Gravity.

1. Introduction

The gravity method is used in exploration geophysics and geology to map **lateral variations in rock density**. Neglecting isostasy, on a geological scale the biggest signals come from the different densities of sedimentary and igneous (basement) rocks. For minerals exploration, metal oxide and sulphide minerals are very much more dense than the host rocks, making gravity "anomalies" a key component of drilling decisions. For petroleum exploration, stratigraphic highs and lows and salt bodies produce gravity signals that can be factored into geological models for drilling decisions. At the smallest scales, highly precise gravity surveys can be used to delineate tunnels, tombs, and other man-made cavities within otherwise solid rock.

The biggest gravity signal, of course, comes from Earth's main gravity field. This means that the variations related to near-surface geological structure have to be measured in a background field that is tens of millions of times larger. Although fairly good ship-borne measurements can be made with automated equipment, and some air-borne gravity is being collected, the great majority of gravity data are collected site by site by people using delicate equipment requiring some specialized knowledge. Another important aspect of gravity surveying is that height needs to be measured to centimeter accuracy for the data to be correctly interpreted, adding another demanding aspect to the method. Nevertheless, most continents have been covered by gravity surveys and it remains one of the basic geophysical tools.

2. Basics: Force and Potential

The Earth's gravitation field is a result of the Earth's internal mass distribution, and so some knowledge of the internal mass may be obtained from surface measurements of the gravity field. This knowledge may be used for mapping regional structure, locating ore bodies, estimating ore reserves, mapping structural traps for petroleum.

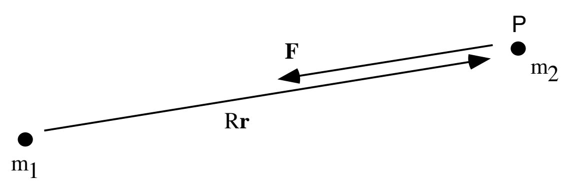

Figure 1: Attraction of point mass m_1 on point mass m_2 located at point P.

We know from Newton's law that masses attract each other, and that the **gravitational force** of attraction on a point mass m_2 due to a point mass m_1 a distance R away is

$$\mathbf{F} = -G\frac{m_1 m_2}{R^2}\mathbf{r}$$

where G is the gravitational constant, 6.6738×10^{-11} m^3 kg^{-1}s^{-2} and **r** is the unit direction vector from m_1 towards m_2. If the force on a body at the Earth's surface is required, $m_1 = m_e = 5.976 \times 10^{24}$ kg and $R = 6.37816 \times 10^6$ m (at the equator). Thus the force on a 1 kg mass is 9.804 kg m s^{-2} (or Newtons). (We have used a result which we shall prove later that says that a spherical mass distribution produces the same gravitational force as a point mass of the same size). The gravitational force per unit mass at P (also called the **gravitational field**), which from $F = ma$ we know to have the units of acceleration, is

$$\mathbf{g} = -G\frac{m_1}{R^2}\mathbf{r} \quad .$$

Thus at the surface of the Earth gravitational acceleration is 9.8 m s^{-2}. In cgs units, this is 980 cm s^{-2}, or 980 gal. In exploration, the customary unit is the **milligal** (1 mgal = 10^{-3} gal). As a result, we have that 1 mgal = 10^{-5} m/s^2. The milligal is slowly being replaced by a more SI-friendly unit, the gravity unit), where 1GU = 10^{-6} m/s^2. A difference of 10 is hard to notice sometimes, so one always needs to check what units are being used.

The first experiment to measure gravitational force in the laboratory was that of Henry Cavendish in 1798, known universally as the Cavendish Experiment. The objective was to measure average Earth density, which Cavendish obtained as 5448 kg/m^3, and he didn't actually compute G. This was done over 70 years later, using Cavendish's results, and was $G = 6.74 \pm .047 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, accurate to 1%.

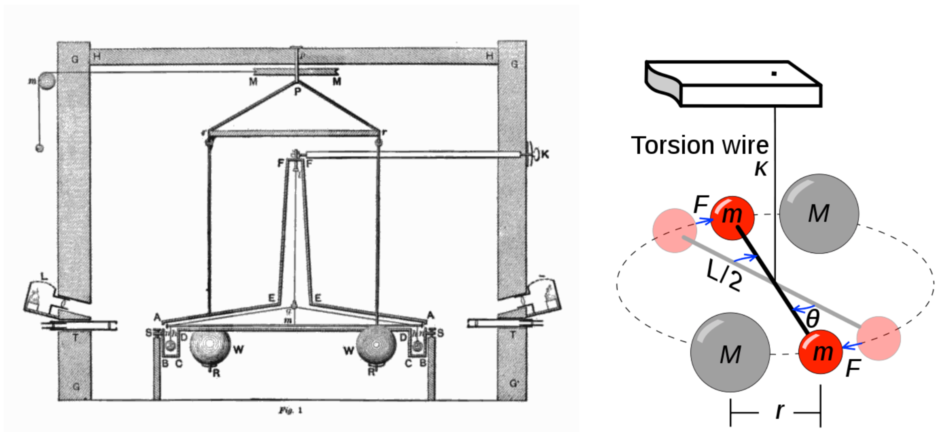

Figure 1b: The Cavendish experiment.

Gravitational potential at a point, P, is defined as the work required to move a unit mass from an infinite distance to the point P. Work is force times distance, so we can compute the potential by integrating the force per unit mass from infinity to P:

$$U_P = \int_{\infty}^{P} \mathbf{g}.d\,\mathbf{r} = Gm_1 \int_{\infty}^{P} \frac{dr}{r^2} = -\frac{Gm_1}{R} \quad .$$

Gravitational potential is always negative because you gain work as you move masses closer to each other. Note that the gravitational force is a **vector** (i.e. it has a magnitude and a direction) but that potential is a **scalar** (i.e. it has magnitude only). Both are functions of three dimensional space. Dealing with potential is often easier than dealing with the field directly. The gravitational field is obtainable from the potential by differentiation:

$$\mathbf{g}_P = -\nabla U_P \quad .$$

The potential is **conservative**, that is, it doesn't matter what path the mass takes to go from point to point, the total work done is the same. Potential fields are also additive, or linear, so the effect of a large mass may be obtained by integrating the effect of all the constituent infinitesimal elements:

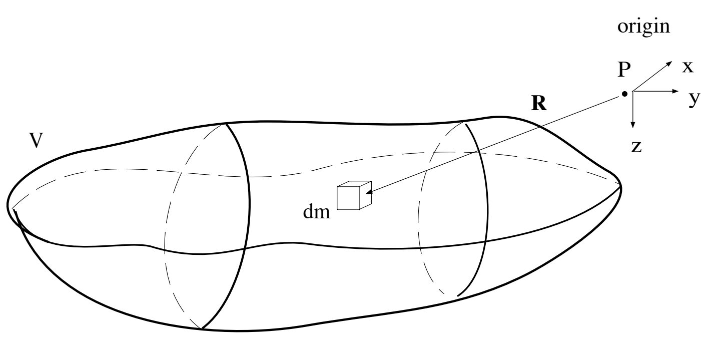

Figure 2: Potential at point P due to the volume element dm is integrated over the volume V to find the effect of the mass on the potential field. We have taken the special case where P is at the origin in order to simplify the mathematics.

To find the potential at P due to the mass in volume V, we integrate the effects of the elements dm:

$$U_P = \int_V dU = -\int_V \frac{G}{|\mathbf{R}|} dm$$

which in Cartesian coordinates is

$$U_P = -\int\int_V\int \frac{G\rho}{R}\, dxdydz\ .$$

We are no longer considering point masses, so we have introduced density to relate volume to mass: mass = volume × density. The SI units of density are kg.m^{-3}, but the cgs units of g.cm^{-3} are very commonly used. Density is usually expressed using ρ as here, but σ is sometimes used. (This is confusing, because σ is better reserved for mass per unit area, or surface density.) If density does not vary with position in the body, it may be taken outside the integration:

$$U_P = -G\rho \int\int\int \frac{dxdydz}{R}.$$

The gravitational field may be obtained from the potential by differentiation, as before. We shall see that it is the vertical (z) component of gravity which is of most interest in exploration, so

$$g_z = -\frac{\partial U}{\partial z} = G\rho \int_V \frac{z}{R^3}\, dxdydz$$

(we have used $R = \sqrt{x^2 + y^2 + z^2}$ here).

