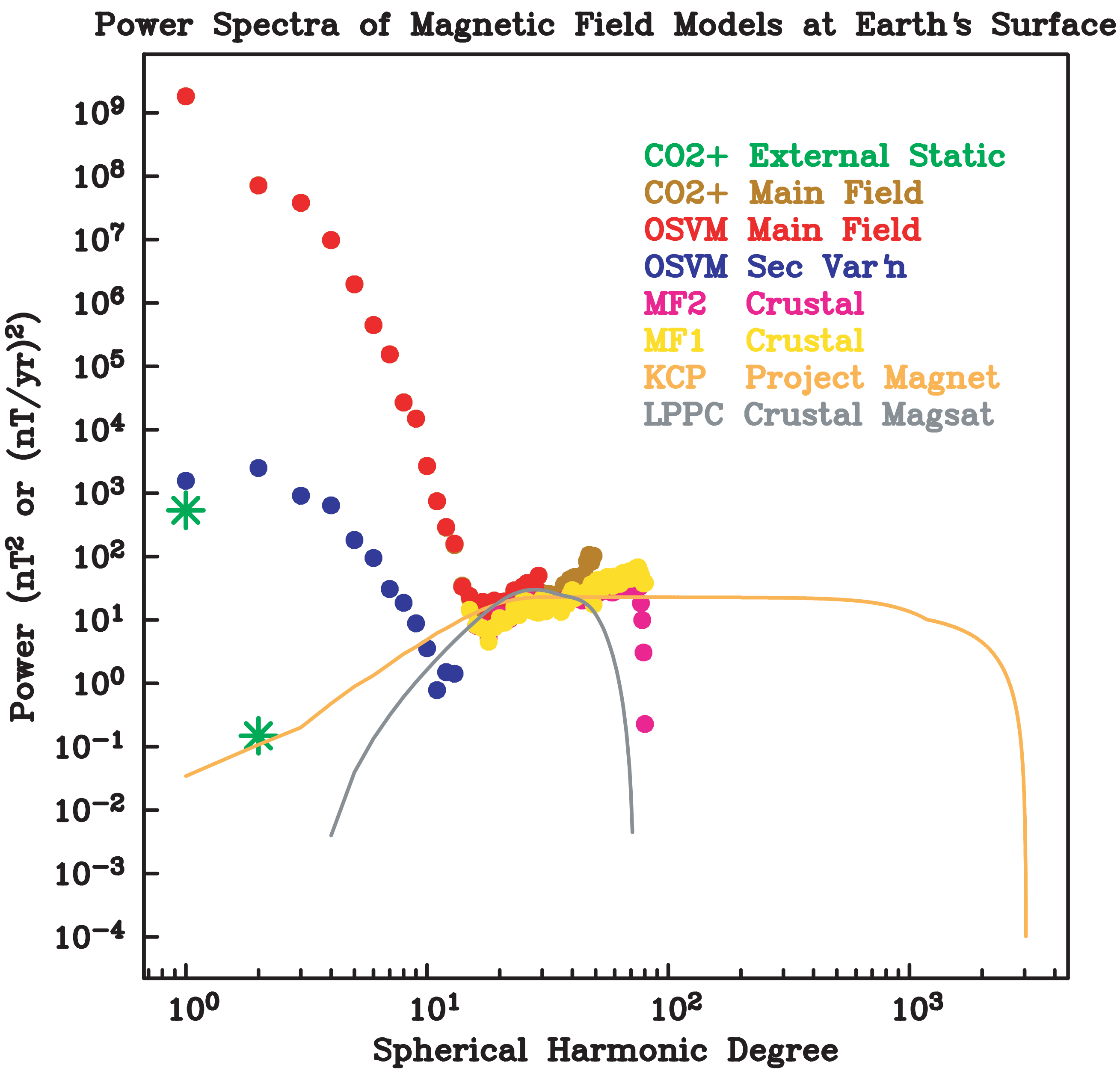
A time-varying magnetic field:



2nd CHAMP Meeting, September 2003

©S.Constable

A space-varying magnetic field:



Using induction to measure Earth conductivity:

- Magnetotelluric (MT) method

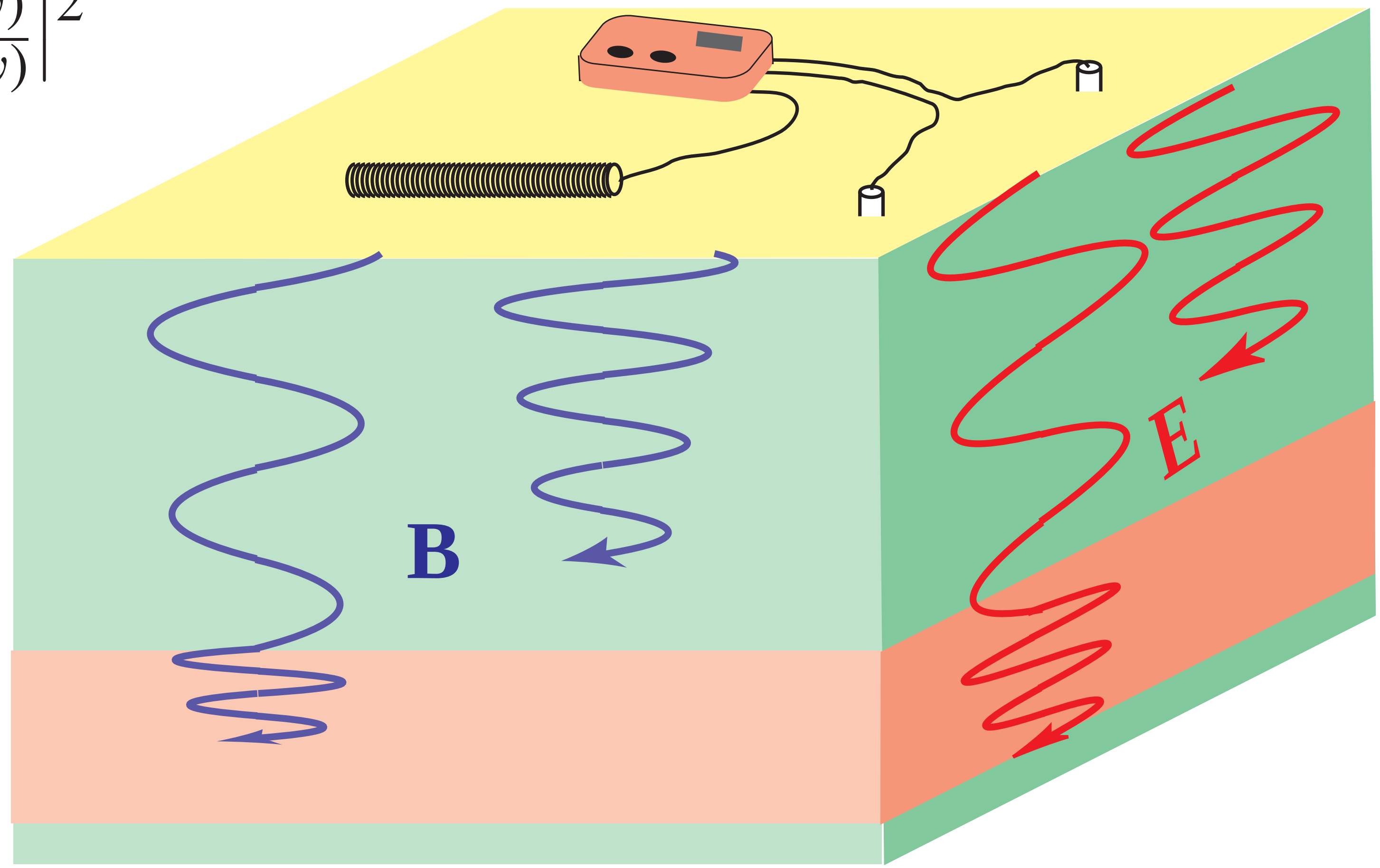
  Measure electric and magnetic fields

- Geomagnetic depth sounding (GDS) method

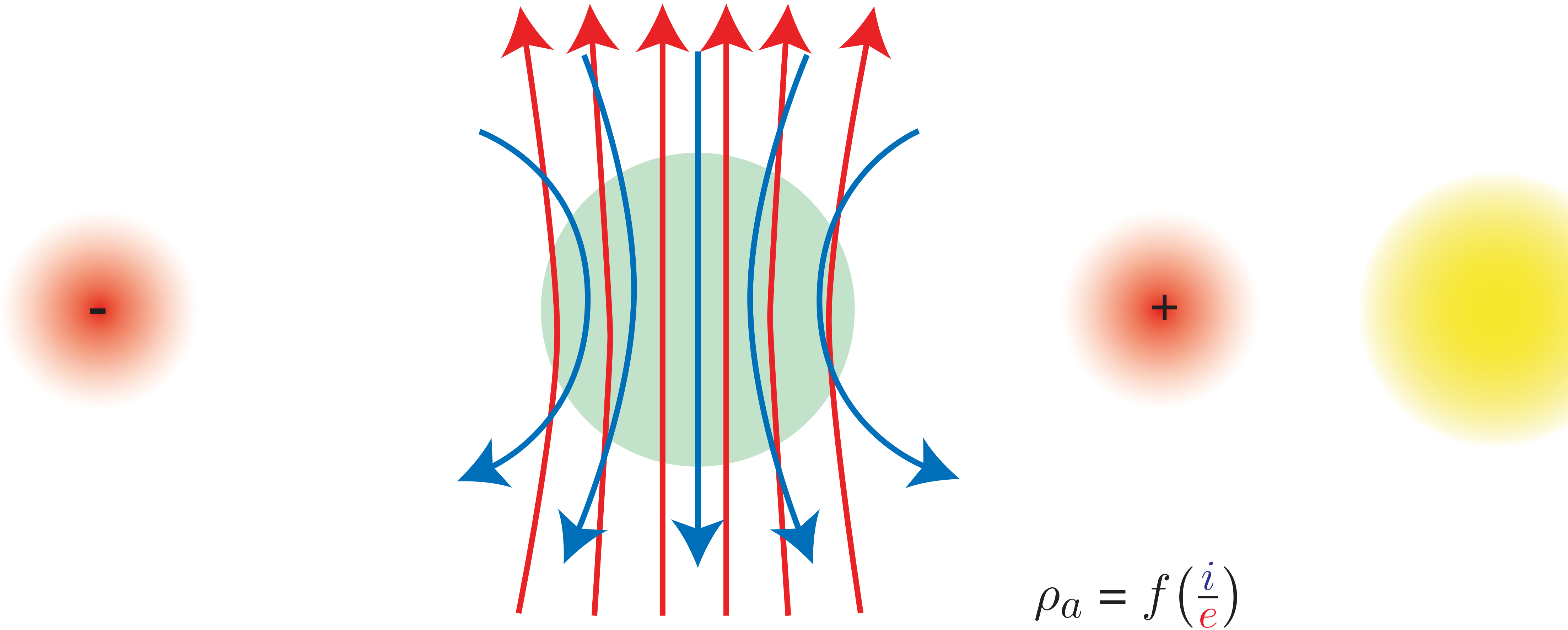
  Measure horizontal and vertical magnetic fields

Magnetelluric method:

$$\rho(\omega) = \frac{T}{2\pi\mu}\left|\frac{E_y(\omega)}{H_x(\omega)}\right|^2$$

