Electromagnetic Methods.

The resistivity method is concerned with the DC resistivity of the Earth, IP measures what is very similar to a capacitive effect, leaving electromagnetic methods to measure the inductive component of the Earth. We remember (don't we?) from simple circuit physics that the impedance of an inductor of magnitude L is ωL, where $\omega = 2\pi f$ is angular frequency, so we see that EM methods necessarily require high frequency transmissions. In practice "high frequency" really just depends on the length scale we are studying. Measurements are made in the 100 Hz - several kHz range for shallow studies, but can go down to 0.001 Hz for deep studies. We will study this relationship soon.

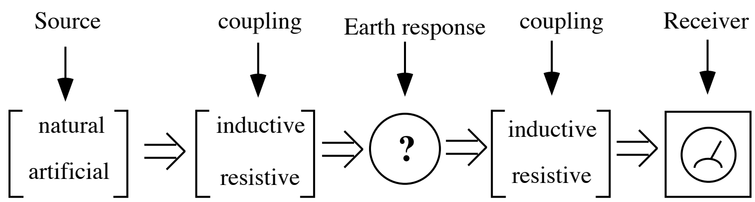


Fig. EM.1: Schematic description of induction within Earth. The objective of the geophysicist is to discern the nature of the Earth response given a knowledge of the source electromagnetic field and the received electromagnetic field.

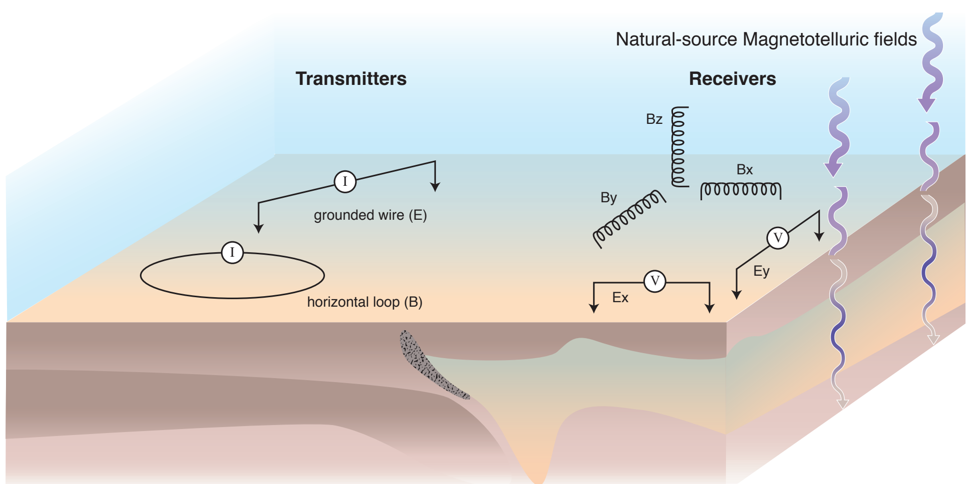


Fig. EM.2: EM transmitters can be galvanically coupled to Earth or inductively coupled. Similarly receivers can be coils or grounded cables. For magnetotelluric sounding, the transmitter is Earth's magnetosphere.

Figures 1 and 2 outline the concepts behind the use of electromagnetic methods to study geology. A source of EM energy excites eddy currents in the earth and the resulting fields are measured by an electromagnetic receiver. The source field may be variations in the Earth's magnetic field (natural) or man-made currents flowing through the earth or a wire loop (artificial). Natural variations in Earth's magnetic field or the magnetic field of a loop of wire must inductively couple to the Earth, but a grounded dipole or bipole making resistive contact to the earth through electrodes may be used, as when executing a resistivity sounding, except now a higher frequency. This is called galvanic coupling.

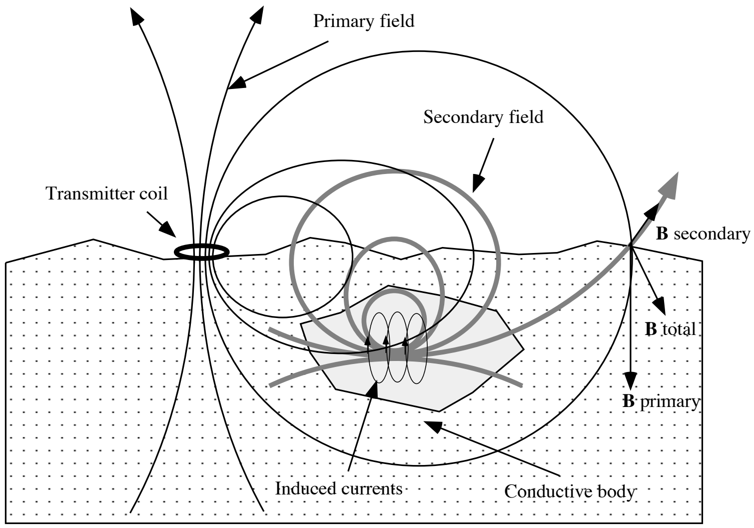


Fig. E.M.3: Currents induced in conductive parts of the earth attempt to appose the primary field within the conductive region. The secondary field associated with the induced currents modifies the primary field everywhere.

If the earth were resistive, as is the atmosphere, then the free-space field of the transmitter (the **primary** field) would be seen by the receiver. However, where the earth is conductive eddy currents are induced (so as to appose the applied field - Lenz's Law) which generate an additional EM field of different geometry and phase to the inducing field. The inducing and induced fields add, giving a response in the receiver which is different from the free-space value. The total and secondary fields in the presence of a conductive body are different from the free-space value in three ways:

a) They are in a different direction,
b) They are of a different magnitude, and
c) They are of different phase (they lag the primary field).

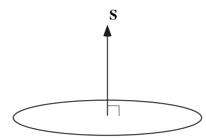
It may be seen from Figure 3 that the direction in space will be different, and we can see from analogy with magnetic anomalies that the secondary field will change the magnitude of the total field. We will see that the induced currents are out of phase with the primary field, so that's where the phase difference comes from. Any technique that measures one or more of the above properties may be used for prospecting, and any combination of inductive and galvanic coupling for the transmitter and receiver can be used, so there are literally hundreds of different electromagnetic prospecting systems to be had. The physics is the same for all...

1. Introductory Theory.

You have met the electromagnetic fields already, **E** is the electric field (V/m), **B** is the magnetic field (T), and **H** is the magnetizing field (A/m).

Integrating over surfaces:

A surface vector, **S**, has a direction that is the *outward normal* to the surface and a magnitude proportional to the area of the surface. An infinitesimal surface element, d**s**, is useful for integrating things over surfaces. In particular, if we have a vector field **A**, then the integral



$$\int_\Omega \mathbf{A} \cdot \mathrm{d}\mathbf{s} = \int_\Omega A.\cos\theta.\mathrm{ds}$$

over surface Ω is the *Flux* through the surface. θ is the angle between **A** and d**s**.

Now we are going to look at Maxwell's equations for electromagnetism. Actually, Maxwell just collected other people's equations and then did a neat trick to describe electromagnetic wave propagation in a vacuum. Indeed, his initial set of equations numbered 20 (in 20 variables) and Heaviside was the one to reduce these to the 4 we use today. We will go through these equations one by one, and present them in both their *integral form* as well as their *differential form*. The integral form is easier in terms of understanding the concepts, but the differential form is more useful when it comes to modeling electromagnetic fields.