3. Rock Densities

Mass is simply density times volume ($m = \rho V$), so for a fixed volume Earth it is variations in density that determine the gravitational field outside the earth.

Densities of rock-forming minerals:

Mineral	Density, kg/m^3
Water	1000
Quartz	2650
Orthoclase	2550
Albite	2620
Anorthite	2760
Olivine	3330
Enstatite	3120
Diopside	3280
Hedenbergite	3550

Densities of oxide minerals:

Magnetite	5000
Chromite	4360
Cuprite	6000

Densities of sulfide minerals:

Galena	7500
Pyrite	5000
Sphalerite	3750

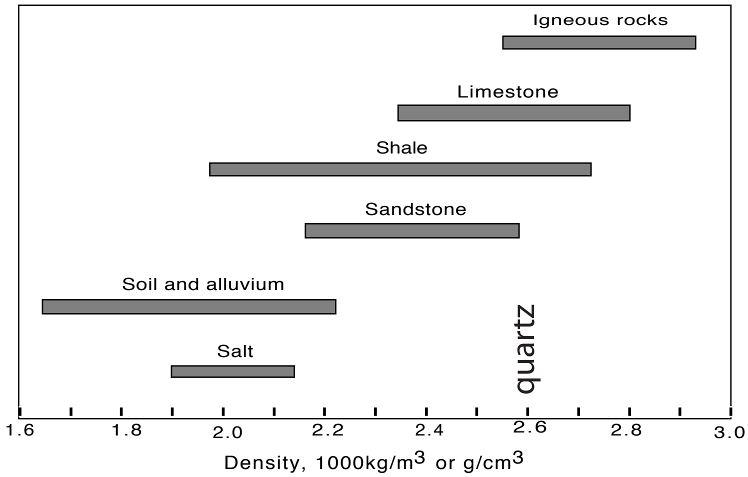

Figure 3: Ranges in density for various rock types (from Grant and West).

Figure 3, from Grant & West, gives a rough idea of the densities of various classes of rock. Note the scatter in the values for similar rock type; estimates from tables such as these will be of limited use for the analysis of specific gravity field data. They give us the approximate relationship

ρ soil $<$ ρ sedimentary rocks $<$ ρ igneous rocks

but there is some overlap between these catagories. The factors which affect rock density are:

a) Porosity and saturation:

ρ dry porous rocks $<$ ρ wet porous rocks $<$ ρ non-porous rocks

If the fractional porosity is ϕ then the density of a saturated rock is

$$\rho_{wet\ rock} = \rho_{grains}(1-\phi) + \rho_{water}.\phi$$

and a dry rock is

$$\rho_{dry\ rock} = \rho_{grains}(1-\phi).$$

If the rock is partially saturated, by a fraction S, then

$$\rho_S = \rho_{grains}(1-\phi) + \rho_{water}.S.\phi$$

b) Age: As a rock gets older, it gets more dense because compaction, cementation and secondary mineral growth in pores reduces porosity.

c) Depth of burial: As depth increases, overburden pressure increases and so compaction increases. It appears that compaction is most important for clays and not important for sands, which mainly lose porosity by cementation. This is seen in Figure 4, illustrating porosity versus depth. Remember that deeper rocks tend to be older, so two factors are being illustrated.

d) The SiO_2 content of igneous rocks: the greater the content of these lighter elements the lower the density.

e) Metamorphism: Increases ρ because of loss of porosity and growth of high pressure (denser) mineral phases.

f) Metal content: Metal oxides and sulphides are very dense.

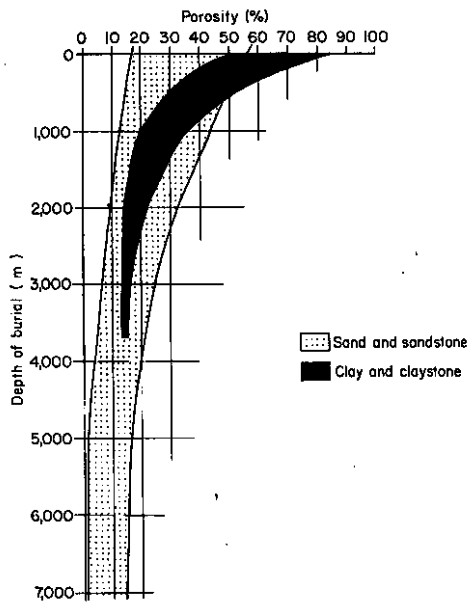

Figure 4: Relationship between porosity and depth of burial in sedimentary basins that have not undergone metamorphism. The rapid loss of porosity in clay is due largely to compaction. The gradual loss of porosity in sands is largely due to cementation. The spreads of data are principally due to variations in thermal and pressure gradients and to mineralogical differences (From Selly, 1976: *An Introduction to Sedimentology*, Acad. Press) .

Methods for estimating the density of rocks are:

a) Laboratory studies. Water saturation must be matched to that *in situ*. One cannot estimate the effect of large cracks, fissures, faults in the laboratory. One must be careful not to use weathered specimens. However, if a core is available and the densities of a number of samples are averaged, a good result is possible.

b) Borehole density logging, see page 794 of Telford *et al.* A gamma ray source is pressed against the wall of a borehole and the number of rays which are scattered into a detector are counted. The gamma rays energetic enough that they may be thought of as particles and 'bounce' off electrons in a billiard ball fashion, transferring momentum. The number of electrons is related to density.

c) Sesimic velocity and the Nafe-Drake relationship (see Figure 5).

d) Borehole gravity measurements. As a gravimeter is lowered down a borehole, **g** gets smaller by an amount proportional to the density of the rocks above the meter.

e) Nettleton's method, which will be discussed later after we have studied the Bouguer correction.

4. Gravity Meters

Gravity measurements may be *absolute*, that is the full value of **g** is measured, or *relative*, where only variations in **g** from place to place are measured. For exploration purposes only relative measurements are sufficient, because comparatively small and shallow features are of interest rather than the deep structure of the Earth, which contributes most to the value of **g**. Torsion balances, pendulums and barometers have all been used to measure relative gravity in the past, but today most instruments are spring-based mechanical devices having a sensitivity of about 0.01 mgal.

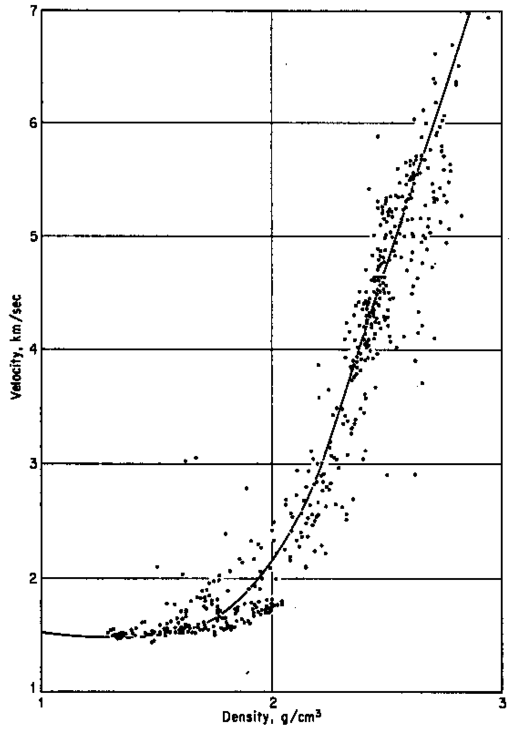

Figure 5: A plot of seismic p-wave velocity against density. The empirical relationship between the two physical parameters is called the Nafe-Drake relationship.

Absolute gravity measurements were made in the past with pendulums, by measuring the period of oscillation:

$$T = 2\pi\sqrt{l/g}$$

where l is the length of the pendulum. Now measurements of absolute **g** are most accurately made by measuring the free fall of a prism in a vacuum using laser interferometery. This method offers the same sensitivity as relative meters (about 0.01 mgal), but is too slow and expensive for exploration. The very best measurements are made using devices which levitate a superconducting mass, but these are not transportable and serve as observatories to measure temporal variations in **g**.