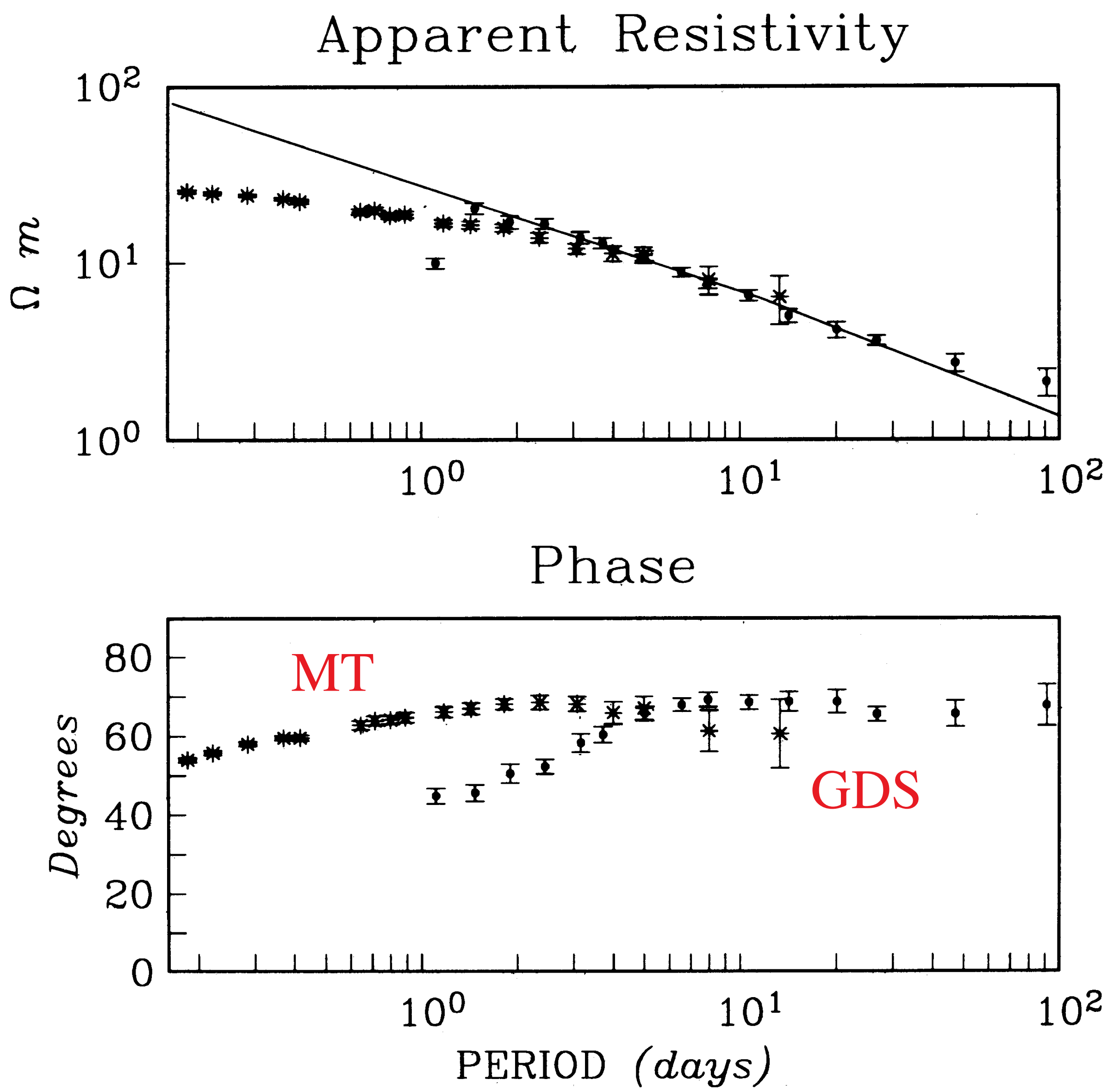


Geomagnetic depth sounding:

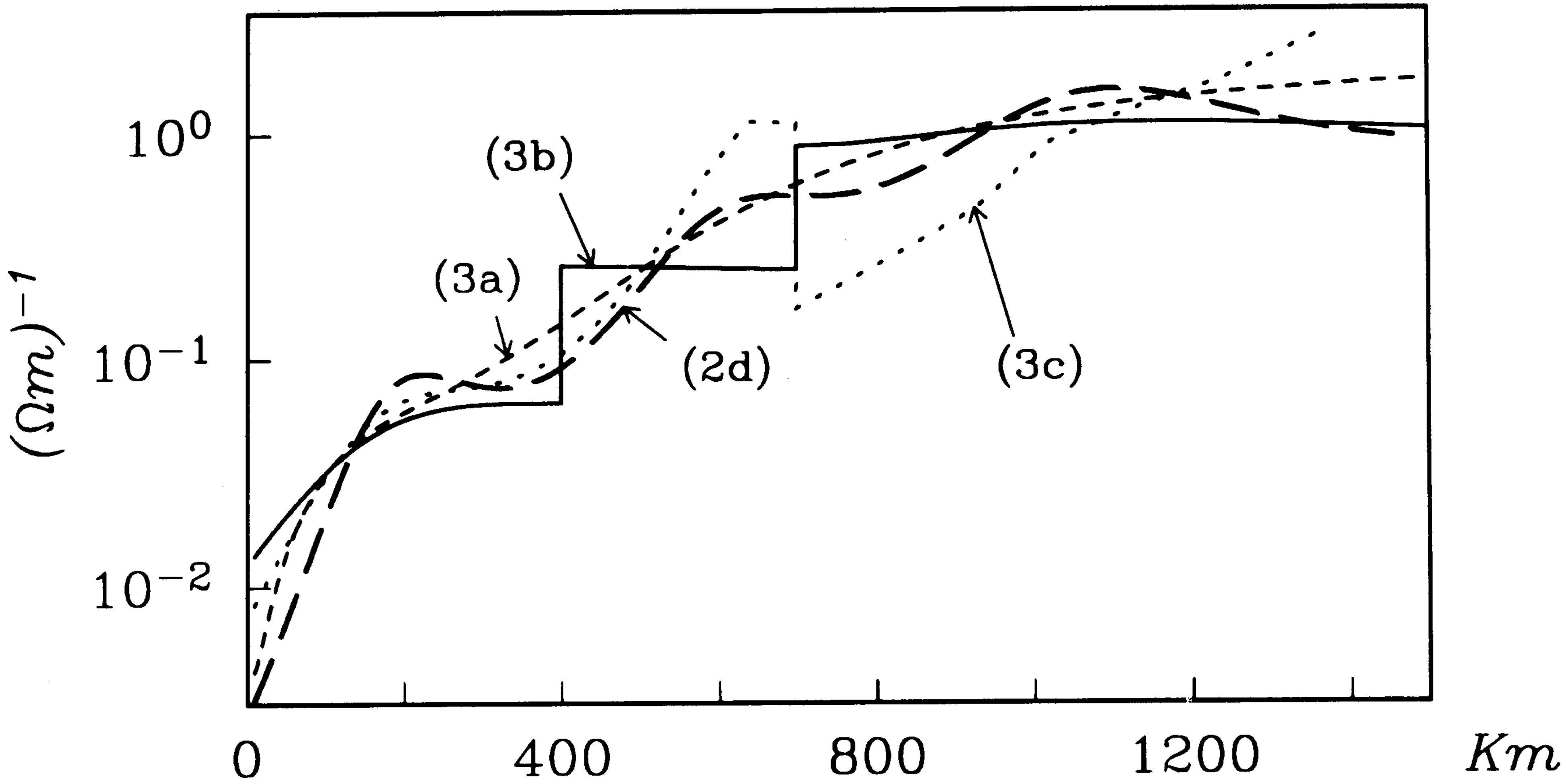


$$\rho_a = f\left(\frac{i}{e}\right)$$

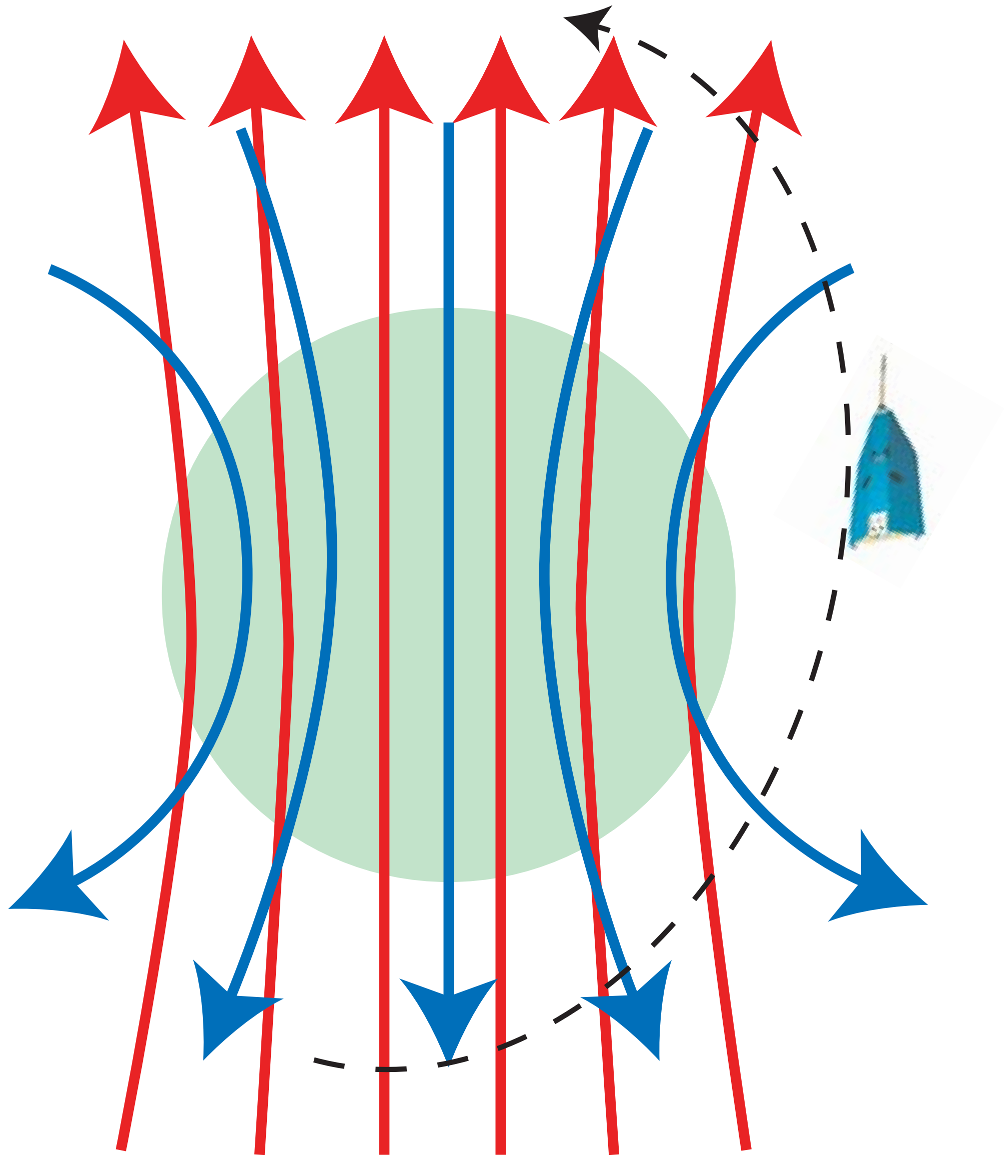
Egbert and Booker: MT and GDS responses



Egbert and Booker: Models



Satellite GDS:



Field geometry for satellite GDS:

Write the observed field as the gradient of a scalar potential,

$$\mathbf{B}(r,\theta,\phi) = -\mu_o \nabla V(r,\theta,\phi)$$

$$V(r,\theta,\phi) \;=\; a_o \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \left\{ i_l^m \left(\frac{a_o}{r}\right)^{l+1} \;+\; e_l^m \left(\frac{r}{a_o}\right)^{l} \right\} P_l^m(\cos\theta) e^{im\phi}$$

with internal coefficients $i_l^m(t)$ and external $e_l^m(t)$.