We are going to need a little calculus, and the use of the ∇ operator.

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

so $\nabla\mathbf{A}$ is just a **gradient**:

$$\nabla\mathbf{A} = \left(\frac{\partial A_x}{\partial x}, \frac{\partial A_y}{\partial y}, \frac{\partial A_z}{\partial z}\right)$$

$\nabla \cdot \mathbf{A}$ is the **divergence**:

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

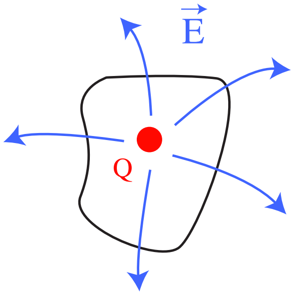
$\nabla \times \mathbf{A}$ is the **curl**,

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

and $\nabla \cdot \nabla\mathbf{A}$ is the **Laplacian**.

$$\nabla \cdot \nabla\mathbf{A} = \nabla^2\mathbf{A} = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}$$

We will start with **Gauss' Law:**

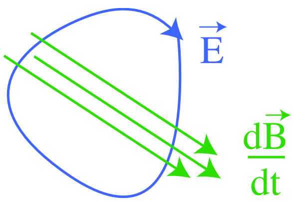


Gauss' Law says that the electric field leaving a volume is proportional to the enclosed charge.

$$\int_{\Omega} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = \frac{Q}{\epsilon_o} \qquad \qquad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

Q is charge, C
ρ is charge density, C/m^3
ϵ_o is permittivity of free space, = 8.895×10^{-12} F/m.

Next we have **Faraday's Law:**

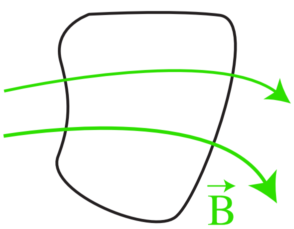


Faraday's Law says that the electric field integrated around a loop (i.e. the voltage) is given by the time rate of change of the enclosed magnetic flux.

$$\oint_c \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \qquad \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Φ_B is the magnetic flux passing though the surface defined by c.

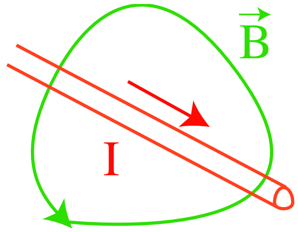
We have another **Gauss' Law, this time for magnetism**:



Gauss' Law for magnetism says that there are no magnetic monopoles. Any flux entering a volume has to leave it.

$$\int_{\Omega} \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 0 \qquad \qquad \nabla \cdot \mathbf{B} = 0$$

Lastly, we have **Ampère's Law:**



Ampère's Law says that an electric current will generate a circulating magnetic field.

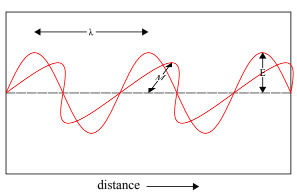
$$\oint_c \mathbf{B} \cdot \mathbf{dl} = \mu_o I \qquad\qquad \nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

I is electric current, A
$\mathbf{J}$ is current density, A/m^2
μ_o is permeability of free space, = $4\pi \times 10^{-8}$ H/m.

Put all these together, and you have Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_o}$$

$$\nabla \times \mathbf{B} = \mu_o \left(\mathbf{J} + \epsilon_o \frac{\partial \mathbf{E}}{\partial t} \right) \qquad\qquad \nabla \cdot \mathbf{B} = 0$$

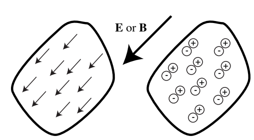


The extra term in Ampère's Law was added by Maxwell. It allows fields to exist without charges or currents, and allows electromagnetic radiation to propagate in a vacuum at speed c, where $c^2 = 1/(\mu_o \epsilon_o)$.

These are the equations in a vacuum. In matter, the electric and magnetic fields interact with the charges and magnetic moments of the material. We have already discussed magnetization $\mathbf{M}$. Charges can be bound to atoms, which results in a polarization $\mathbf{P}$ similar to magnetization, or free to move, which results in an electric current density $\mathbf{J}$:

$$\mathbf{P} = \epsilon_o \chi_E \mathbf{E} \qquad\qquad \mathbf{M} = \frac{\chi_M}{\mu_o} \mathbf{B} \qquad\qquad \mathbf{J} = \sigma \mathbf{E}$$

χ_E is electric susceptibility
χ_M is magnetic susceptibility
σ electrical conductivity, S/m



The last equation is, of course, Ohm's Law, but these are approximations! Matter doesn't have to be linear and isotropic. Clearly, there will be saturation phenomena, as we have already seen for magnetism.

From these relationships we can obtain Maxwell's equations in matter:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad\qquad \nabla \cdot \mathbf{B} = 0$$

where

$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P} = \epsilon_o(1 + \chi_E \mathbf{E})$ is the electric displacement field, and
$\mathbf{H} = \mathbf{B}/\mu_o - \mathbf{M}$ is the magnetizing field.

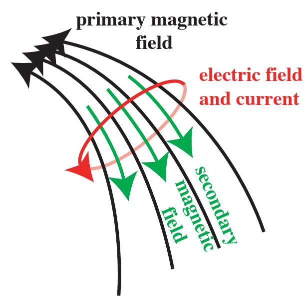
Generally in electromagnetic methods we don't concern ourselves with polarizable media, so $\mathbf{E} = \mathbf{D}/\epsilon_o$ and $\mathbf{B} = \mu_o \mathbf{H}$. (This is rarely a bad approximation – the only time I have seen it matter is the detection of shallow unexploded ordinance in magnetite rich soils in Hawaii.)

$\partial \mathbf{D}/\partial t$ is called the displacement current, and can be ignored at the frequencies, length scales, and conductivities that are relevant to geomagnetic induction. (Why? Because in rocks σ is orders of magnitude bigger than ϵ_o.) We now have the tools to qualitatively describe **electromagnetic induction**:

Faraday's Law tells us that a time varying magnetic field will induce electric fields in a circuit around the flux.

Ohm's Law says that in a conductor the electric field will generate a current.

Ampere's Law says that this current will generate a secondary magnetic field.



The secondary magnetic field opposes the changes in the primary magnetic field – this is a consequence of the minus sign in Faraday's Law (but also known as Lenz's Law). The consequence of this is that conductive rocks absorb more variation in the applied magnetic fields than resistive rocks. In uniform conductors, the rate at which the primary field is absorbed is exponential, and the characteristic length scale is given by the skin depth, which we will now derive from Maxwell's equations.