The sensitivity of a mechanical gravimeter is given by dx/dg, where x is the deformation of the spring system used to make the reading. Gravimeters are classed as stable or unstable, depending on whether dx/dg is a constant or depends on g (that is, whether x is a linear or non-linear function of g). The terms static and astatic are also used.

<u>Linear meters:</u> Linear meters have a mass, M, suspended from a spring with spring constant k. They are not used today but serve as an introduction to gravimeter design. Balancing the spring force against gravity

$$k(x - x_0) = Mg \quad .$$

Differentiating yields

$$\frac{dx}{dg} = \frac{M}{k} \qquad or \qquad dx = \frac{M}{k}dg \quad .$$

To achieve a sensitivity of 0.1 mgal, or $10^{-7} \times g$, a variation of 10^{-6} cm (0.2 times the wavelength of visible light) must be measured in a 10 cm spring.

A gravimeter's sensitivity may be related to its period of oscillation (*c.f.* a pendulum). For a mass suspended from a spring,

$$T = 2\pi\sqrt{\frac{M}{k}} \quad .$$

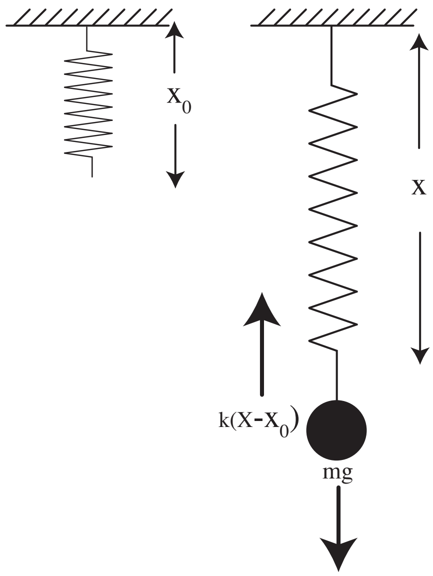

Figure 6. Linear gravity device (mass on a spring).

That is

$$\frac{M}{k} = \frac{T^2}{4\pi^2} \quad .$$

Substituting into the sensitivity expression:

$$dx = \frac{T^2}{4\pi^2} dg \quad .$$

Thus we see that T must be made as large as possible. For a linear system this means making M/k large (i.e. large masses and long springs). Clearly, there is a limit to this, but T can be increased by going to a non-linear system.

Non-linear Gravimeters: Figure 7 represents the principles of operation of a Lacoste-Romberg gravimeter. Here x_o is again the length of the spring at no load, and the apparatus is assumed to be at equilibrium. Balancing the torques due to the mass' weight and the spring force we have

$$aMg = bk(x - x_o) \cos\phi = bk(x - x_o)\frac{y}{x} \quad .$$

$$g = \frac{bky}{Max}(x - x_o) \quad .$$

(when the bar is horizontal). Observe what happens when g gets smaller. The spring will lift the arm up because of the imbalance in the torques and ϕ will get smaller, so $\cos\phi$ gets larger and a greater proportion of the spring force is transferred to the beam, lifting the arm up even more. We may demonstrate this increased sensitivity by differentiating the above expression:

$$\frac{dg}{dx} = \frac{bkx_o y}{Max^2}$$

$$dx = \frac{aMx^2}{bkx_o y} dg$$

The sensitivity may be increased only so much by making the ratio aMx^2/bky large, but x_o may be made very small by using a *zero length spring*. Such a spring is wound under tension as described in Telford *et al.* so that the residual

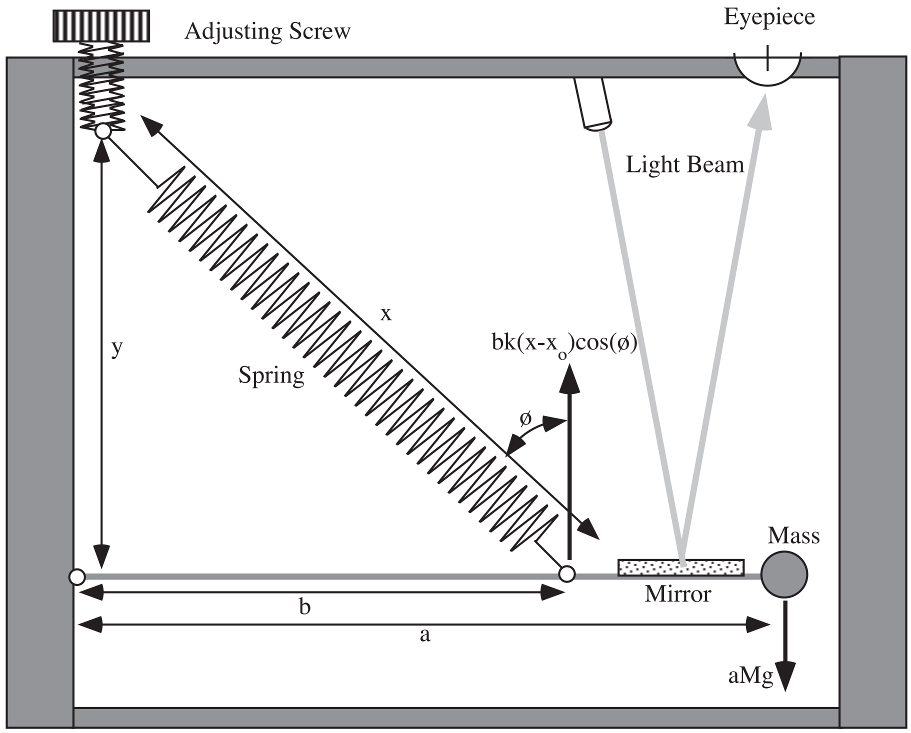

Figure 7: Schematic diagram showing the operation of a non-linear gravimeter of the LaCoste-Romberg type.

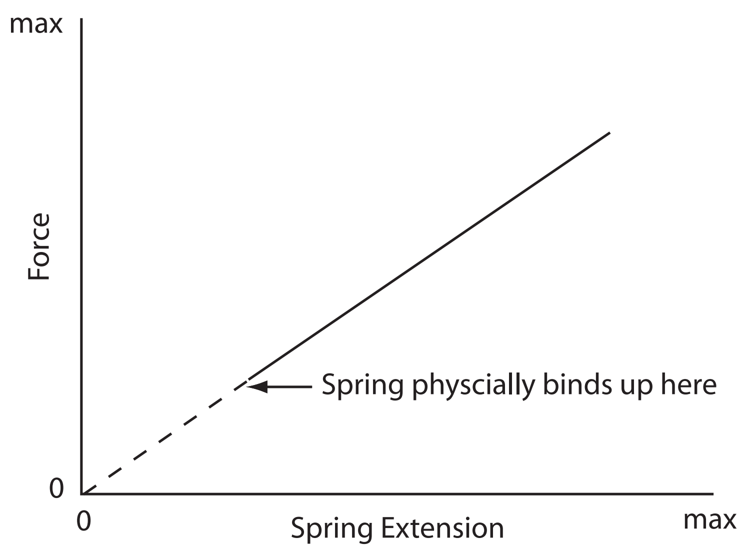

Figure 8: Concept of a zero-length spring. Spring force is proportional to extension, and at some point the spring coils physically bind up. However, if the force versus extension curve is extrapolated to zero force, it passes though the origin and zero extension.

force exerted by the spring when it is fully contracted is equal to the spring constant times the length of the contracted spring.

In practice such a device is operated as a null instrument, that is the actual displacement, which will no longer be linearly dependent on g, is not measured but rather the amount of adjustment, to y and x in this case, required to bring the arm back to the horizontal state is measured. The precise position of the arm is monitored by bouncing a light beam off a mirror and into a graduated eyepiece. Such sensitive instuments, capable of measuring 1 part of g in 10^8, are easily affected by environmental changes. Changes in air pressure will alter the buoyancy of the masses, so the case must be sealed. Temperature has a huge effect because of the thermal expansion and contraction of the mechanism, so the case is thermostatically heated in the Lacoste-Romberg meter. (Another meter once in general use, the Worden, has temperature compensating bimetallic springs and is housed in a dewar bottle.) As dislocations in the construction material move, the mechanism creeps, producing sudden offsets or tares. Needless to say bumping such a device is very bad, and dropping a gravimeter? Well ...