Keeping only the P_1^0 contribution and with r, θ, ϕ in geomagnetic coordinates

$$V_1^0(r,\theta,\phi) = a_o\{i_1^0(\frac{a_o}{r})^2 + e_1^0(\frac{r}{a_o})\}P_1^0(\cos\theta)$$

$$B_r = \left[-e_1^0 + 2i_1^0\left(\frac{a}{r}\right)^3\right]cos(\theta)$$

$$B_\theta = \left[e_1^0 + i_1^0\left(\frac{a}{r}\right)^3\right]sin(\theta)$$

or

$$\begin{bmatrix} -cos(\theta) & 2(\frac{a}{r})^3 cos(\theta) \\ sin(\theta) & (\frac{a}{r})^3 sin(\theta) \end{bmatrix} \begin{bmatrix} e_1^0 \\ i_1^0 \end{bmatrix} = \begin{bmatrix} B_r \\ B_\theta \end{bmatrix}$$

BUT.... magnetic satellites measure more than just induced fields.

Have to remove:

The main (core) field, and its secular variation.

The crustal field due to remanent and induced magnetization.

Ionospheric currents (daily variation)

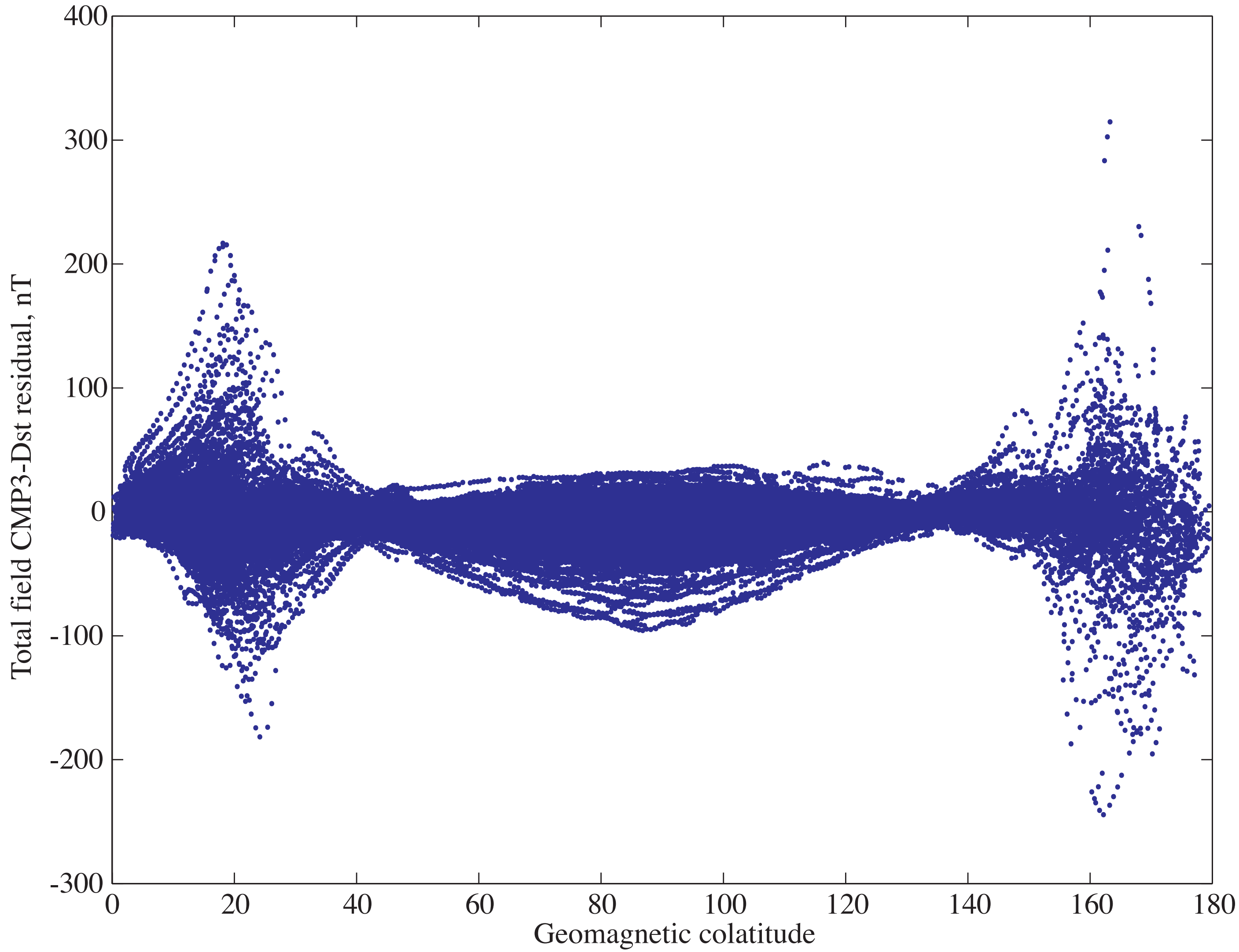
Field aligned and meridional currents, and seasonal variations.

Equatorial electrojet.

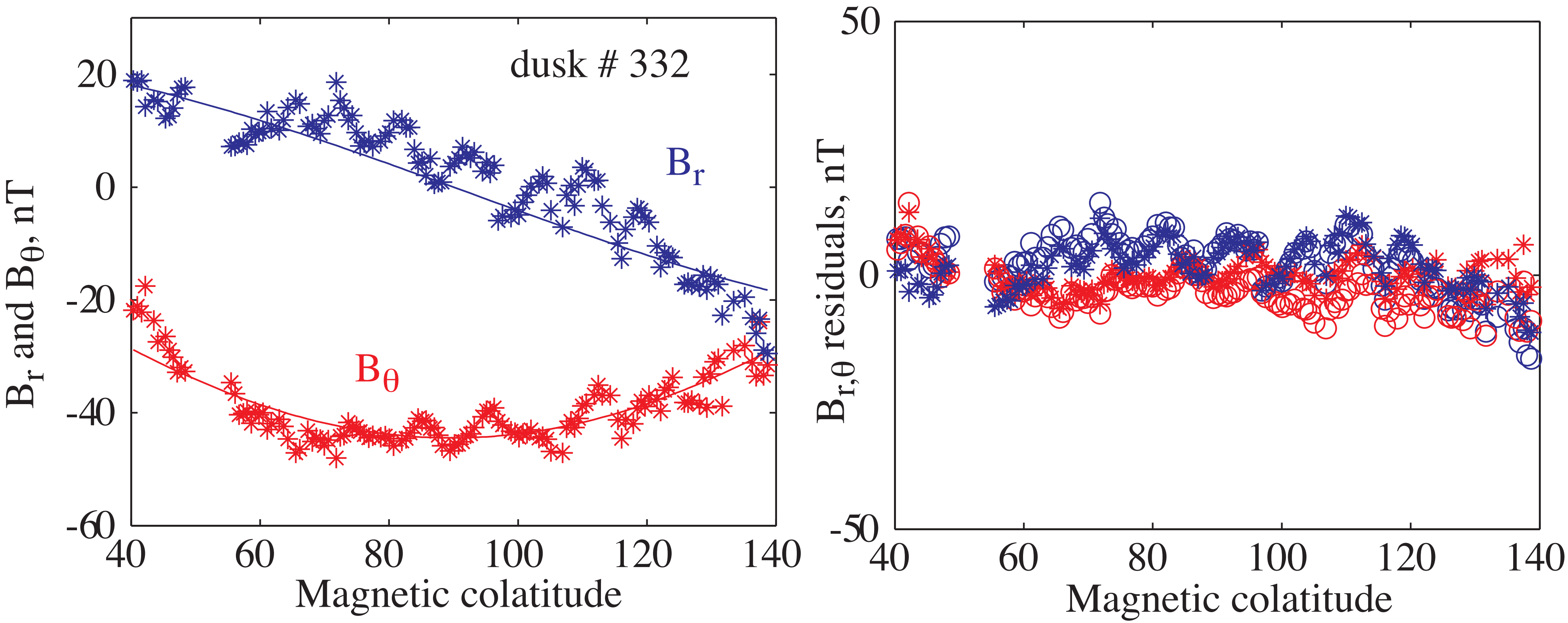
Coupling and induction of the above.

for which we use CMP3 (Sabaka *et al.*, 2000; 2002) on Magsat.

Residuals (Magsat - CMP3) for 100,000 data:



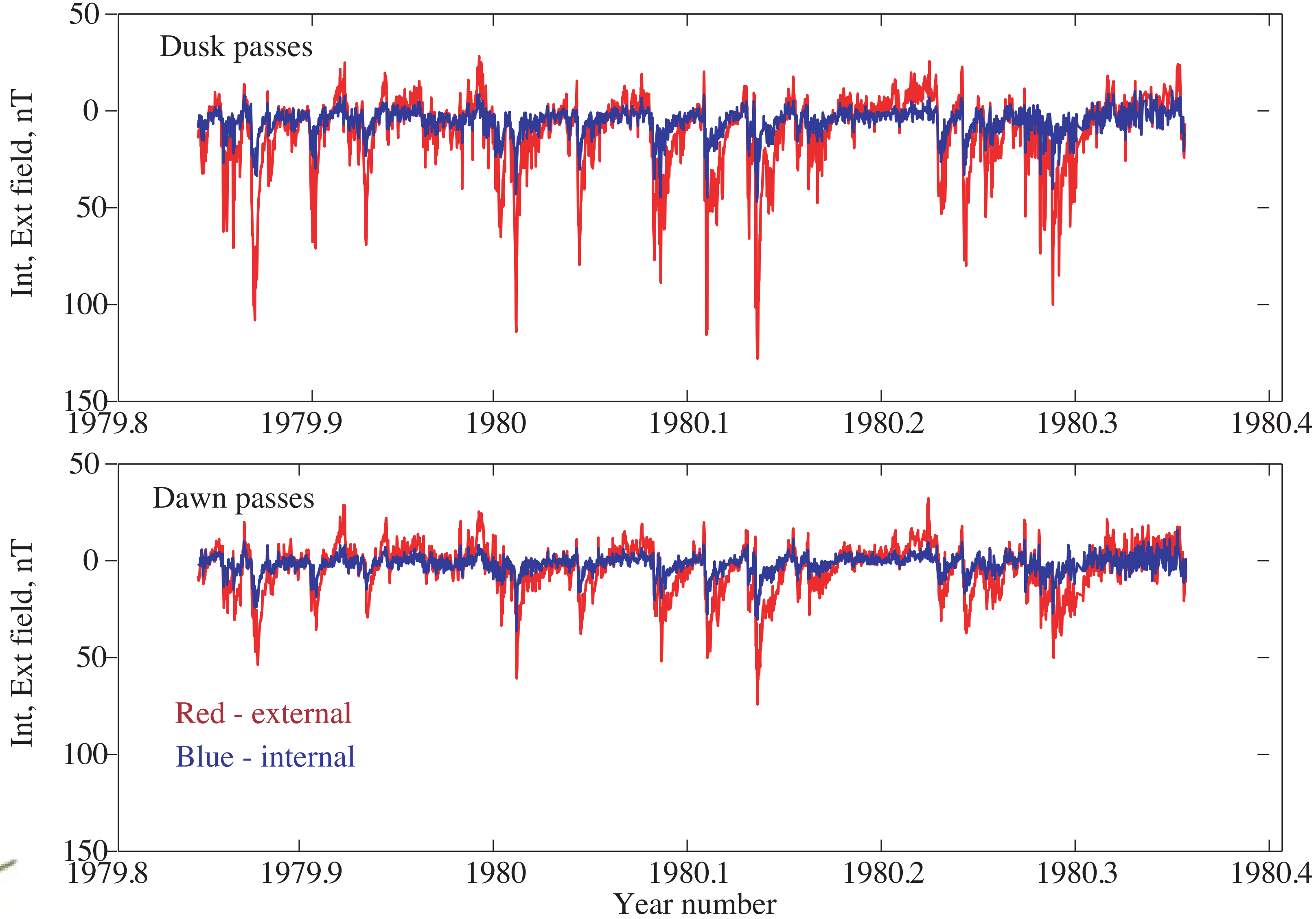
Fits to an individual pass:



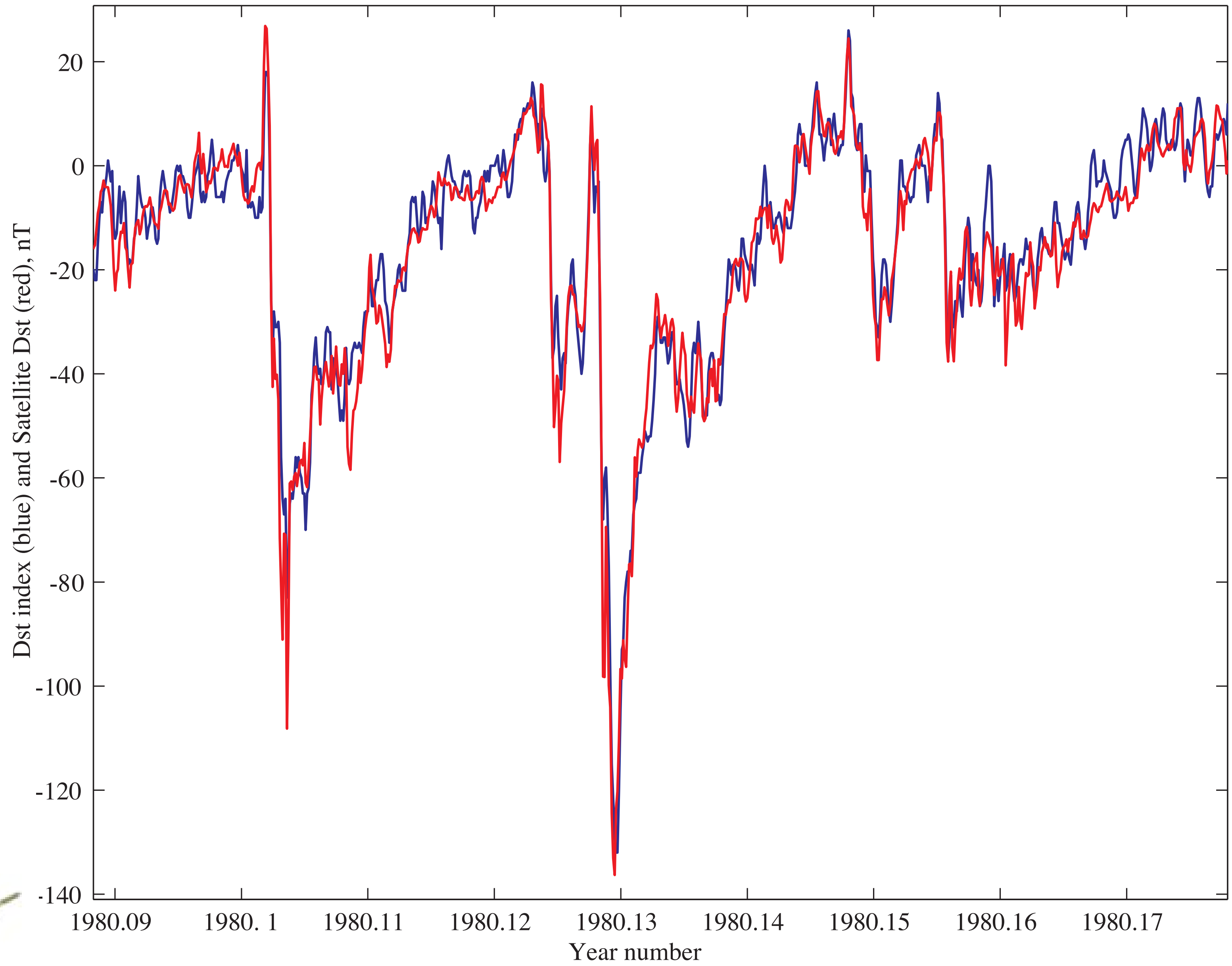
RMS residuals to 900 dawn and 900 dusk passes

	dawn	dusk
B_θ	5.30 nT	6.15 nT
B_ϕ	8.40 nT	11.03 nT
B_r	6.05 nT	5.47 nT
CMP3 B_θ	11.16 nT	19.53 nT
CMP3 B_r	6.64 nT	6.57 nT
N	107689	99301

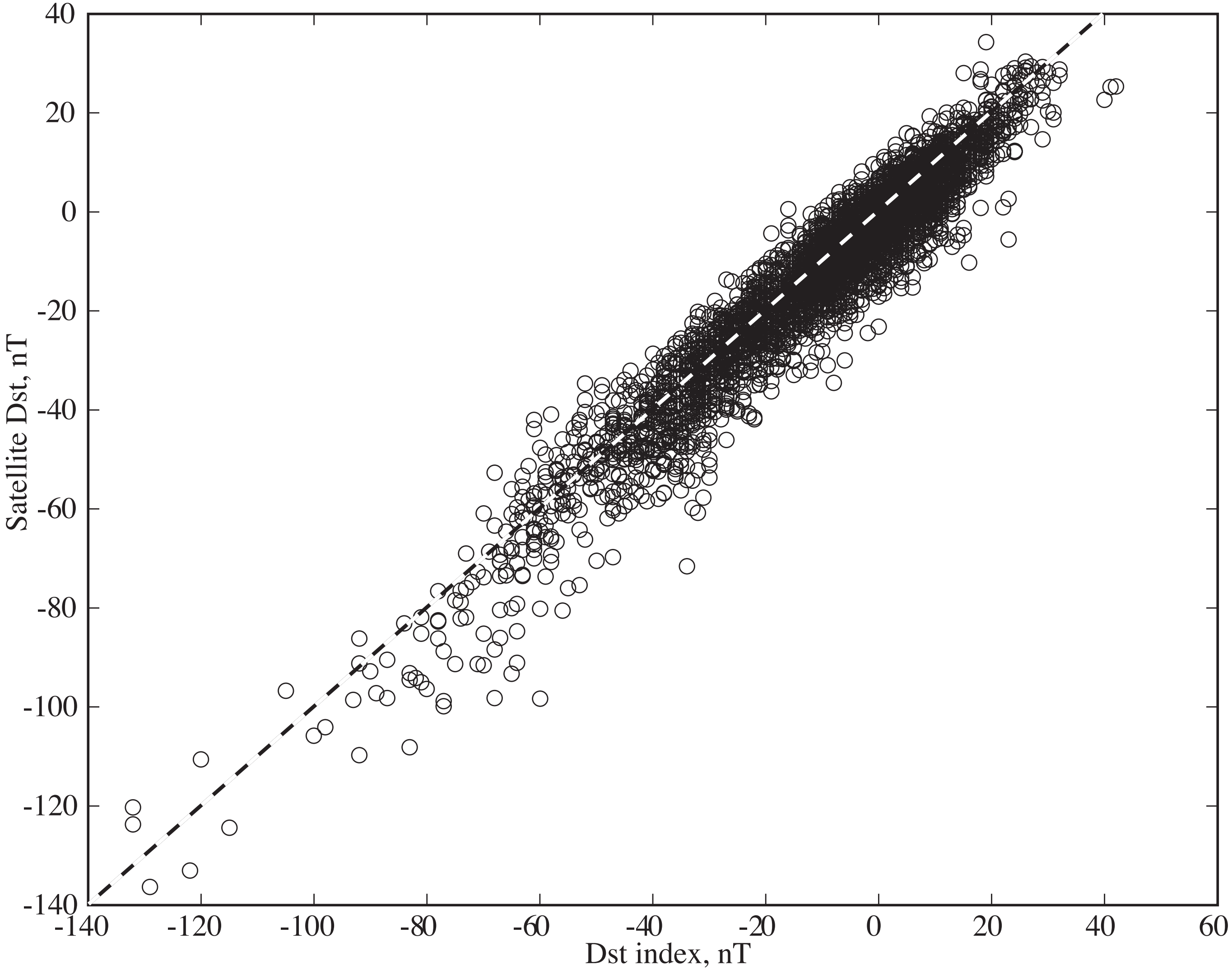
Timseries of individual i and e estimates:



Satellite Dst (red) and observatory Dst index (blue):



The agreement is statistically good:



(Sat. Dst - Dst index):

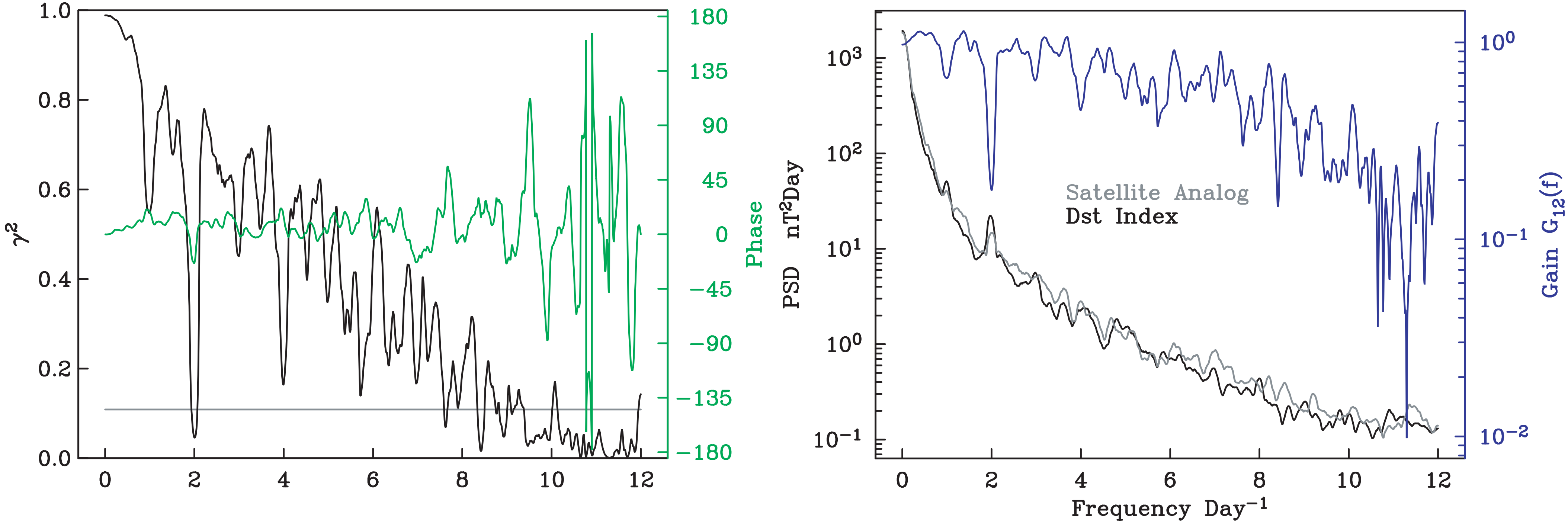
std. deviation = 5.7 nT

mean = 2.4 nT

(+25 nT from CMP3)