If we substitute Ohm's Law $\mathbf{J} = \sigma \mathbf{E}$ into Ampère's Law $\nabla \times \mathbf{B} = \mu_o \mathbf{J}$ we get :

$$\nabla \times \mathbf{B} = \mu_o \sigma \mathbf{E}$$

We take the curl of this and use Faraday's Law $(\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t)$:

$$\nabla \times \nabla \times \mathbf{B} = \mu_o \sigma \nabla \times \mathbf{E} \quad \rightarrow \quad \nabla \times \nabla \times \mathbf{B} = -\mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

Now we need the vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ to get

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

But, the no-monopoles law (Gauss' Law for magnetism) says that $\nabla \cdot \mathbf{B} = 0$, so ...

$$\nabla^2 \mathbf{B} = \mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

Similarly, we can take the curl of Faraday's Law and substitute Ampère's and Ohm's Laws to get

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) \quad \rightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\mu_o \sigma \frac{\partial \mathbf{E}}{\partial t}$$

To pull the same vector identity trick we need to use $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ on Ampère's Law ($\nabla \times \mathbf{B} = \mu_o \mathbf{J}$) to get

$$\nabla \cdot \mathbf{J} = 0 \quad \text{which for constant } \sigma_o \text{ gives} \quad \nabla \cdot \mathbf{E} = 0$$

so

$$\nabla^2 \mathbf{E} = \mu_o \sigma_o \frac{\partial \mathbf{E}}{\partial t}$$

Again, note that this last result only holds in regions of constant conductivity (which is not a big limitation, since regions with varying conductivity can be divided into subregions of constant conductivity with boundary conditions applied between them).

These two equations in **B** and **E**

$$\nabla^2 \mathbf{E} = \mu_o \sigma_o \frac{\partial \mathbf{E}}{\partial t} \qquad\qquad \nabla^2 \mathbf{B} = \mu_o \sigma \frac{\partial \mathbf{B}}{\partial t}$$

are **diffusion equations**. What happened to the electromagnetic waves that Maxwell's equations describe? Well, they went away with the displacement current. The result is that electromagnetic induction methods can only detect conductivity variations that are of comparable size to the scale of the source–receiver geometry, or depth of burial. If $\sigma \approx 0$ (in air) or $\omega = 0$ (DC resistivity) the equations reduce to Laplace's equation.

Now it is time to consider a single frequency $\omega = 2\pi f$, so

$$\mathbf{B}(t) = \mathbf{B}e^{i\omega t} \quad \text{and} \quad \frac{\partial \mathbf{B}}{\partial t} = i\omega \mathbf{B}$$

and the same for **E**, so our diffusion equations become

$$\nabla^2 \mathbf{E} = i\omega \mu_o \sigma_o \mathbf{E} \qquad\qquad \nabla^2 \mathbf{B} = i\omega \mu_o \sigma_o \mathbf{B}$$

Let us consider an external source of **B**, at Earth's surface **B**, which is purely horizontal and uniform with frequency ω. Remember, we have had to assume that conductivity σ is constant, so in this case the earth is what we call a half-space. So we have

$$\mathbf{B} = B_o e^{iwt}$$

Going back to our definition of the Laplacian, we have that

$$\nabla^2 \mathbf{B} = \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2}$$

but for a uniform horizontal field the first two derivatives in x and y must be zero, and **B** can only vary with z, so our diffusion equation becomes

$$\frac{d^2B}{dz^2} = i\omega\mu_o\sigma_o B(z)$$

The next step will be simpler if we define a **complex wavenumber** $k^2 = i\omega\mu_o\sigma_o$ so that

$$\frac{d^2B}{dz^2} = k^2 B(z)$$

This is a second order linear ODE with solutions of the form

$$B(z) = c_1 e^{kz} + c_2 e^{-kz}$$

The first term grows with depth so $c_1 = 0$, and for $z = 0$ we can infer that $c_2 = B_o e^{i\omega t}$. We can write k as

$$k = \sqrt{i\omega\mu_o\sigma_o} = (1+i)\sqrt{\frac{\omega\mu_o\sigma_o}{2}} = \frac{(1+i)}{z_o}$$

where

$$z_o = \sqrt{\frac{2}{\omega\mu_o\sigma_o}}$$

is called the **skin depth**. We finally have

$$B(z) \quad = \quad B_o e^{i\omega t} e^{-(1+i)z/z_o} \quad = \quad B_o e^{-z/z_o}\big(\cos(\omega t - \frac{z}{z_o}) + i\sin(\omega t - \frac{z}{z_o})\big)$$

So $B(z)$ falls off exponentially with a characteristic distance of z_o, and with a phase shift of one radian or 57° every multiple of z_o.

The skin depth of EM energy determines how deeply it will penetrate the rocks, since at every skin depth the field has decayed to $1/e$ (37%) times its previous value. An easy way to remember the skin depth relationship is to convert conductivity to resistivity ρ and angular frequency to period in seconds T:

$$\text{skin depth} = \sqrt{\frac{2\rho}{\omega\mu_o}} = \sqrt{\frac{2\rho}{2\pi f\mu_o}} \approx 500\sqrt{\rho T} \quad \text{metres}$$

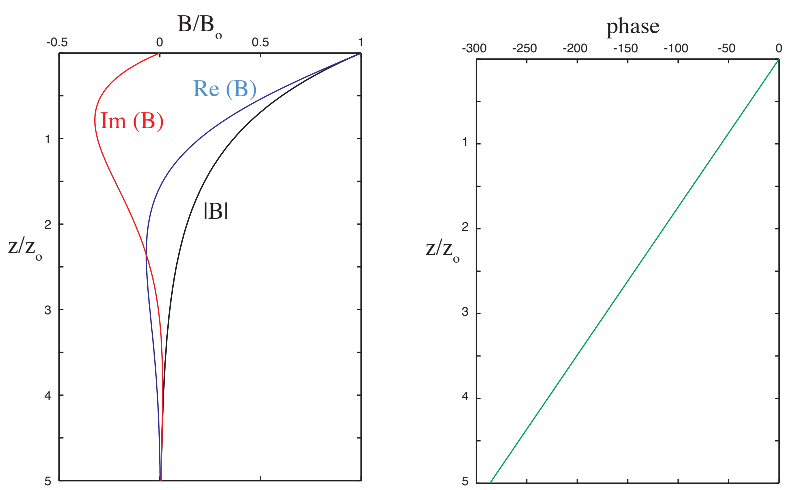


Fig. EM.4: Skin depth attenuation in a homogenous half-space.

The skin depth is important in all types of EM prospecting, because it determines the depth of penetration that can be expected at a given frequency. The higher the frequency and the higher the conductivity, the shallower the induced currents. The lower the frequency and higher the resistivity, the deeper. Substituting a few numbers into the equation shows that skin depths cover all geophysically useful length scales from less than a meter for conductive rocks and kilohertz frequencies to thousands of kilometers in mantle rocks and periods of days. Thus if one where prospecting for ore bodies in a basement which was buried beneath 100 m of 10 Ωm sediment, no matter what sort of EM method was being employed, a frequency lower than 250 Hz (which has a skin depth in the sediment of 100 m) would have to be used. Operating at 1 kHz would fail to discover anything in the basement rocks. On the other hand, using energy at a period of 1,000 seconds on a granite batholith of resistivity 1,000 Ωm will allow energy to penetrate more than 500 km into Earth (although the granite would give way to more conductive mantle rocks long before this distance was reached).

We are now ready to apply this result to the magnetotelluric method.