Any motion of a gravimeter during measurement is problematic, as vertical acceleration makes the beam bounce up and down and horizontal acceleration couples badly into the beam as well. In spite of this, shipborne measurements can be made by putting the meter at the center of motion of the ship, mounting it on a gimbaled platform, and heavily damping the mechanisms. Shipborne meters are read automatically, rather than by human operation.

Gravimeters are calibrated by occupying two or more sites at which g is known, often from absolute measurements, and assuming a linear relationship between number of screw turns and Δg.

A gravity survey consists of measurements made with a gravimeter in a pattern over the surface of the earth. The station spacing depends on the problem being solved. Regional surveys may have station spacings of 10 km or so, while detailed engineering or archaelogical work might require spacings of only a few meters. If variations in a lithological interface are being mapped, the station spacing will need to be half the depth to the interface or less. If the target structure is considered to be two dimensional (2D) then only a line of stations might be occupied, otherwise the stations will be spaced on a 2D grid. However, before using gravity to study geological structure or map ore bodies, variations in gravity due to other factors must be understood. The gravity field of the Earth varies with

a) earth tides

b) and drift over time due to creep and temperature changes.

c) surface density and topography

d) change in elevation

e) change in latitude

All these effects may be much larger than the anomalies petroleum and mining geophysicists are interested in, so they have to be estimated and removed from raw gravity data. To understand the latitude correction we need to understand the gravity field of the Earth.

5. Earth's gravity

The geoid. An equipotential surface is is simply a surface over which the potential is everywhere equal. So, by definition, it requires no work to move over an equipotential surface, and it follows that **g** is always perpendicular to the equipotential surface. We have seen that for a point or spherically symmetric masses the equipotentials are spheres, easily verified by setting

$$U = \frac{Gm}{R} = \text{constant}$$

so $R =$ constant also, defining spheres. Gravitational acceleration, **g**, is also constant. However, it is not necessarily true that **g** is constant across equipotential surfaces, as it is for a sphere.

On Earth there is a special equipotential surface called the geoid, the equipotential corresponding to mean sea level. It

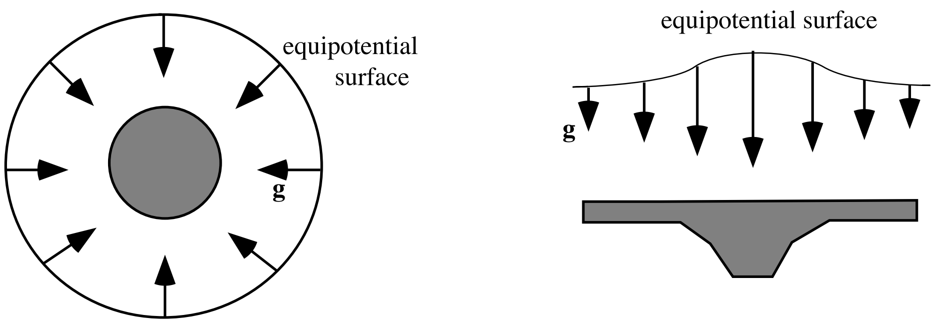

Figure 9: Equipotentials and gravity over spherical and non-spherical mass distributions. **g** is constant over spherical equipotenials, but not over equipotentials associated with more complicated mass distributions.

is clear that the sea surface is an equipotential, because if there were a potential *gradient* along the surface there would be a gravitational force along the surface and the water would flow sideways. The geoid is defined on land, and may be visualized as the mean sea level of water in narrow canals connected to the oceans. The geoid is very complicated, and includes the effects of all anomalies at all size scales, as well as the effect of Earth's rotation. A simpler surface, which we can use to remove the effects of Earth's shape and rotation from gravity data, is the *reference spheroid or ellipsoid*. This is the oblate spheroid which fits the geoid as well as possible, so **g** is very nearly perpendicular to it everywhere and is given by the *international gravity formula*:

$$g = 978031.846(1 + .005278895 \sin^2 \lambda + .000023462 \sin^4 \lambda) \text{ mgal}$$

where the direction of g is assumed to be downwards, and λ is the latitude being considered (this is the 1967 version that appears in Telford et al., others exist). Figure 10, from Dobrin, shows the difference between the geoid and the sheroid. One can see directly from the gravity formula that sea-level gravity varies 0.5% (5,000 mgal) over the surface of Earth, from 9.7803 m/s^2 at the equator to 9.8322 m/s^2 at the poles with a mean value of 9.806 m/s^2. Gravity is lower at the equator because of the combined effect of Earth's oblateness (the equatorial radius is 22 km larger than the polar radius) and rotation (the outwardly directed centrifugal force opposes gravity).

6. Gravity Reductions

We are now in a position to consider the processing required to convert raw gravity measurements into useful measurements (i.e. those that we can interpret). The gravimeter is only a relative instrument which will give us the difference in gravity between two points, so one of the stations in a survey must be designated a base station. This may be a point at which the absolute gravity is known, or simply a point which will be designated as having zero gravity anomaly.

Drift and tide corrections: If a single gravity station were occupied continuously, the reading of the gravimeter would not remain fixed but rather would drift. The drift is due to tides, which have a maximum amplitude of 0.3 mgal, temperature changes, and creep in the meter's mechanism. The temperature coefficient and mechanism drift depend very much on individual meters.

The tidal variations are mainly caused by the varying gravitational effects of the sun and moon, but also include the Earth's deformation in response to these effects (several tenths of a meter!). This latter effect accounts for abut 20% of the tidal vatiations. The tidal variations in gravity may be predicted, but often are incorporated in the drift curve. In order to monitor the drift of a meter, base stations must be occupied every hour or so during a survey. Base station readings may be made less frequently if some form of leapfrogging is carried out, but still need to be occupied regularly to eliminate closure errors. The drift between base station readings may be assumed to be linear, or models of tidal variations may be used to draw smooth curves through the drift data. The accurate removal of drift necessitates the recording of time during every gravity measurement.

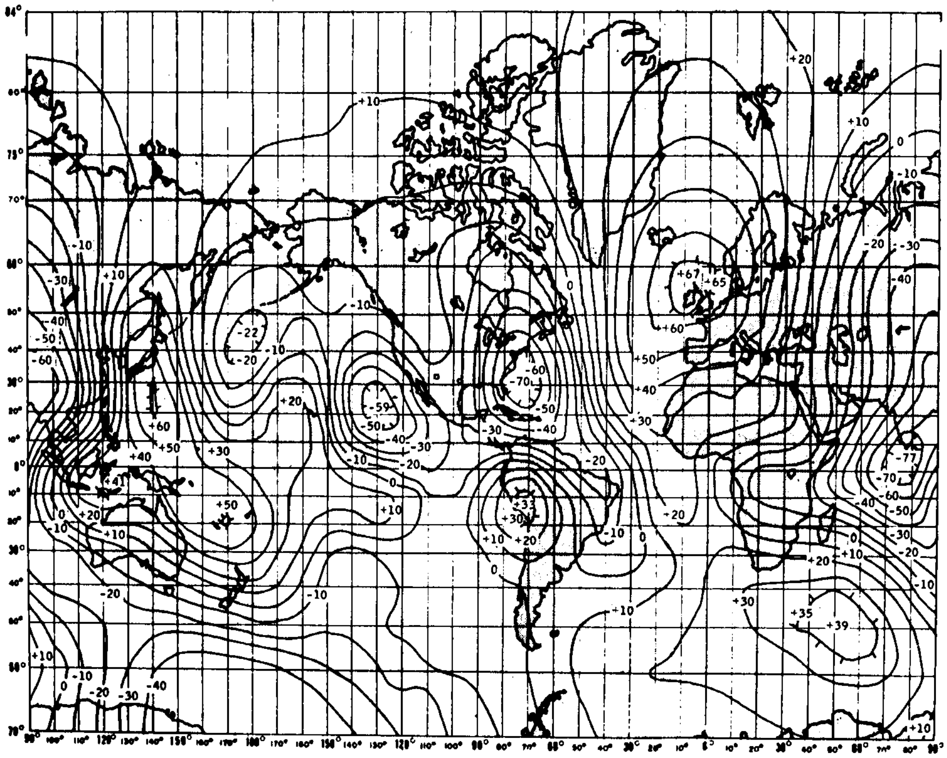

Figure 10: Difference between the geoid and reference spheroid, in metres. (from Dobrin).