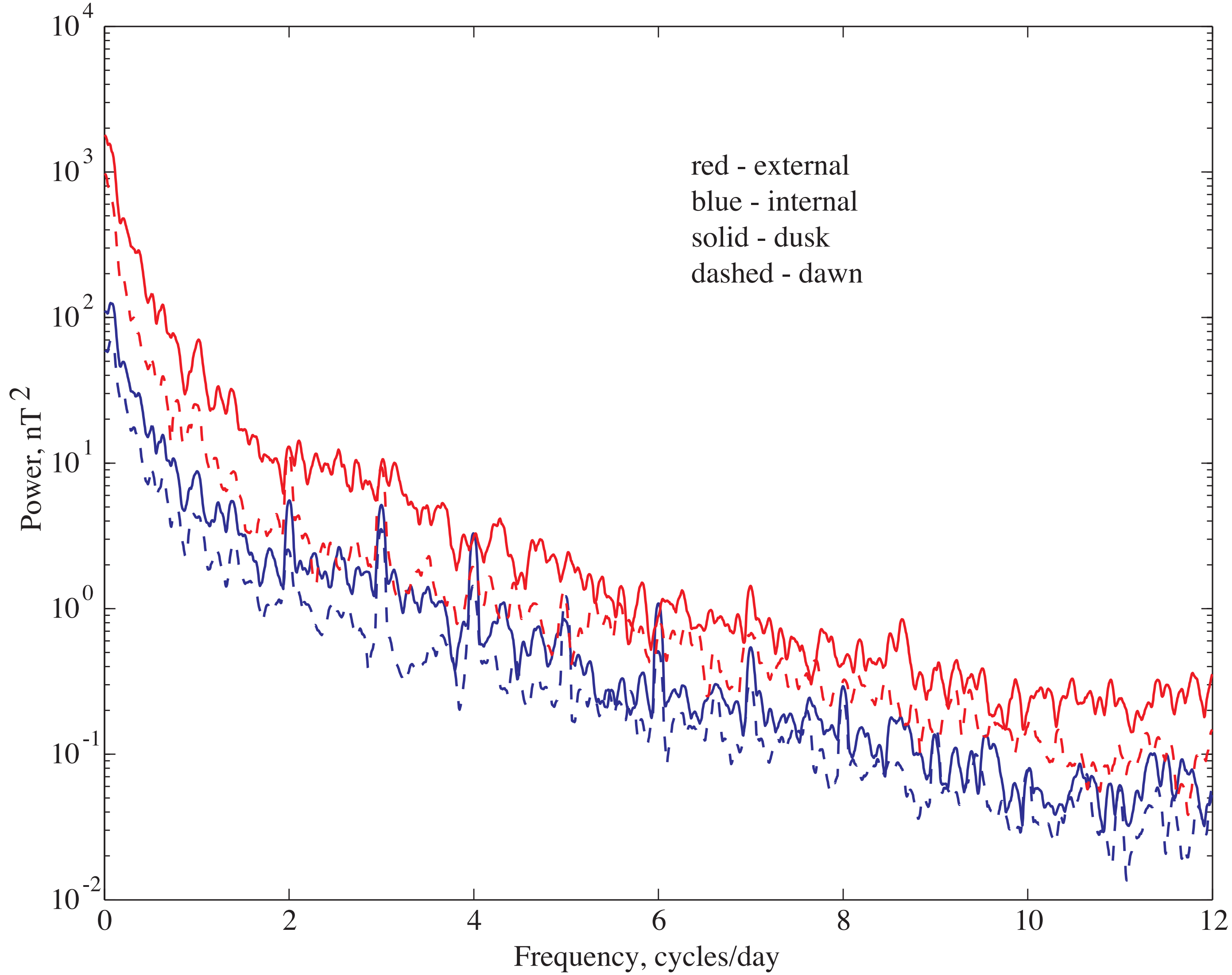
Spectra of Dst and satellite Dst:

Power in Dst estimated from ground and Magsat

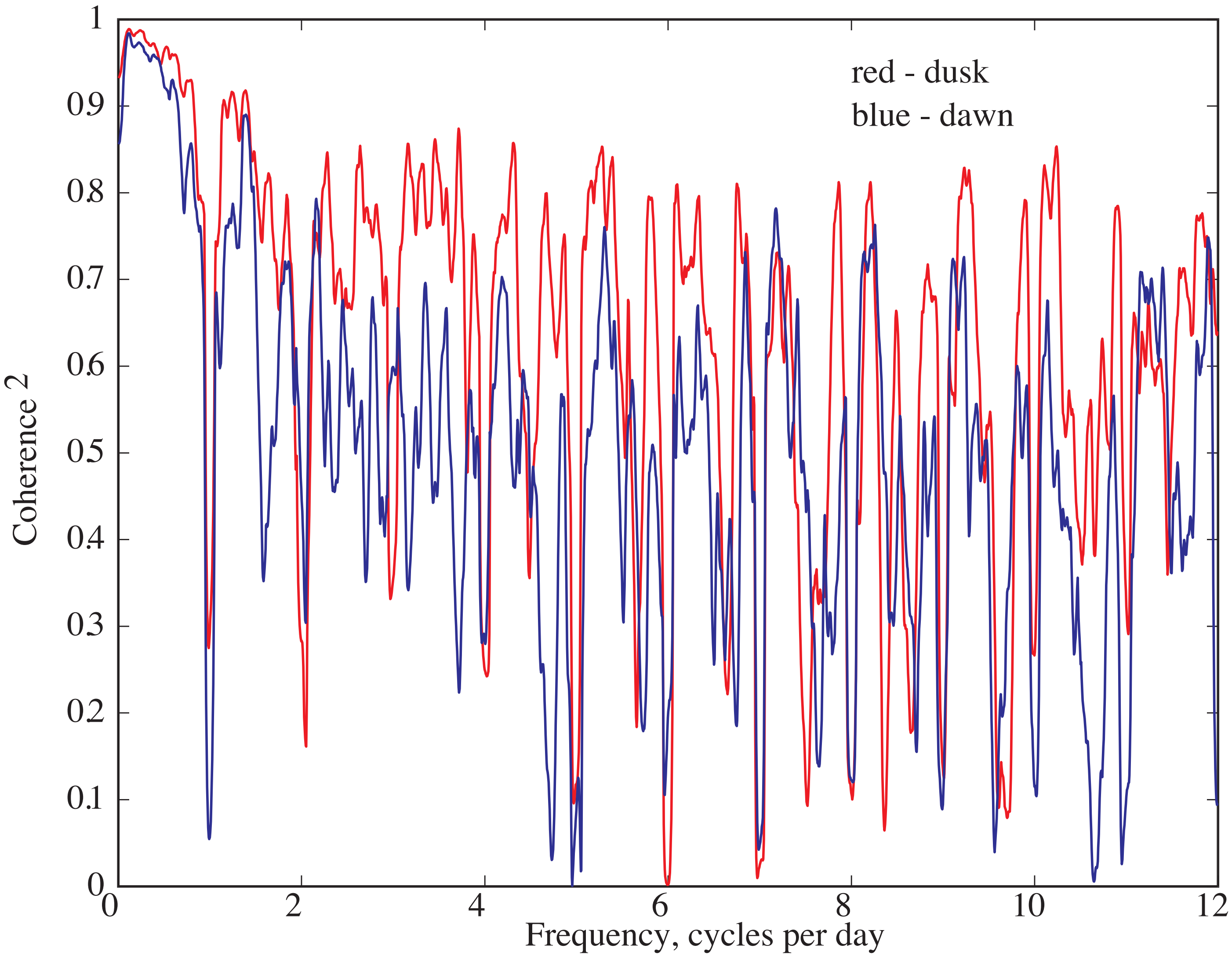


2nd CHAMP Meeting, September 2003

Use multi-taper time series analysis to obtain the transfer function between e and i.



Coherence^2 between e and i.



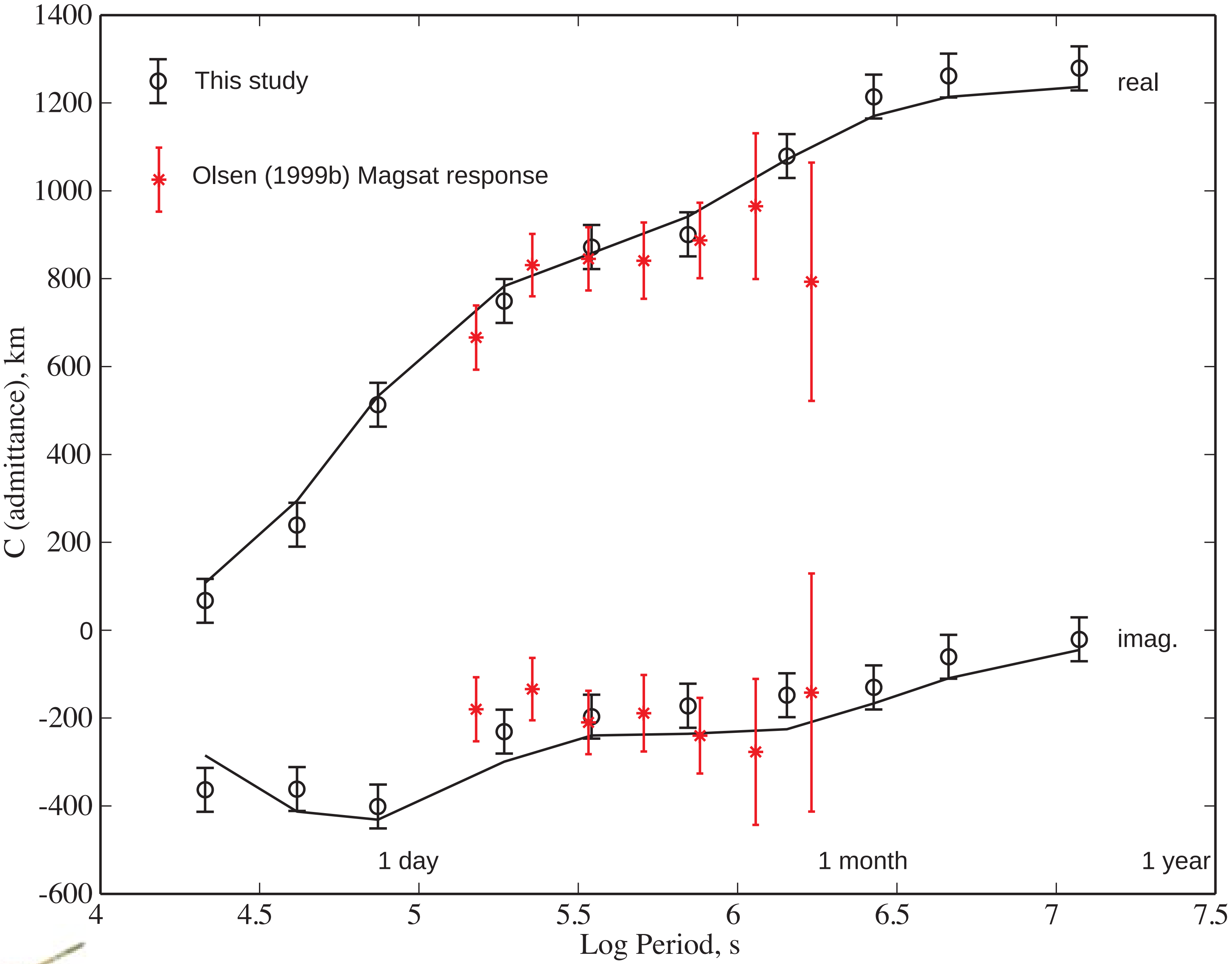
Dusk passes have more power, better coherence. Use band averaging and coherence weighting of transfer function to estimate