2. The Magnetotelluric Method

For the magnetotelluric method, natural variations in Earth's magnetic field are used as the source. These variations are a consequence of the interaction of the solar wind with Earth's internal magnetic field (0.0001 Hz to 10 Hz), along with higher frequency energy excited by lightning in the atmosphere (10–100 Hz). At even higher frequencies man-made radio signals can be used. In every case the magnetic source field can be considered horizontal, just as we have above in deriving the skin depth formula. Induced electromagnetic fields are measured by grounded electrodes making an electric field measurement.

Recall that substituting Ohm's Law into Ampere's Law we got

$$\nabla \times \mathbf{B} = \mu_o \sigma \mathbf{E}$$

If the horizontal magnetic field as derived above is purely in the x direction, the only component of the curl operator that isn't zero is $\partial B_x/\partial z$, which appears in the y-component of the curl, so

$$E_y = \frac{1}{\mu_o \sigma_o} \frac{dB_x}{dz} = -\frac{1+i}{\mu_o \sigma_o z_o} B_x = -\frac{k}{\mu_o \sigma_o} B_x$$

Similarly,

$$E_x = \frac{1}{\mu_o \sigma_o} \frac{-dB_y}{dz} = \frac{1+i}{\mu_o \sigma_o z_o} B_y = \frac{k}{\mu_o \sigma_o} B_y$$

noting that the x-component of the curl is $-\partial B_y/\partial z$. This equation is valid for any depth z, but in practice we are only interested in the surface where $z = 0$ and $B_x = B_o e^{iwt}$. We can take the ratio of the electric to magnetic field at any particular frequency to obtain an expression for half-space resistivity:

$$\left| \frac{E_y}{B_x} \right|^2 = \left(\frac{k}{\mu_o \sigma_o} \right)^2 = \frac{\omega \mu_o \sigma_o}{(\mu_o \sigma_o)^2} = \frac{\omega}{\mu_o \sigma_o}$$

$$\rho = \frac{\mu_o}{\omega} \left| \frac{E_y}{B_x} \right|^2$$

This is the MT equation made famous in Cagniard's 1953 paper. The phase between E and B is given by the $-(1+i)$ term, which is -45°.

Just as in the DC resistivity method, even though the earth is not a half-space, we make the resistivity calculation and call it *apparent resistivity*, and compute the phase between E and B, which may well not be 45°. Because the earth may in fact be 3 dimensional (or if 2D may not line up with the measurement directions x and y), there may be cross coupling between E_x and B_x (and the y components), so in practice a 2×2 *impedance* matrix is calculated:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}$$

Note that it is conventional to use H, not B, for the impedance matrix, so the apparent resistivity and phase formulas now become

$$\rho_a = \frac{1}{\omega\mu_o}\left|\frac{E}{H}\right|^2 \qquad \phi = \tan^{-1}\left(\frac{E}{H}\right)$$

Remember, all these values are functions of frequency, and in practice the elements of **Z** are obtained by taking cross-spectra between the magnetic and electric field measurements.

In the magnetotelluric, or MT, method, equipment is placed at one location and time series recordings of the horizontal electric and magnetic fields are made. Through the use of Fourier techniques, these measurements may be transformed into estimates of amplitude and phase of the **E** and **B** fields as a function of frequency (usually between about 0.001 and 100 Hz, but possibly over an even broader band). By making use of the fact that EM fields propagate more deeply into a conductive body at lower frequencies, the data can be interpreted to determine resistivity with depth (similar to resistivity sounding). MT is used to study deep structure for petroleum exploration, but can be used for mining geophysics if the frequency is made high enough (this is sometimes accomplished using a controlled source of energy). Such a variation of the method used for shallow engineering studies uses very high frequencies into the audio-frequency range (audio-frequency MT, or AMT).

The estimation of geomagnetic (purely magnetic) response functions and their interpretation in terms of mantle electrical conductivity structure dates from the end of the last century, but the use of both electric and magnetic fields dates back to only 1956.

The MT measurements can be subject to the same electrode effects that we saw in DC resistivity sounding, which results in a *static shift*, but the phase measurements are immune to these.

Over a layered earth the magnetotelluric response is easily computed using a recurrence relationship similar to the one used in resistivity. If we introduce an inductive scale length, c, then for N layers

$$c = \frac{1}{k_1}\coth[k_1 t_1 + \coth^{-1}\left(\frac{k_1}{k_2}\coth[k_2 t_2 ...\right.$$

$$\left.... \coth[k_{N-1}t_{N-1} + \coth^{-1}\left(\frac{k_{N-1}}{k_N}\right)...)\right]$$

where k_j is an inductive impedance in the jth layer (similar to m above) given by

$$k_j = \sqrt{i\omega\mu_o\sigma_j} = \sqrt{i2\pi f\mu_o\sigma_j}$$

and the t_j and σ_j are the layer thicknesses and conductivities. One starts at the deepest layer, and propagates the magnetotelluric response up through the impedance contrasts at each layer boundary (the k_j/k_{j+1}) and through the inductive thickness of each layer (the $k_j t_j$). The MT response of interest, of course, is the one at the top of the uppermost layer; the Earth's surface.

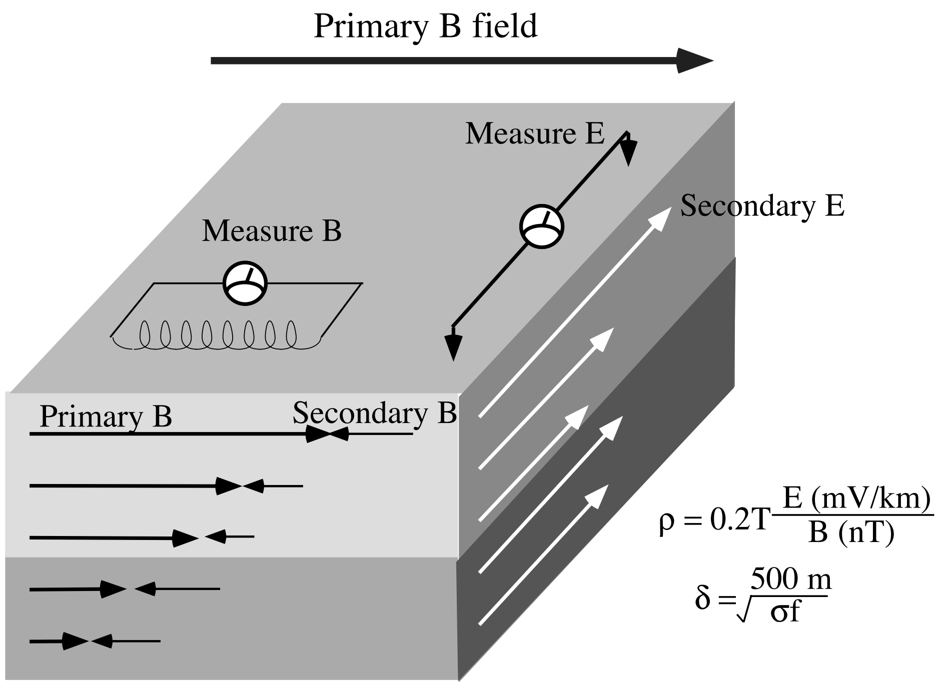


Fig. EM.5: Field setup for MT sounding, along with schematic representation of the fields.