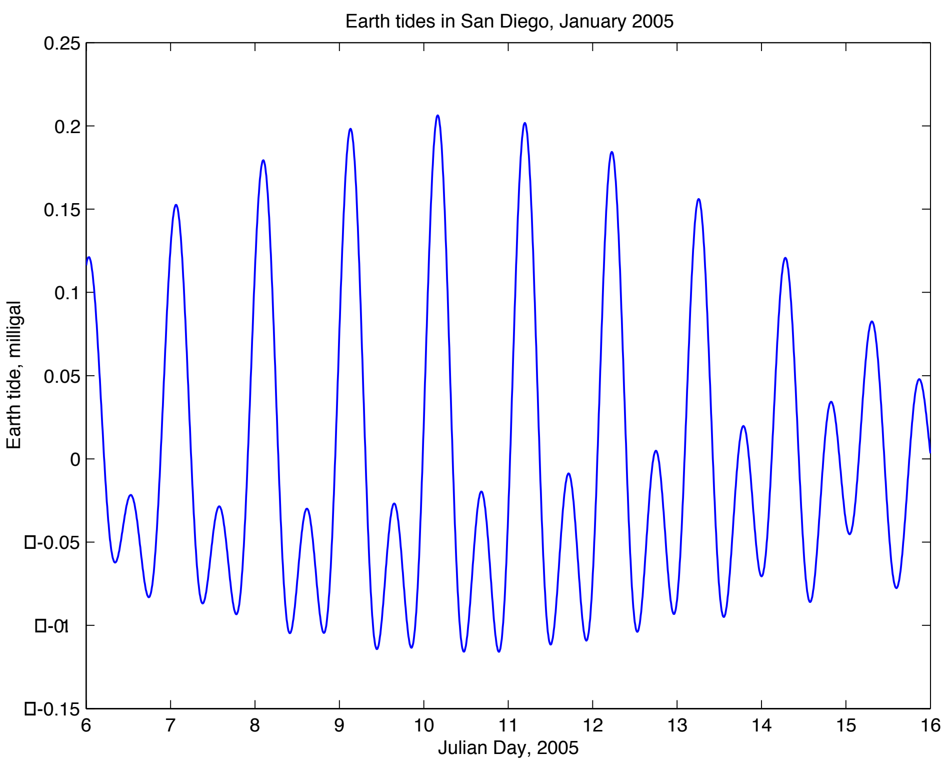

Figure 11. Tides for mid-January 2005 at the latitude and longitude of San Diego.

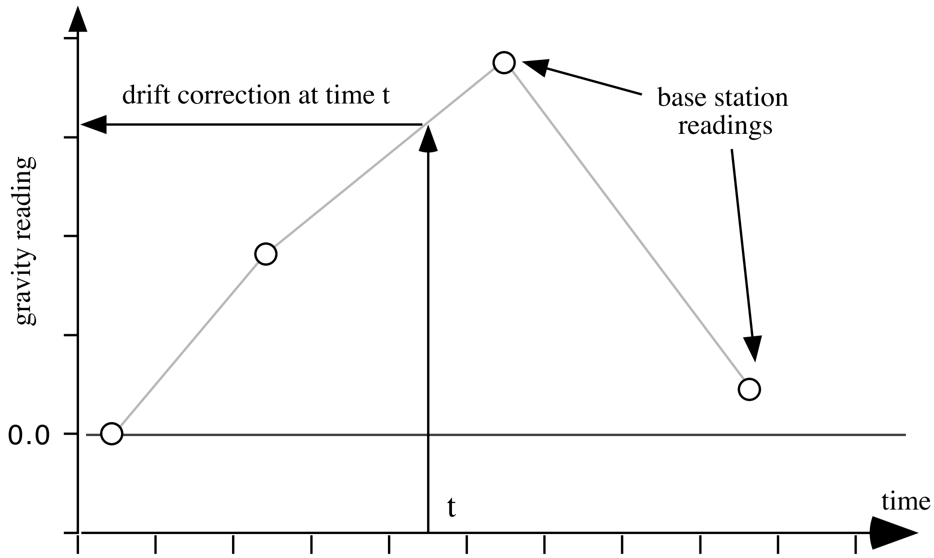

Figure 12: A drift curve. In this example 4 base-station readings have been made. Any other gravity reading, made at a time between base station measurements, can have the drift correction estimated by linear interpolation.

Figure 12 illustrates a drift curve. A base station is occupied at the start of a gravity survey and re-occupied every hour or so. The meter readings are plotted in relation to the first measurement and a curve drawn through the base station data. Now the drift at any time can be estimated, and then removed, from any gravity reading.

If a gravity survey is conducted over several days, the base station must be read at the start of each day, and also after any breaks. If the survey extends into an area too far from the original base station, a new base station must be tied to the first by repeated measurements between the two stations, so that measurements tied to the second station may ultimately be tied to the master base station and thus the rest of the survey.

If a meter is knocked or jolted, it is possible to produce a sudden offset, or tare. Unlike drift due to tides and creep, the effect of a tare should not be distributed evenly over data between base measurements. Rather, if the meter is jolted, the last station should be occupied immediately to check for a tare and, if necessary, estimate it. This then becomes a correction to be applied to all future readings, in addition to the drift correction. If a base station reading is found to be much different from previous readings, a tare should be suspected and the last set of gravity data searched for a sudden offset.

Latitude correction. We have seen that Earth's gravity varies 5000 mgal as one goes from the equator to the poles. The international gravity formula could be evaluated for every station in a gravity survey, but unless one is conducting a regional study it is more convenient to *linearize* the formula to obtain a correction for each kilometer of north-south distance from the base station. We can write the gravity formula as

$$g = g_o(1 + \alpha \sin^2 \lambda + \beta \sin^4 \lambda)$$

then

$$\frac{dg}{d\lambda} = 2g_o\alpha \sin \lambda . \cos \lambda + 4g_o\beta \sin^3 \lambda . \cos \lambda$$

If we neglect the term in β and use $2 \sin \lambda \cos \lambda = \sin 2\lambda$ we get $dg/d\lambda = g_o\alpha \sin 2\lambda$. To obtain a formula in north-south distance s, rather than latitude, we have

$$\frac{dg}{ds} = \frac{d\lambda}{ds}\frac{dg}{d\lambda} = \frac{g_o\alpha}{r} \sin 2\lambda$$

Substituting $g_o = 978$ gal and $r = 6378$ km then

$$\frac{dg}{ds} = 0.812 \sin 2\lambda \text{ mgal/km}$$

This correction is subtracted from stations further from the equator (i.e. more northernly stations in the northern hemisphere).

Free air correction. Variations in elevation result in variations in gravity as a consequences of the inverse square law. The gravity at a distance r from Earth's center, is

$$g = G\frac{m}{r^2}$$

Again, we can obtain a correction for small variations in altitude by differentiating:

$$\frac{\partial g}{\partial r} = -2\frac{Gm}{r^3} = -2g_o/r$$

This time we substitute $r = 6.367 \times 10^6$ m (an average value) and $g_o = 980629$ mgal (from the gravity formula at 45°) we obtain -0.3086 mgal/m. (Equatorial values for r and g yield -0.3067 mgal/m and polar values -0.3093 mgal/m, but this small (1%) effect is usually ignored, as is the correction for extreme elevations.) Because of this dependence on altitude, some level must be designated as the datum level and all measurements corrected to that height. The negative sign implies that gravity gets smaller with altitude, so the correction of 0.3 mgal/m is added to measurements made above the datum. Raw gravity corrected for latitude and elevation is called the free air gravity or free air anomaly. Note that to preserve an accuracy of 0.01 mgal, elevation must be known to about 3 cm. For a regional gravity survey, aneroid barometers may be used to measure elevation, but since these devices are precise to 0.5 m at best, they are not useful for mining or petroleum gravity surveys. To achieve the required elevation control, all stations must be occupied by a surveying team; an operation which usually costs many times that of collecting the gravity data. Today the GPS system may be just good enough for gravity surveying – although not optimized to provide elevation (c.f. lateral position), centimeter level accuracy can be achieved using differential, kinematic GPS surveys.