$$Q_1 = \frac{i_1}{e_1}$$

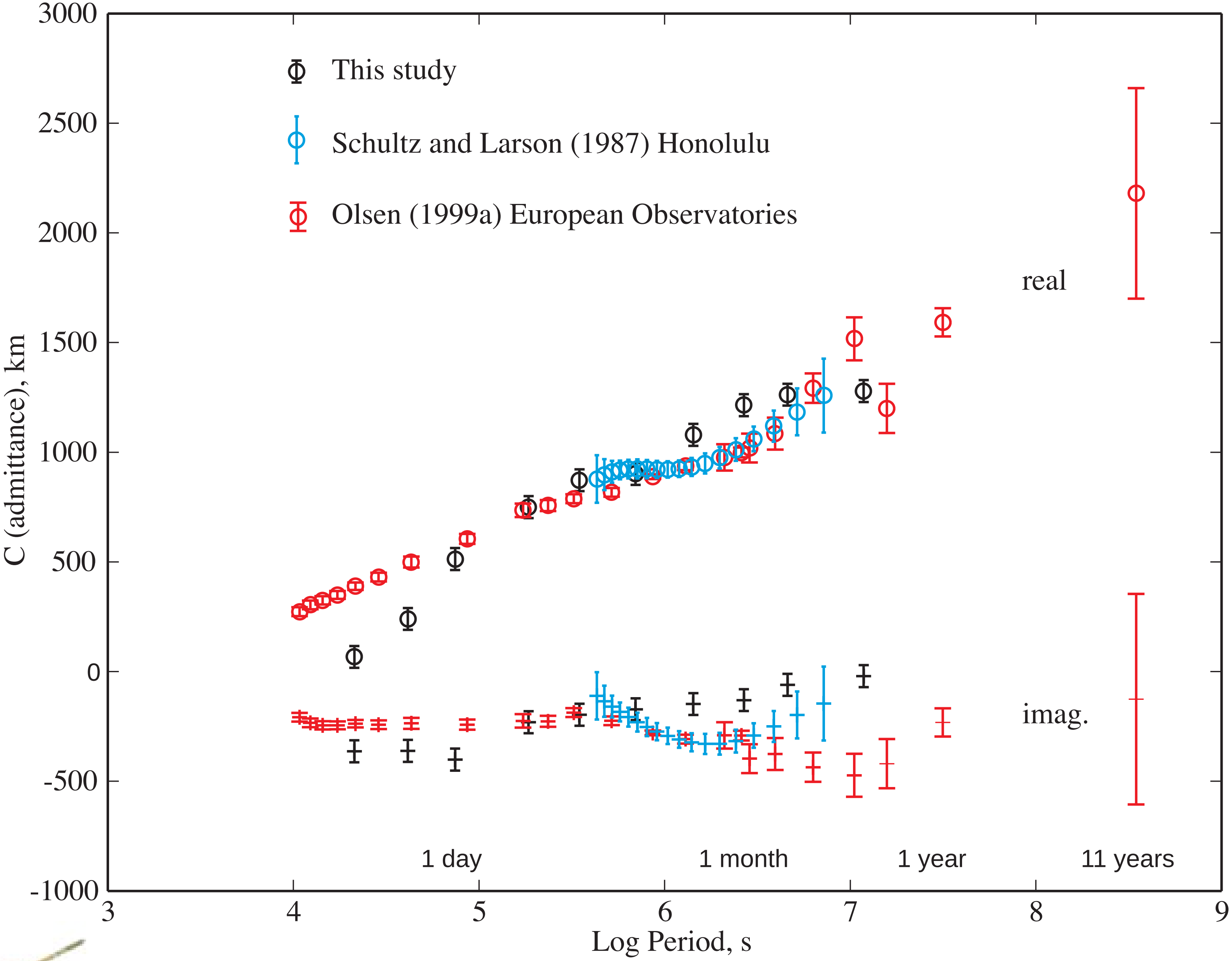
Convert Q_1 to complex admittance c using

$$c = a \, \frac{l - (l+1)Q_l}{l(l+1)(1+Q_l)}$$

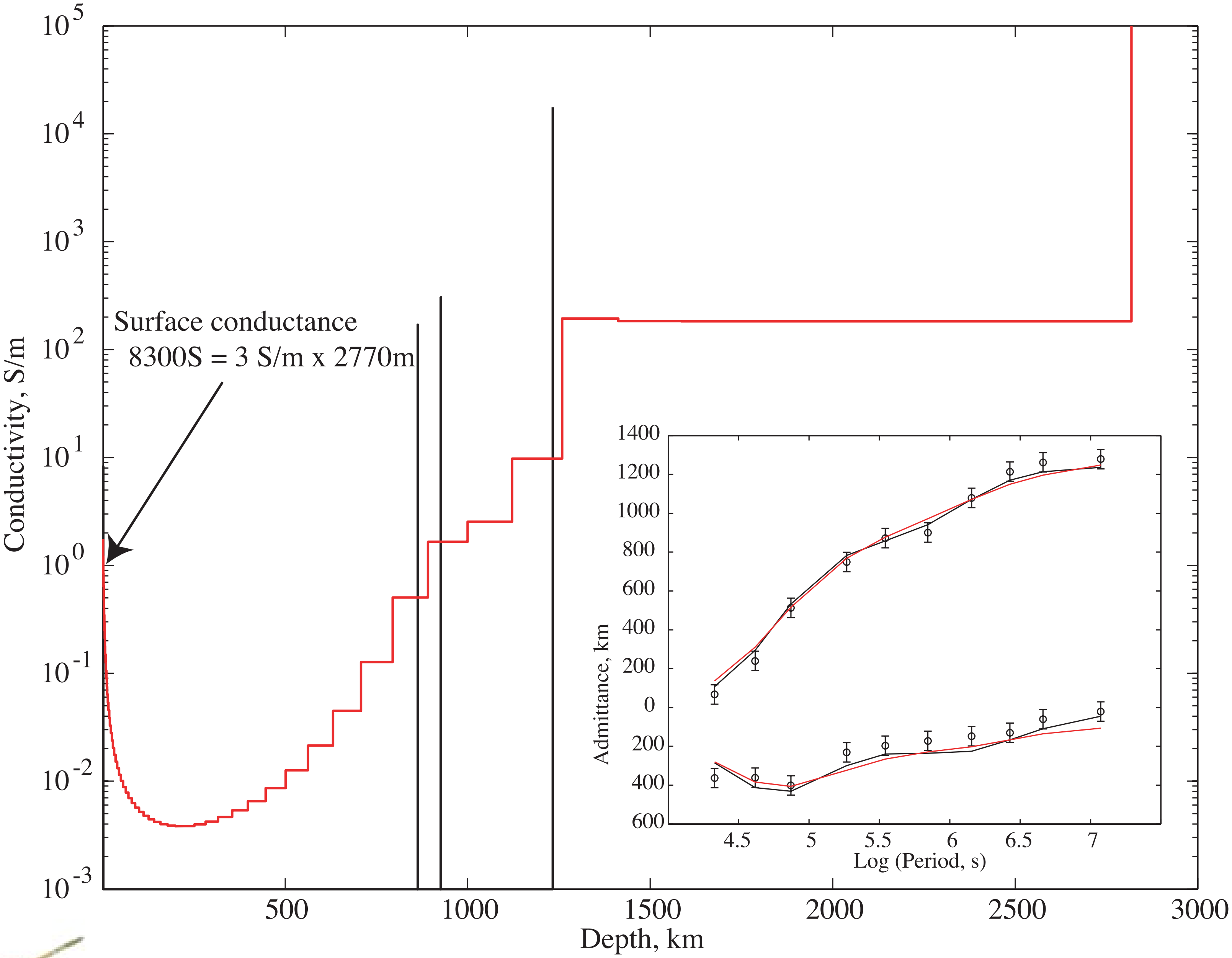
(a is the radius of Earth and $l = 1$ is the order of the spherical harmonic expansion).