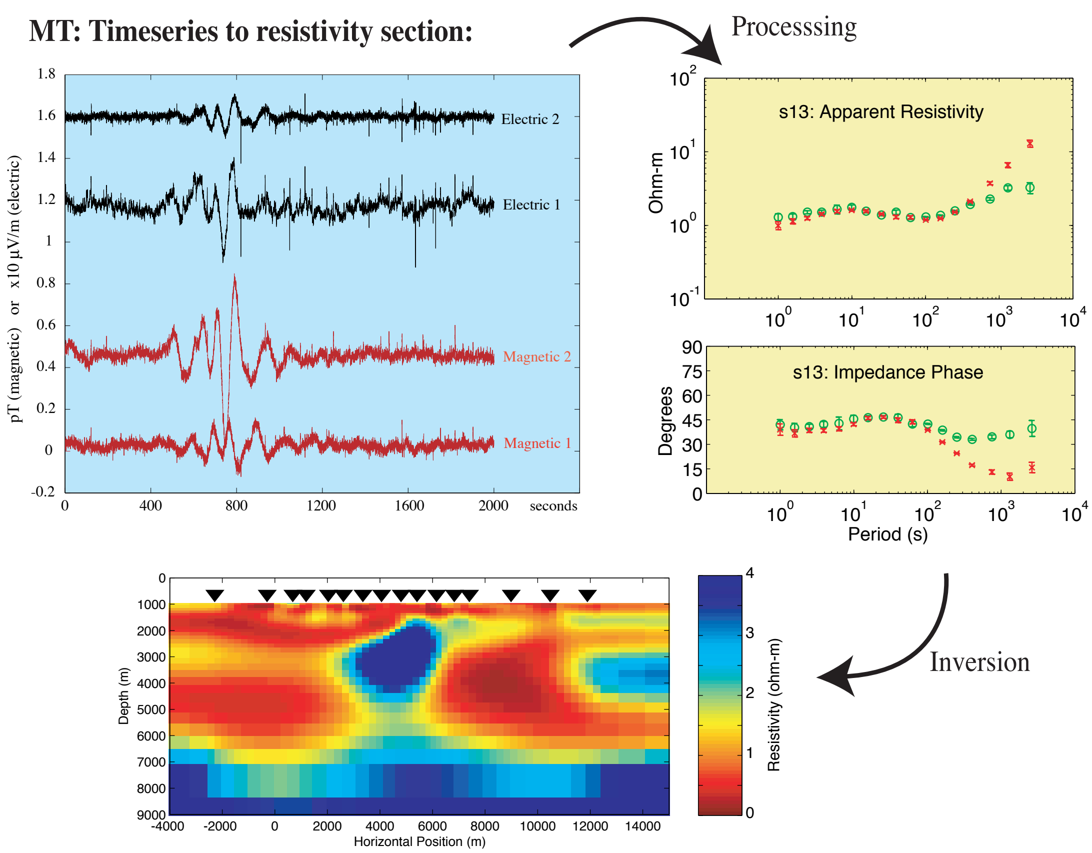


Fig. EM.6: Data, MT responses, and model interpretation for marine MT data.

The apparent resistivity and phase are given by

$$\rho_a = 2\pi f \mu_o c^2, \qquad \phi = \tan^{-1}\left[\frac{\text{Real}(c)}{\text{Imag}(c)}\right]$$

The magnetometer of choice for magnetotelluric sounding is an induction coil; many turns of wire wrapped around a core of permeable material (ferrite or soft iron). A voltage is induced in the coil that is proportional to $\partial B/\partial t$. This is clearly proportional to the frequency of the magnetic field variations, and so induction coils are better sensors than fluxgates or PPMs above about 1 Hz. Furthermore, the natural spectrum of magnetic variations is *red*, that is the lower the frequency the greater the power. Thus an induction coil tends to flatten the natural spectrum. Electrodes are the same as for resistivity sounding, except that electrode spacings of 50 to several hundreds of meters are used to increase the signal to noise ratio (now the noise of resistivity sounding is the signal we desire).

In modern practice, a *remote reference* magnetic station is used to reduce the noise in the magnetic field measurement. The magnetic field variations have long spatial wavelengths, that is they do not vary over distances of several hundred kilometers, and so the remote site is expected to be correlated with the magnetic signal at the sounding site.

Cagniard, L., 1953. Basic theory of the magneto-telluric method of geophysical prospecting. *Geophysics*, **18**, 605-635.

Two other natural source methods related to the magnetotelluric method are:

Telluric method. The amplitude of horizontal electric fields are mapped in much the same way as in a self-potential survey, except that instead of DC measurements being made electric fields are measured at several chosen frequencies.

Audiofrequency magnetics (AFMAG) uses the noise in the 100 - 500 Hz range generated by lighting and trapped in the waveguide formed by Earth-atmosphere-ionosphere system (*sferics*). The amplitude and phase of the primary field cannot be known, indeed it will vary all of the time, so the direction of the maximum field is measured with a portable two-coil system. For an Earth devoid of conductive bodies the maximum field is generally horizontal. When there is a conductor the induced secondary fields include a vertical component. This method can be used for mineral exploration and operated from an aircraft. Depths of penetration of 10–100 m have been claimed.

3. Controlled Source Methods

Controlled source methods use man-made (controlled) sources. Generally this means that the geophysicist deploys the transmitter and knows the amplitude and phase characteristics of the primary field. Measurements can be made at fixed frequencies (frequency domain) or after rapid turn-off of the transmitter (time domain, also called transient EM).

The *VLF method* uses 15-30 kHz transmissions from U.S. military communication and navigation antennae as a source. Because of the lack of control over the transmitters, the method uses tilt-angle measurements of the magnetic field just as in AFMAG surveys. The source fields are more stable than for AFMAG, but the higher operating frequency limits the depth of penetration.

Phase and amplitude systems. Once the transmitter, a grounded wire or, more usually (because of the greater portability), a coil, is under the control of the geophysicist then the amplitude and phase of the seconary field relative to the primary field may be measured and mapped. Any electric fields induced in conductive ground will be 90° behind the exciting field. This is simply a consequence of Faraday's Law:

$$E = -\frac{\partial \Phi}{\partial t}$$

where Φ is the magnetic flux cutting the conductor:

$$\Phi = \int_{surface} \mathbf{B}.d\mathbf{s}$$

If the exciting field is sinusoidal, the derivative is also sinusoidal but 90° behind. Any inductance, L, in the conductive body will produce an additional phase lag of $0 - 90°$ in the induced current which creates the secondary magnetic field:

$$\theta_L = \tan^{-1}\left(\frac{2\pi f L}{R}\right)$$

where R denotes the resistance of the conductor. Thus the total phase shift of the secondary field is $90 - 180°$. (Note that the above formula is for a simple series circuit, not the distributed conductivity of the Earth, but serves to illustrate the situation). Resistive rocks will produce a secondary field which is 90° out of phase with the primary signal (because L is small and R is large so L/R is small and therefore θ_L is also small) but conductive rocks have a larger L and (obviously) smaller R so θ_L is larger, resulting in a secondary field which approaches 90°.