Bouguer correction. For two stations at different altitudes, it is not only the difference in distance from the Earth's centre that affects gravity, but also the different amounts of mass between the stations and the datum level.

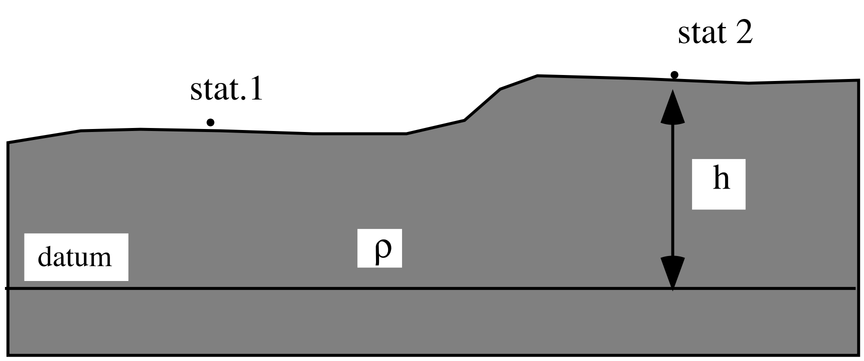

Figure 13: Illustration showing the need for the Bouguer correction, to compensate for the attraction of the mass beneath station 2 that does not exist beneath station 1.

To correct for this matter, we must compute its effect. The Bouguer correction makes the approximation that the Earth's surface is flat and at the same elevation as the station being considered, i.e. a slab of infinite extent (deviations from this approximation are corrected, if necessary, by making the terrain correction described below). Recall from page 2 that

$$U = -\int_V \frac{G\rho}{R}\, dxdydz\ .$$

We will do the computation in cylindrical coordinates, remembering that the volume element is now $r.dr.d\theta.dz$. Thus

$$U = -G\rho\int_V \frac{r}{R}drd\theta dz = -G\rho\int_V \frac{r}{\sqrt{r^2+z^2}}drd\theta dz$$

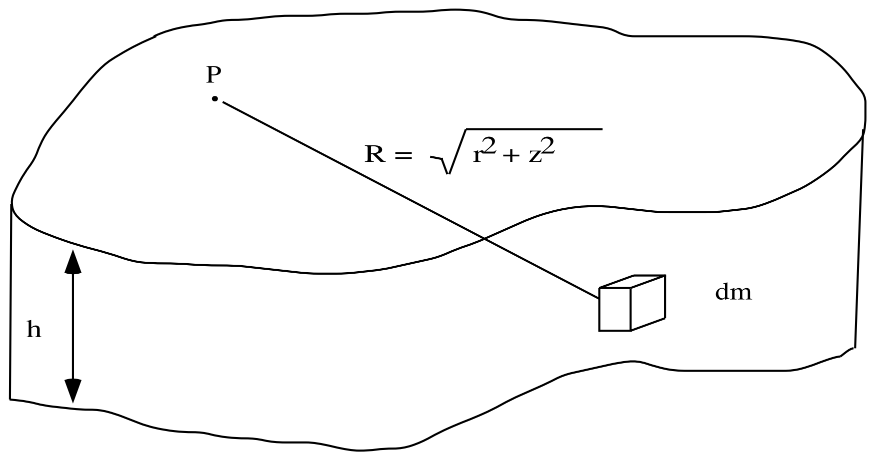

Figure 14: To compute the Bouguer effect, we integrate the effect of an infinite slab of vertical thickness h on the point P, on the surface of the slab. This is most easily accomplished in a cylindrical coordinate system.

$$g_z = -\frac{dU}{dz} = G\rho \int_V \frac{zr}{(r^2+z^2)^{3/2}} dr d\theta dz$$

so using integration limits which define an infinte slab we have

$$g_z = G\rho \int_0^h dz \int_0^\infty dr \int_0^{2\pi} \frac{zr}{(r^2+z^2)^{\frac{3}{2}}} d\theta$$

There is no dependence on θ so

$$g_z = G\rho \int_0^h dz \int_0^\infty dr \left. \frac{zr\theta}{(r^2+z^2)^{\frac{3}{2}}} \right|_{\theta=0}^{2\pi}$$

$$= 2\pi G\rho \int_0^h dz \int_0^\infty \frac{zr}{(r^2+z^2)^{\frac{3}{2}}} dr$$

$$= -2\pi G\rho \int_0^h dz \left. \frac{z}{(r^2+z^2)^{\frac{1}{2}}} \right|_{r=0}^{\infty}$$

$$= 2\pi G\rho \int_0^h dz$$

$$= 2\pi G\rho z|_{z=0}^h = 2\pi G\rho h$$

or 0.04188ρ mgal/m if ρ is expressed in the cgs units of g.cm^{-3}. Since we are trying to remove the effect of the slab, this correction is *subtracted* when the station is above the datum. The average crustal density is 2.67 g/cm^3, and this value may be used if no sensible estimate of the near-surface density is made. However, it is most advisable to make a better guess than this, and the most effective way is using *Nettleton's method*:

A gravity profile is conducted over a small hill. The smallness of the hill supposedly ensures that variations in gravity due to geological structure are also small. Elevation and Bouguer corrections are made to the data using a series of

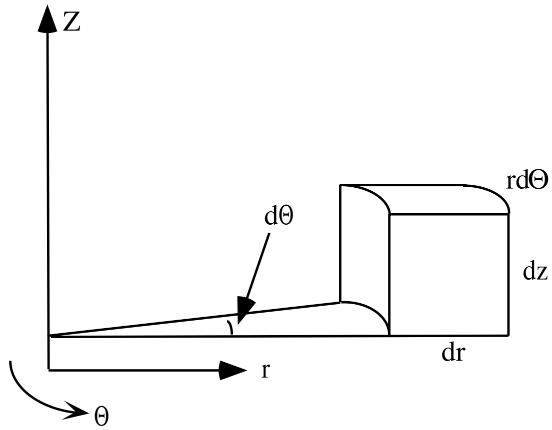

Figure 15: Volume element in cylindrical coordinates.

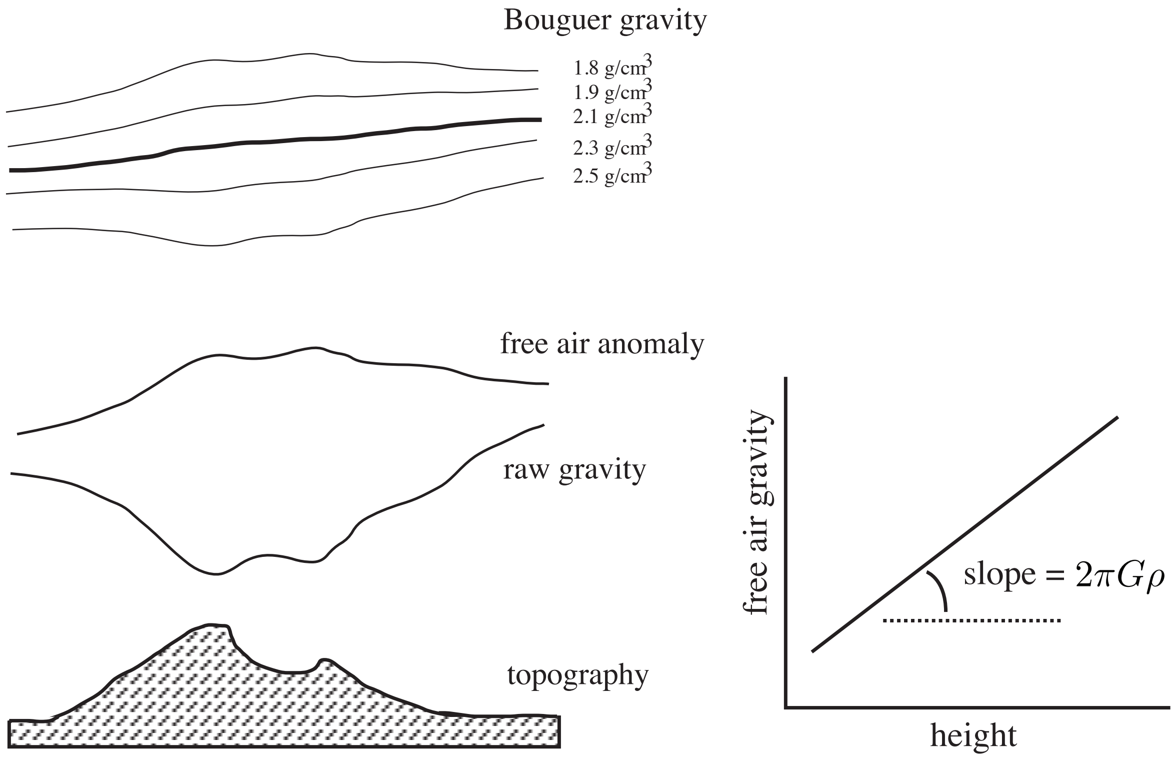

Figure 16: Nettleton's method. A gravity profile is made over a small hill and the Bouguer corrections made using various values of density. The density corresponding to that of the hill will have least evidence of topography in the signal. Alternatively, one can plot free-air gravity against elevation – the slope is just the Bouguer correction.

values for ρ. The profile dispaying the least correlation with topography was reduced using the most correct value of surface density. This may be chosen by eye, or the correlation minimised using statistical methods.