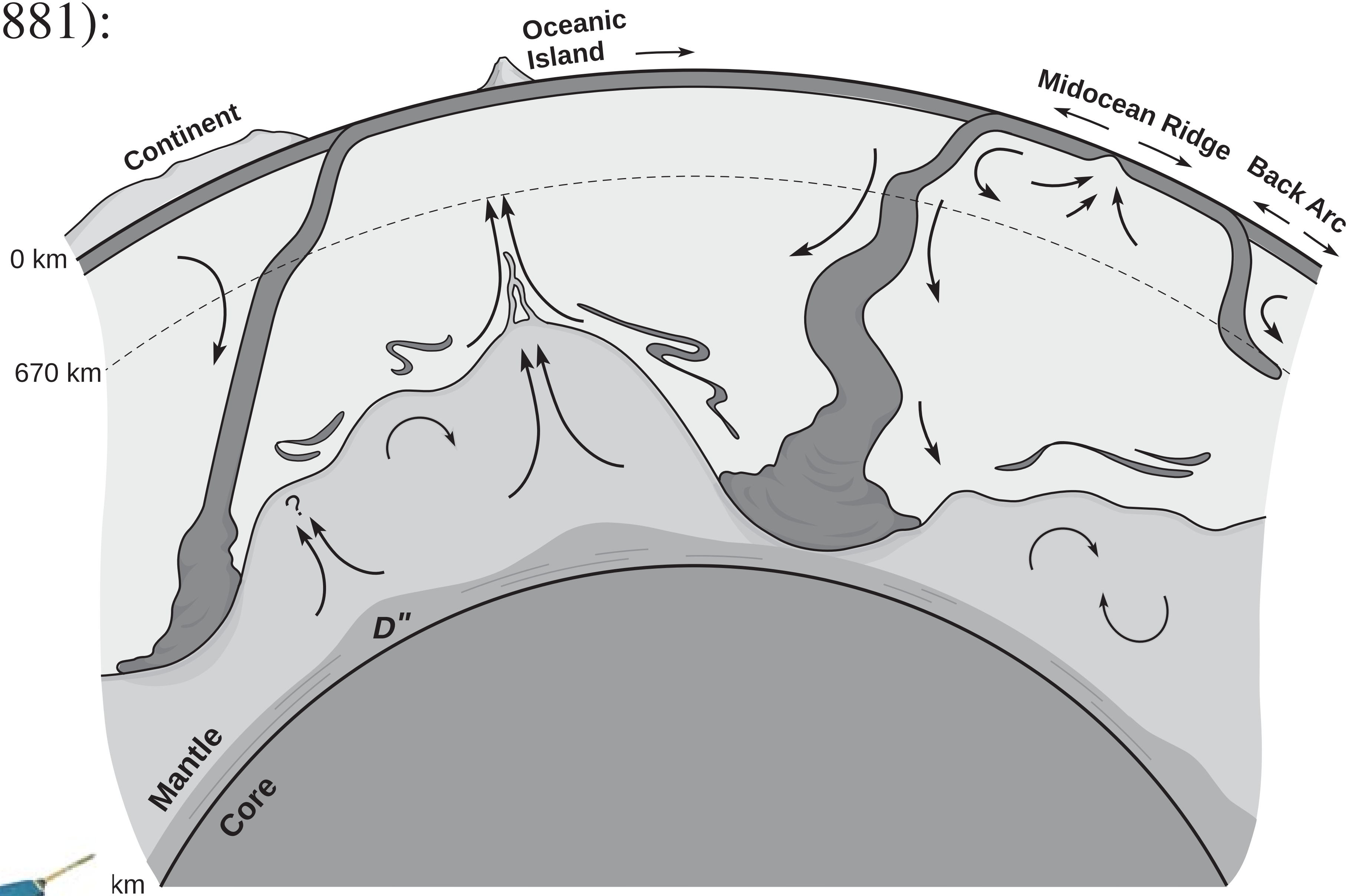
2nd CHAMP Meeting, September 2003



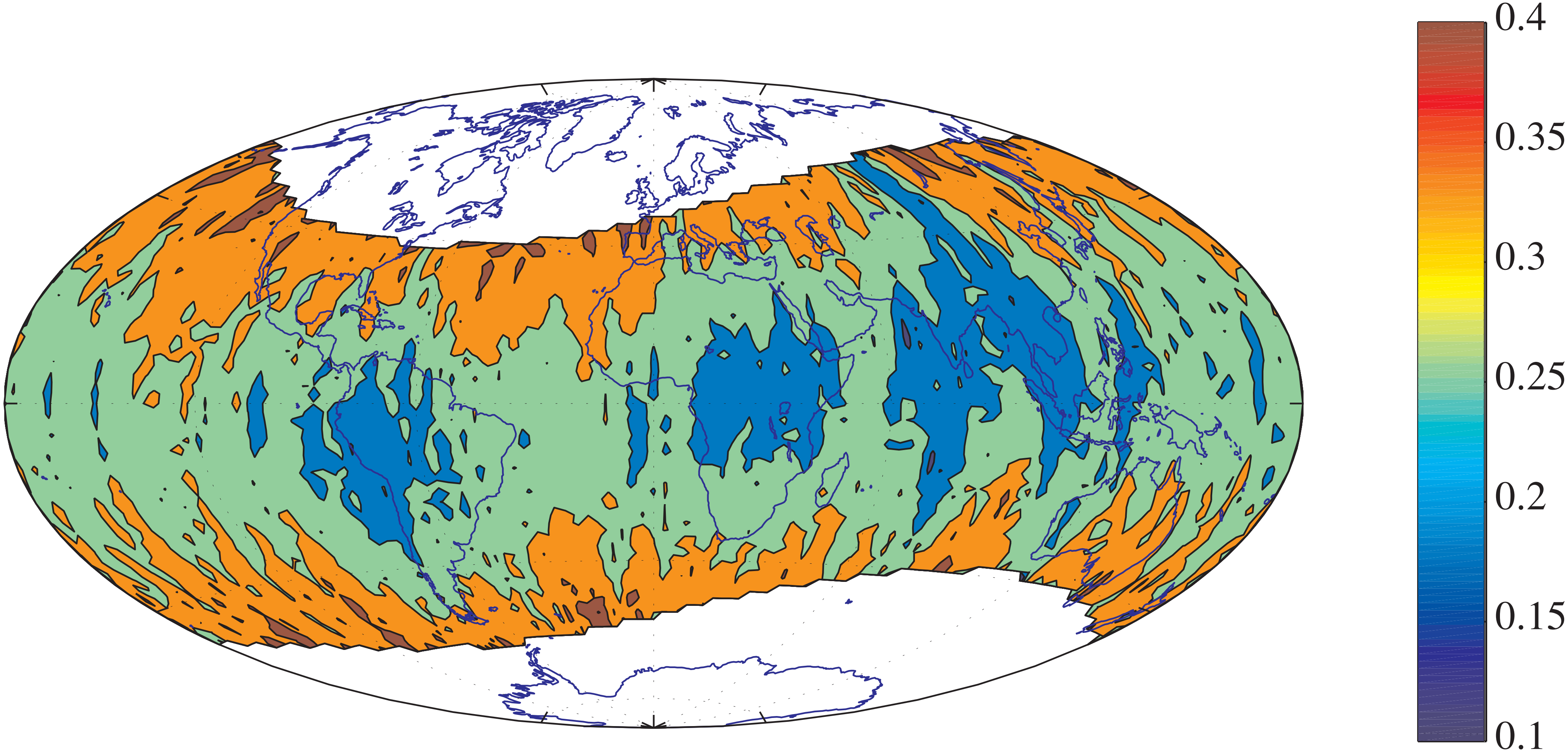
2nd CHAMP Meeting, September 2003


Surface conductance
8300S = 3 S/m x 2770m
Conductivity, S/m
Depth, km
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
500
1000
1500
2000
2500
3000
Admittance, km
Log (Period, s)
1400
1200
1000
800
600
400
200
0
200
400
600
4.5
5
5.5
6
6.5
7

Dynamics of a dense lower mantle layer (Kellogg *et al*. (1999) Science, 283, 1881):

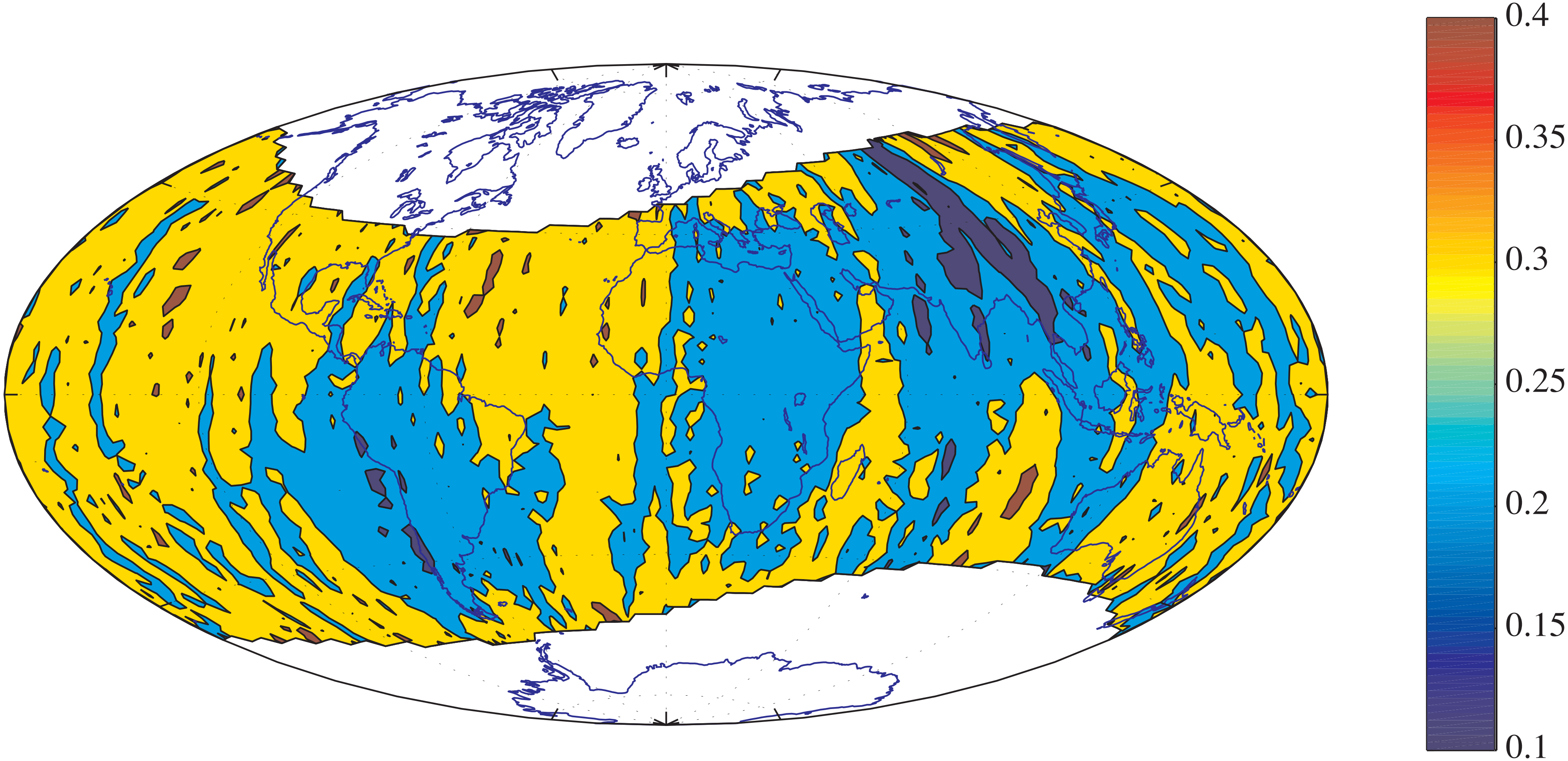


Internal fields as a fraction of external field:

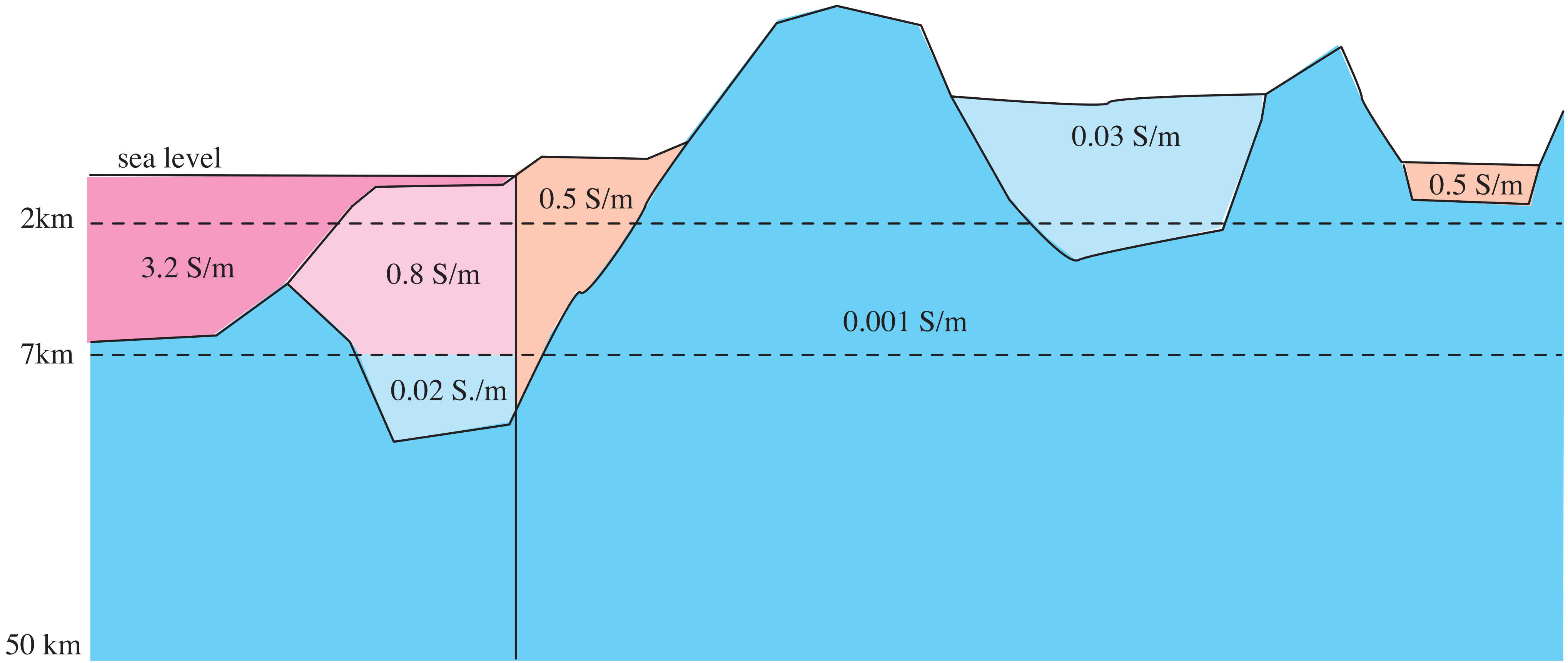


2nd CHAMP Meeting, September 2003

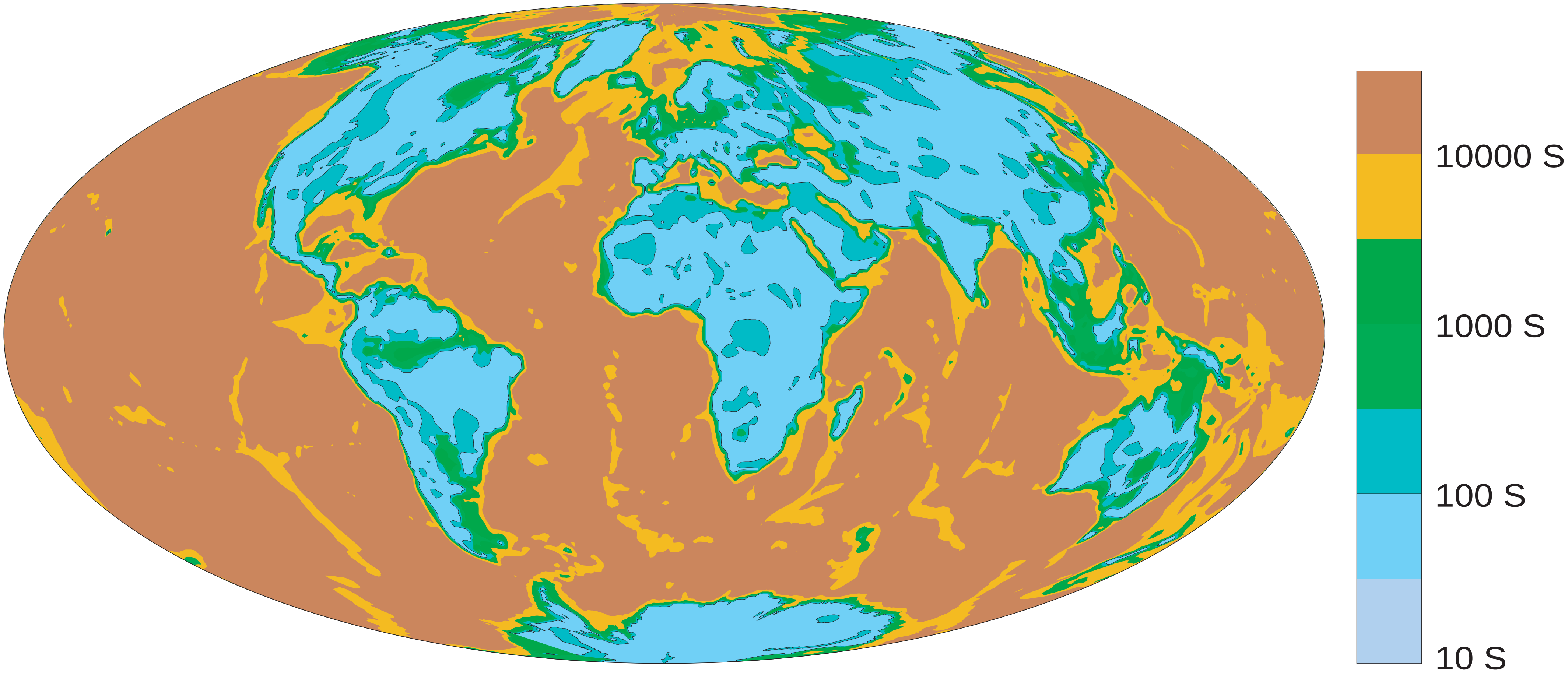
Average latitudinal dependence removed:



Turn CRUST 5.1 into conductivity map:

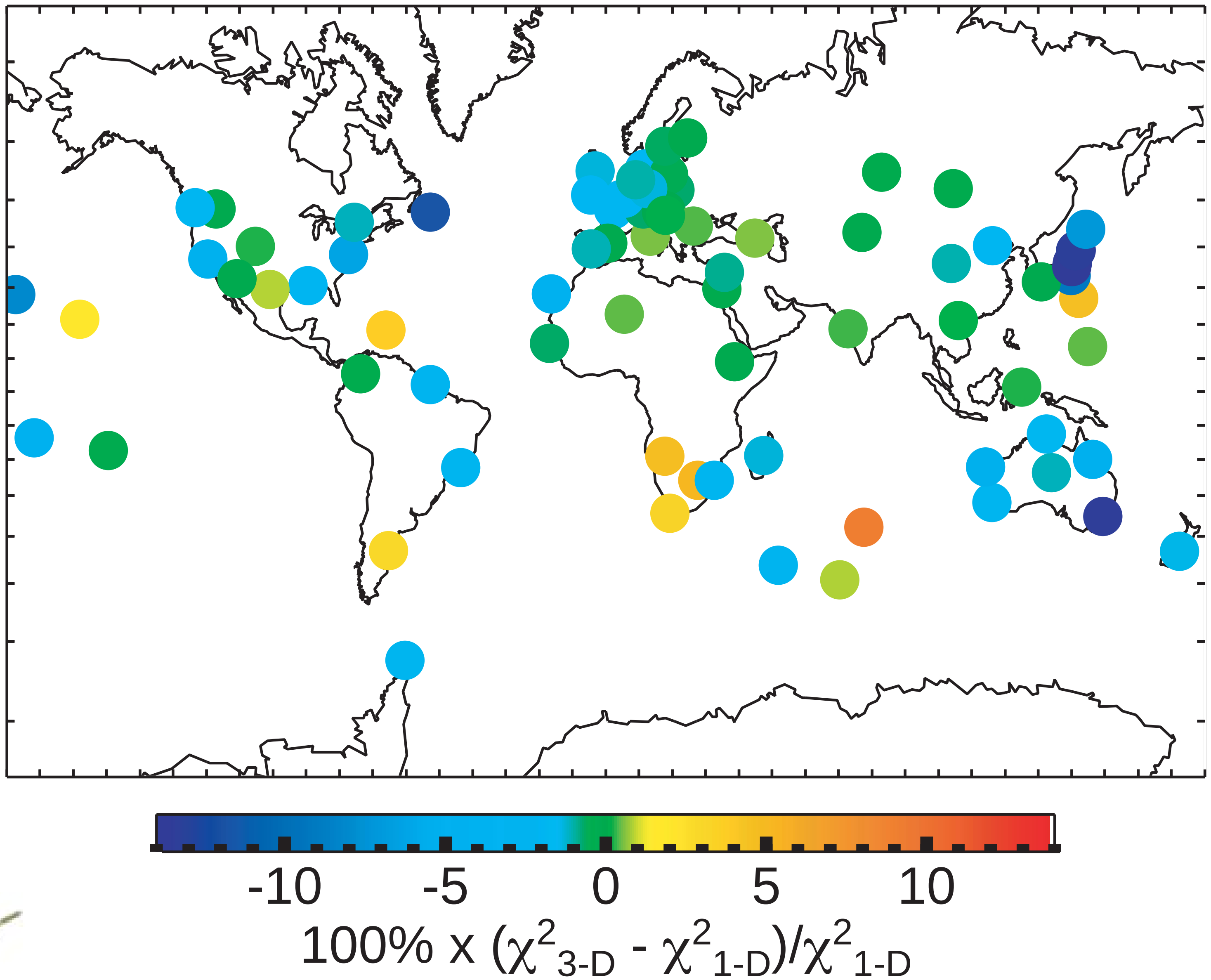


Global conductance map:

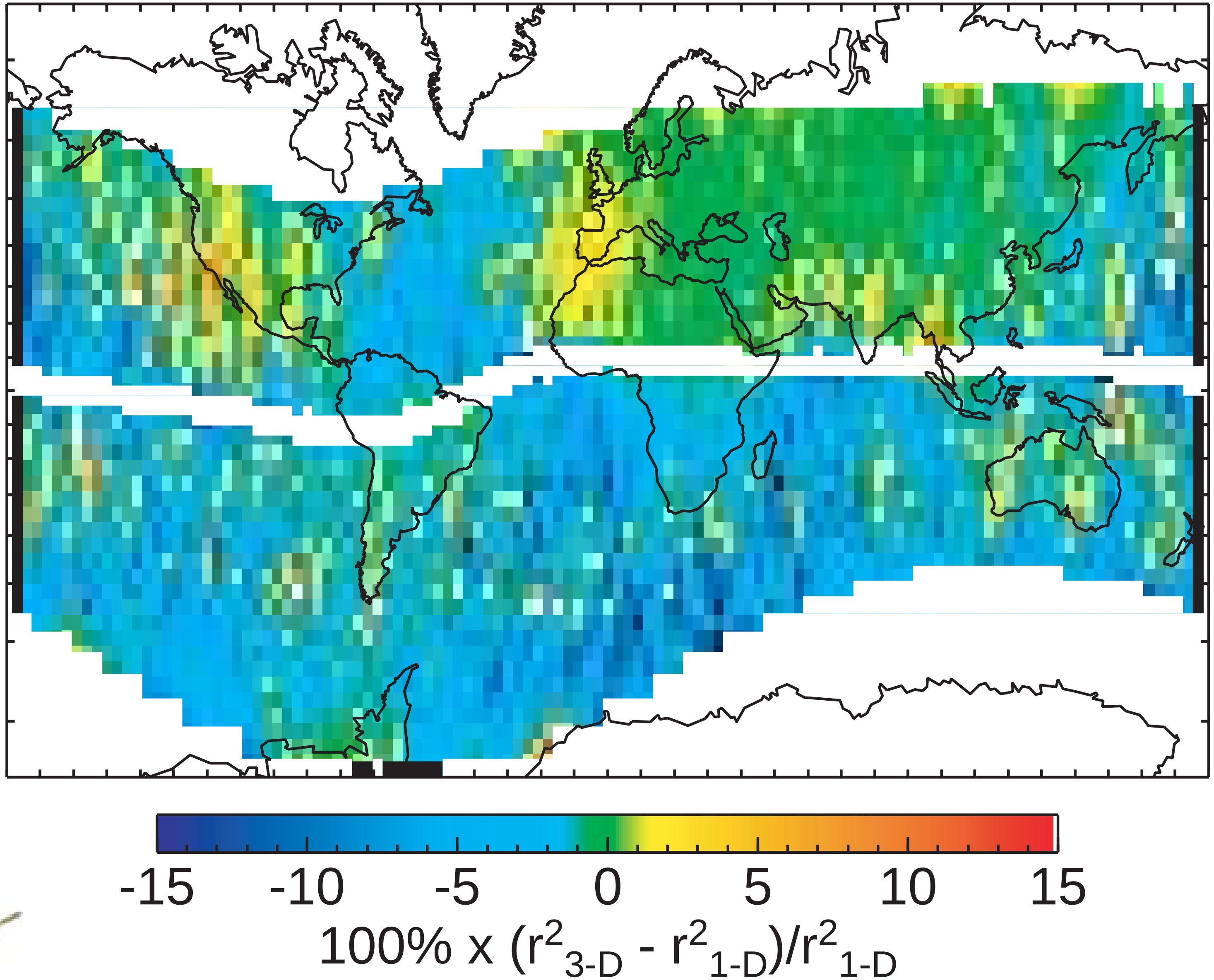


2nd CHAMP Meeting, September 2003

Improvement in time domain fits observatory data (Velimsky and Everett):



Improvement to Oersted data (Velimsky and Everett):



Conclusions:

Electrical conductivity provides important information on temperature, volatiles, and phases in the mantle

Even short magnetic missions provide broad spectrum EM responses using modern methods

Satellite data identify both shallow and deep features not seen in observatory data sets

(Copies of the paper and talk can be obtained from

http://mahi.ucsd.edu/Steve/MDAT)

Where next?

Surface conductivity needs to be well understood before 3D mantle conductivity can be mapped

Comprehensive model is a very important tool for induction

Time domain is likely to be a good, maybe even the best, way to image 3D mantle structure

Data analysis should include higher order internal fields and surface conductivity *a priori*