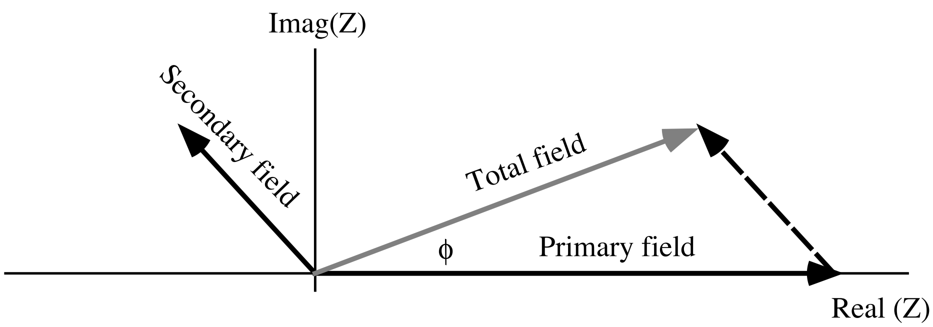


Fig. EM.7: Phase and magnitude relationships demonstrated by means of the complex plane.

One cycle of a sinusoid corresponds to 360°, so 90° phase shift refers to $1/4$ of a cycle. Any arbitrary sinusoid can be expressed as the sum of a sinusoid of 0° phase shift (the in-phase, or real component) and a sinusoid of 90° phase shift (the out-of-phase, imaginary or quadrature component). Note that a phase shift of 180° produces a wave which is in phase again, but has a reversed sign (something not normally detectable using exploration apparatus).

For the systems we have considered the secondary field must be measured in the presence of the primary field, which is always in-phase by definition, and is very much larger than the secondary field. The addition of the primary and secondary fields to obtain the resultant or total field (that which is measured) can be visualized by plotting the EM wave as a vector, using the x-axis for the real component and the y-axis for the imaginary component (figure E.20).

Real(Z) and Imag(Z) denote the real (in-phase) and imaginary (out-of-phase, or quadrature) components of the complex impedance. The phase angle is just the counter-clockwise rotation of a vector from the real axis. Observe that the secondary field is indeed drawn with a phase of between 90 and 180°, and that the phase of the total field will approach 0° as the phase of the secondary field approaches 180°. Thus it should be clear that the measurement of the phase of the total field or the ratio $Re(total)/Im(total)$ (which amounts to measuring phase anyway) is the basis for a method of detecting good conductors. However, it is useful to measure the in-phase component of the total field as a percentage of the primary signal (which can be computed or measured over resistive ground), to distinguish between small phase angles due small secondary fields (poor conductors) and small phase angles due to large, but almost completely out-of-phase, secondary fields (good conductors).

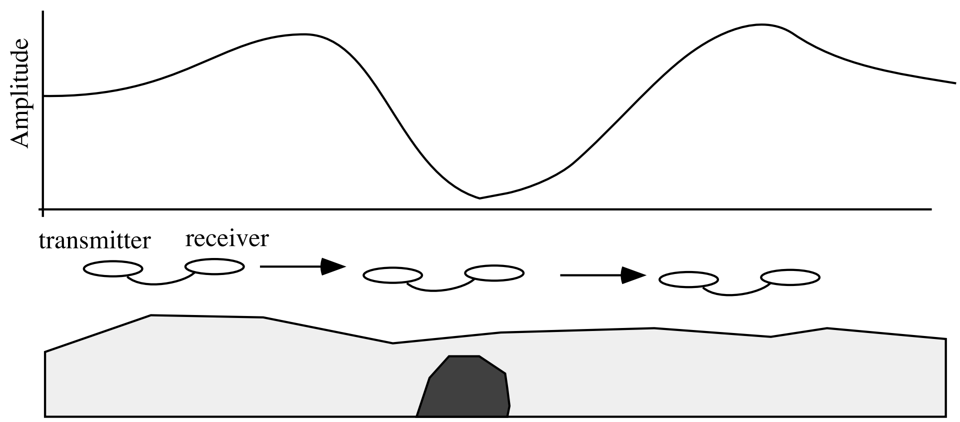


Fig. EM.8: A horizontal loop-loop system designed for use by two pedestrians, in which the amplitude and phase characteristics of the transmitter are communicated to the receiver by a fixed wire. Over a conductive body, the secondary field tends to cancel the primary field and reduce the signal amplitude. To the sides of the body, currents induced in the conductor tend to reinforce the primary air-wave.

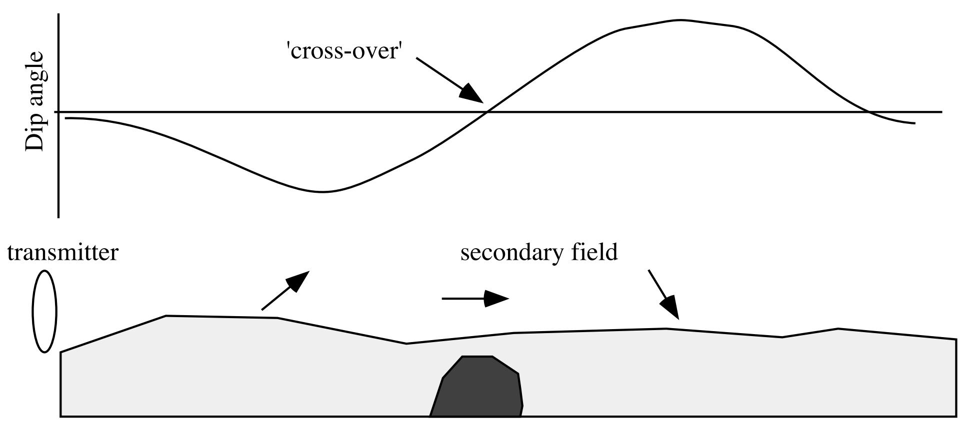


Fig. EM.9: The angle of the field due to a fixed vertical loop transmitter can be mapped. When passing over a conductive body with the receiver, the field direction changes very rapidly, producing a 'cross-over'. This is also the operational principle of the VLF system, in which the transmitter is a powerful communications antenna a long distance away.

Figure E.21 illustrates a portable EM system designed to be carried by two people. The connecting cable conveys information about the amplitude and phase of the primary signal and helps maintain a fixed separation between source and receiver. The orientation and separation must be maintained if the amplitude of the secondary field is being measured, as variations in the primary field will be indistinguishable from the secondary field. However, if the transmitter is fixed to an aircraft and the reciever is towed on a cable behind the craft, measurements of phase will be insensitive to variations in the reciever position and orientation. The transmitter and receiver coils may both be towed behind the aircraft in a rigid frame to overcome the problem of differential motion between transmitter and reciever. Figure E.22 shows the dip-angle type of measurement. A fixed transmitter generates a primary field with a horizontal field orientation at the surface of the ground. Induced currents distort the field and the direction of the resulting fields can be mapped by moving a receiver coil until the received field is either a maximum or minimum. The orientation of the total field varies most rapidly over conductive features.

Transient EM systems. The measurement of a small secondary field in the presence of a large primary field may be overcome by using a transient, or time domain, system. In a TEM system a steady current is passed through the transmitter and then switched off abruptly. The primary signal passing through the atmosphere travels at the speed of light, and so the primary signal disappears in the receiver the instant the primary current ceases. However, the electric currents induced in the ground take time to decay; the more conductive the ground the longer it takes to dissapate the electrical energy as heat through I^2R type heating. (In the limiting case of a superconductor the currents would last forever.) The magnetic field associated with the decaying eddy currents are seen a transient decay in the recceiver.