Valleys should not be used for Nettleton's method, because the sediments which accumulate at the bottoms of valleys are not usually typical of the surrounding hills. Data which are reduced using the latitude, elevation and Bouguer corrections are termed the Bouger anomaly or gravity. The choice of surface density can be checked after all the data in a survey have been reduced by again looking for a correlation of Bouguer gravity and topography. Beware, however, of correlations in geology and topography, which could well produce genuine gravity anomalies.

Terrain corrections: If the topography is severe a correction may be required for the effect of the surrounding hills and valleys, both of which *reduce* the gravity from the value which would be measured over a flat Earth. Corrections may be made manually by marking the station on an accurate topographic map and centering a transparent template, upon which is printed radially symmetric zones, over the station. The average elevation in each zone is estimated and tabulated and multiplied by a factor for each zone size before summing the effects of all the zones. The factors giving the gravitational effect of the radial zones are easily derived by integrating over the appropriate limits in cylindrical coordinates. Because both hills and valleys reduce gravity, the correction is always positive. Such manual computation of terrain corrections is very tedious. The calculations may be done by computer, but the elevations must be digitised, using a very fine grid spacing close to the gravity stations. The effort required to enter the topographic data makes the use of a computer prohibitive unless an extensive survey is being conducted or digital topography is available from some other source.

The free air correction, Bouguer correction, and terrain correction are all really about the same thing– correction for height. Why not combine them? Well, the free air correction is used for marine gravity, because the ocean is flat and the effects of topography on the seafloor are often unknown (in fact, satellite estimates of gravity are used to recover seafloor topography). Terrain corrections are complicated to make, and are not important except for extreme topography, and are usually ignored.

Regionals and residuals: Removal of regional gravity gradients is possibly the most important part of gravity interpretation. The part of the gravity data that is going to be interpreted in terms of structure, usually local structure, is often termed a gravity anomaly. The regional gravity field the becomes, by definition, that part of the gravity field which is not of primary interest. The object is to remove this part of the field to reveal the residual, the variation in gravity due to the structure which we hope to map, so that our interpretation and modelling reflect the local structure.

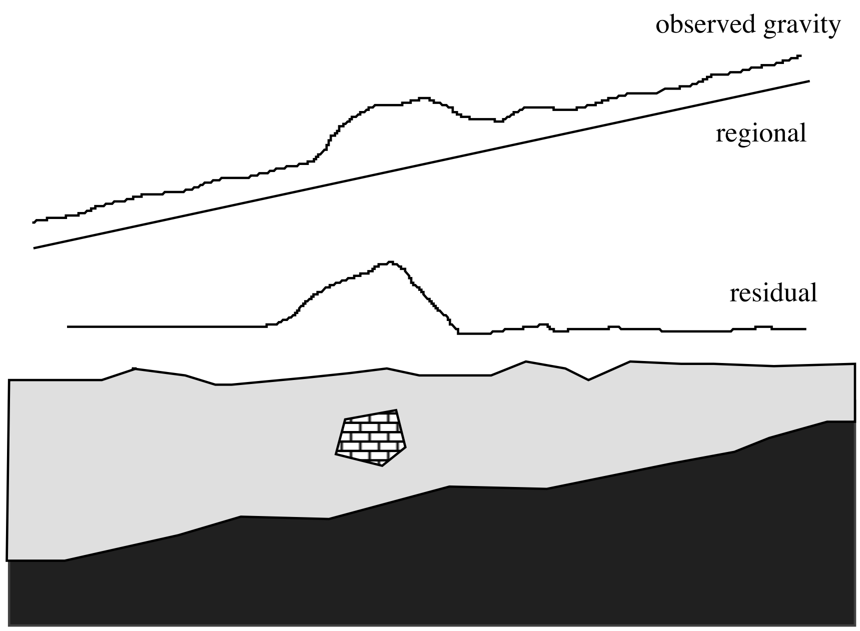

Figure 17: Distinction between regional gravity and residual gravity. The regional gravity field is caused by large-scale geological features, and imposes a trend on the small-scale anomalies that are of interest in local gravity surveys. The regional trends must be removed from the local surveys before modeling or interpretation is carried out.

The removal of the regional gravity assumes a knowledge we are not likely to posess, that is, a relatively complete understanding of large scale density structure. This makes the problem of regional removal very poorly defined and therefore difficult to solve. Regional gravity maps can provide a guide to trends in the gravitational field that can be removed from local gravity surveys. There are also several mathematical schemes used to separate regional variations from the observed gravity. These schemes are of assistance in the interpretation of gravity surveys, especially contoured maps (as apposed to profiles). However, the skill and knowledge of the geophysicist, who we expect to be familiar with the large scale variations in geological structure which are responsible for the regional variations in gravity, is ultimately the most reliable tool in the separation of regional and residual.

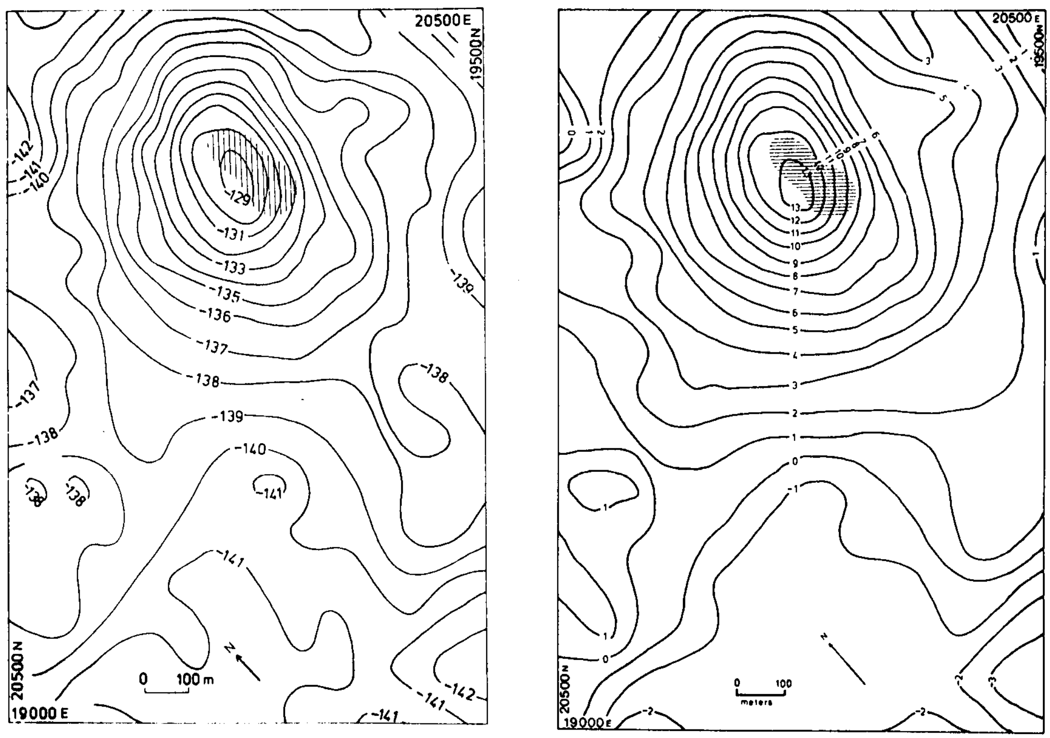

Figure 18: Bouguer gravity (left) over massive sulphide ore body (in southeastern Australia) and the same anomaly after removal of a regional gradient (right). The position of the ore body, determined by drilling, is marked by the shading.