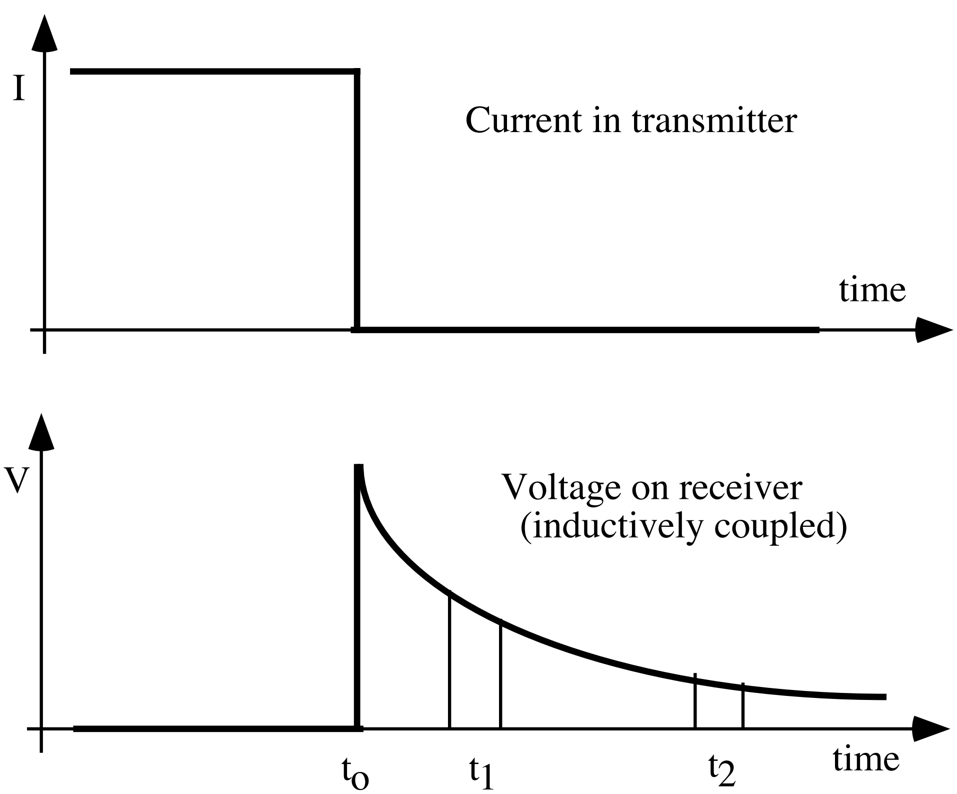


Fig. EM.10: In a transient EM system, the current in the transmitter is suddenly turned off. In an inductively coupled receiver, there will be a transient decay associated with decaying currents in the earth. These are measured as voltages at various times after the current is interrupted.

The transient fields are very much smaller than the fields associated with a continuous frequency method, but they directly represent the conductivity of the ground. In practice the pulse is repeated to form a square wave and the received signal stacked in phase with the primary waveform to improve the SNR of the transient. Transient methods are very much less sensitive to variations in transmitter position, making them useful for airborne EM (one such system goes by the trade name INPUT). The data from a transient system are displayed as decay voltage at several times after t_o as a function of position.

4. Ground Penetrating Radar

What was until recently an exotic method for probing very resistive materials such as ice and salt has, with the evolution of electronics and computerized instrument control, become a state-of-the-art technique for shallow archaeological, engineering, environmental and mineral (mainly coal) exploration. Unlike all the electromagnetic methods described so far, ground penetrating radar (GPR) is based on *wave* propagation of electromagnetic energy, instead of diffusion. In modern apparatus a short radar pulse is generated at the ground's surface and reflected energy measured on a receiving antenna.

From Maxwell's equations we have that

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

which is similar to the equation derived above for MT, but has a second term describing a *displacement current* with the physical parameter ϵ which is electric permittivity. The second term is small at frequencies of a few kilohertz or less, which is all we have considered so far, leaving the first term, describing diffusion propagation, to dominate. However, at sufficiently high frequencies the second term, which describes wave propagation, dominates (we will assume that relative permeability, μ/μ_o, is one). This is the distinguishing feature of radar, which normally operates in the range of 10 to 1000 MHz, typically around 100 MHz.

This equation has a solution of the form

$$\mathbf{E}(z,t) = \mathbf{E}_o e^{-\alpha z} e^{-i(\omega t - \beta z)}$$

where ω is angular frequency ($2\pi f$), z is depth, t is time, an attenuation constant is given by

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\epsilon/\epsilon_o}{2}\left(\sqrt{1+P^2} - 1\right)}$$

and a phase constant

$$\beta = \frac{\omega}{c}\sqrt{\frac{\epsilon/\epsilon_o}{2}\left(\sqrt{1+P^2} + 1\right)}$$

which includes the relative permittivity or dielectric constant, ϵ/ϵ_o. Notice that $1/\alpha$ is a skin depth and $1/\beta$ is a phase velocity.

The ratio of conduction current to displacement current is given by the loss factor

$$P = \frac{\sigma}{\omega\epsilon}$$

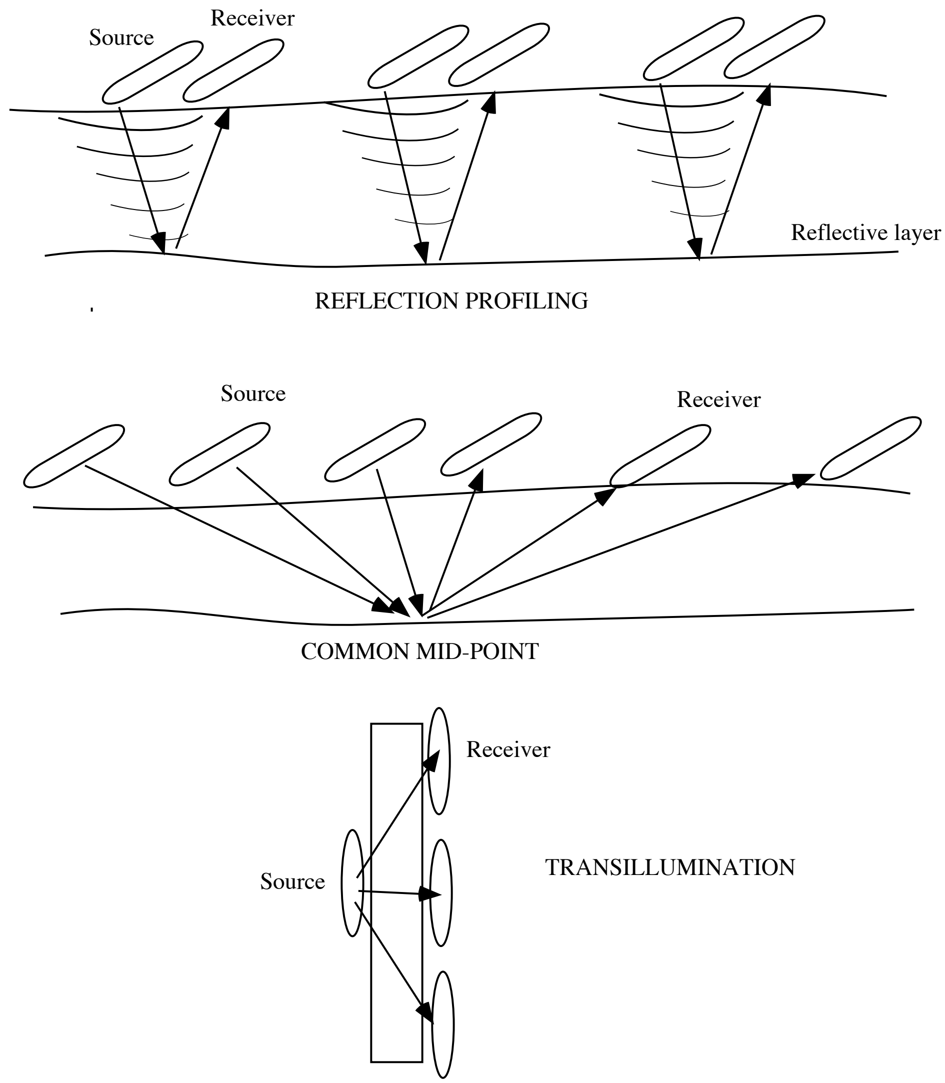


Fig. EM.11: Schematic view of radar used in various configurations.

For good conductors, displacement currents are negligible compared with conduction, and the above equations reduce to the diffusion equation, considered in MT. When frequencies are high, the displacement current dominates and wave propagation persists. Propagation is by means of a damped wave of phase velocity

$$V = \frac{\omega}{\beta} = c/\sqrt{\frac{\epsilon/\epsilon_o}{2}\left(\sqrt{1+P^2}+1\right)}$$

which for low conductivity reduces to

$$V = \frac{c}{\sqrt{\epsilon/\epsilon_o}} = \frac{c}{\sqrt{\epsilon_r}}$$

where c is the speed of light.

Relative permittivity $\epsilon_r = \epsilon/\epsilon_o$, or dielectric constant, is a maximum of about 80 in water and typically 5-10 in minerals. Typical soil velocities are around 0.1 m/ns and depth of penetration (given approximately by $1/\alpha$) would be about 1 m for 100 Ωm soil. A reflection coefficient at a layer is given by

$$R = \frac{Z_2 \cos\theta_1 - Z_1 \cos\theta_2}{Z_2 \cos\theta_1 + Z_1 \cos\theta_2}$$

where the complex electromagnetic impedance is

$$Z = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\epsilon}}$$

For normal incidence, $\cos\theta = 1$, and if σ is small and μ does not vary then R reduces to a ratio of velocities:

$$R = \frac{V_2 - V_1}{V_2 + V_1}$$

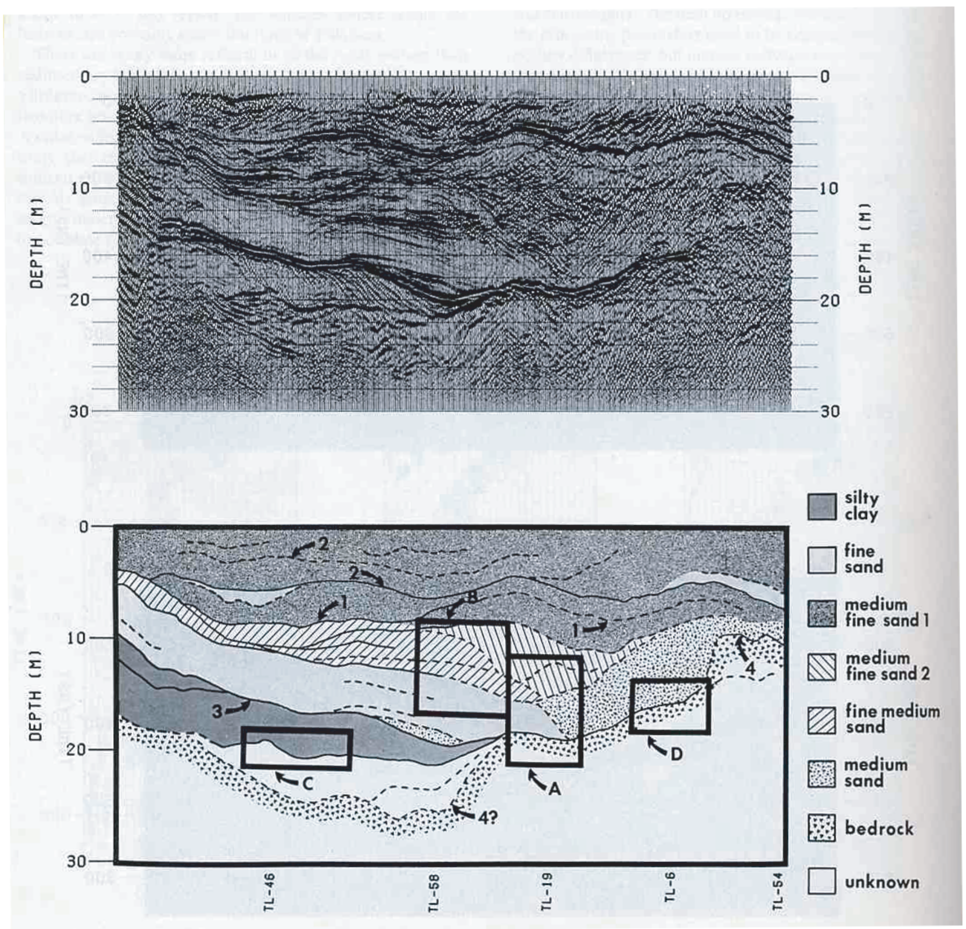


Fig. EM.12: Example of ground penetrating radar over buried river channel. The 30 m vertical scale corresponds to two-way travel times of about 900 ns.

Here is a table of typical attenuation factors and velocities:

	ϵ_r	$\sigma(S/m)$	$\rho(\Omega m)$	V (m/ns)	α (dB/m)
air	1	0		0.3	0
fresh water	80	5×10^{-4}	2000	0.033	0.1

seawater	80	3	0.3	0.01	1000
granite/sand	3-5	1×10^{-5}	100,000	0.15	0.01
wet sand	20-30	$10^{-4} - 10^{-5}$	1,000-10,000	0.06	0.03-0.3

Radar can be used in three modes (Figure 27), reflection profiling, common mid-point gathering, or transillumination. Reflection profiling is most commonly used to map subsurface structure. However, it give no information about velocities, which must be assumed or estimated independently. Common mid-point gathers, on the other hand, are used to estimate velocity as a function of depth. Finally, transillumination is used in mines, boreholes, or on two sides of concrete structures to examine internal structure.

The equipment can transmit a continuous radar frequency, but most modern equipment transmits short (20 ns or so) pulses of radar energy and looks for discrete reflections between pulses. Because transmission speeds are so great, a very large number of reflections can be stacked during a measurement lasting only a few seconds. Penetration depths are on the order of a few 10's of meters, or much less in very conductive sediment or saltwater.