7. Review of field practice

So, the steps in the collection and reduction of gravity data are (in order):

a) Collect the gravity data, recording time and position at each reading and returning periodically to the base station.

b) Convert the gravity readings to mgal.

c) Correct the gravity readings for drift and tides.

d) Correct the data for latitude, or N-S distance.

e) Correct the data for height using both free-air and bouger corrections and some assumption or measurement of bouger density.

f) Correct the data for topography, if needs be.

g) Remove regional gradients, if needs be.

At this point the gravity map or profile is available for interpretation, which is our next subject.

8. Gauss' Law

We are now going to establish a result which will be used later to compute the total mass of, say, an ore body, from gravity data. This is an extremely powerful use of the gravity method, and is the reason why gravity measurements might be made over an ore body that is already quite well known from drilling data (to estimate the total reserve).

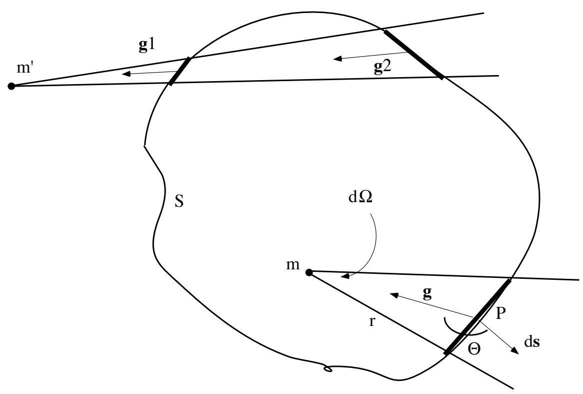

Figure 19: A closed surface S containing mass. Mass element m is inside S and mass element $m\prime$ is outside the surface.

Consider a closed surface, S, enclosing a point mass m (Figure 19). At point P, we can form an infinitesimal element of the surface S. A planar surface may be described by a vector whose magnitude is the area of the surface and whose direction is the (outward) normal to the surface, so in the above case $d\mathbf{s}$ describes the surface element.

Now we wish to introduce the concept of **flux**. The surface integral of a vector field **F**,

$$\int_S \mathbf{F} \cdot d\mathbf{s}$$

is called the flux of **F** through the surface S. Note that the dot product ($\mathbf{x} \cdot \mathbf{y} = x.y.\cos\theta$, where θ is the angle between **x** and **y**) has the effect of taking the component of **F** perpendicular to the surface. We want to compute the total flux through the surface S surrounding the mass m:

$$\int_S \mathbf{g} \cdot d\mathbf{s} = \int_S g.\cos\theta ds = \int_S \frac{G.m.\cos\theta.ds}{r^2}$$

Before performing the integration we note that

$$\frac{-\cos\theta.ds}{r^2} = d\Omega,$$

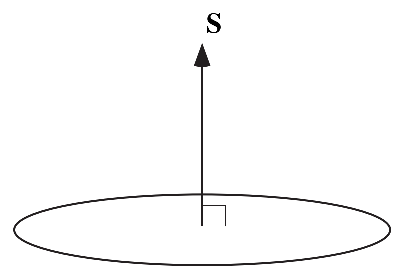

Figure 20: A planar surface may be described by a vector, **S**, whose magnitude is the area of the surface and whose direction is the (outward) normal to the surface.

where $d\Omega$ is the solid angle subtended at m by the surface element $d\mathbf{s}$. (The minus sign comes from the fact that θ is always $> 90^o$). Now we have

$$\int_S \mathbf{g} \cdot d\mathbf{s} = \int_{\text{unit sphere}} -G.m.d\Omega = -4\pi Gm.$$

We can ignore the contributions to the flux of all masses outside the surface S; referring to the diagram it may be seen that masses outside the surface subtend two surface elements with identical solid angles but with flux of opposite signs, cancelling. Thus we have the important result, *Gauss' Law*, that the total flux through a closed surface is equal to $-4\pi G$ times the mass enclosed by the surface. For an extended mass of density ρ

$$\int_S \mathbf{g} \cdot d\mathbf{s} = -4\pi G \int_V \rho \, dv$$

The equation derived above is very useful. For example, we can show that outside a spherical distribution of mass, the gravitational field is the same as though the mass were all concentrated at a single, central point:

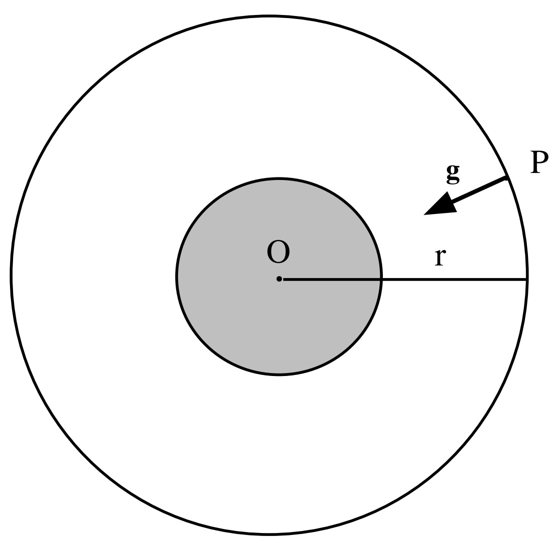

Figure 21: Spherically symmetric mass centered on O. An observation of **g** is made at point P, a distance r from the center.

Consider a spherically symmetric distribution of mass with centre O. If we take a point P a distance r from the mass' centre, a sphere through P centred on O clearly has **g** perpendicular and equal over its entire surface, so it is easy to compute the surface integral;

$$\int_S \mathbf{g} \cdot d\mathbf{s} = -4\pi r^2 g$$

Applying Gauss' Law this is equal to

$$-4\pi GM = -4\pi r^2 g$$

where we have set the total mass of the sphere to M. Thus the gravitational attraction anywhere outside the sphere is given by

$$g = G\frac{M}{r^2}$$

which is the same expression for a point mass at O. Remember that we used this result to compute the gravity at the surface of the Earth knowing only the Earth's total mass. But now consider the problem of predicting the above mass distribution from gravity measurements. One cannot distinguish any symmetric mass distribution from another. This is an example of **non-uniqueness** in geophysical prospecting. There are many others. As a general rule, geophysics cannot tell us exactly what the internal structure of the Earth is, and this must be remembered at all times during the interpretation of geophysical data. Geophysics is useful when a sensible assumption may be made (e.g. that the above sphere is of uniform density), when a unique parameter such as total mass may be estimated, or when additional data is required to distinguish between several geologically reasonable hypotheses.

This exercise illustrates another aspect of gravity surveying. It is only **lateral variations** in rock mass or density that are discernible. Layered or radially symmetric variations in density are never resolvable from the outside of the mass distribution, which is where we always are at the surface of the earth.

We also note that since the mass of Earth is large, and it is mostly radially symmetric in density, the field is dominantly vertical at the surface. Indeed, the direction of gravity is normally used to define the vertical. Thus, when we consider the effect of lateral variations in the gravity field, we will be looking only at the effect on the vertical field.

9. Excess mass calculation in gravity.

From Gauss' Law (pages 4-6) we have that the normal component of gravitational acceleration integrated over a closed surface is proportional to the total mass enclosed within that surface:

$$\int_S \mathbf{g} \cdot ds = 4\pi \mathrm{GM}$$

In geology we are not working with masses in free space, but rather differences in density between host rocks and the structure of interest (*e.g.* a metalliferous ore body), so instead of total mass we must consider excess mass, that is, how much more mass is represented by the structure than would be there if only the host rock existed. Quantitatively,

$$\text{excess mass} = \int_V (\text{density}_{\text{structure}} - \text{density}_{\text{host}})\, dv$$

The excess mass computation, if correctly done, yields a unique result which does not depend on any assumptions about the shape of the body. It is a very powerful tool for esimating tonnages of ore or the total size (in the sense of mass) of a structure. The known total mass may be used as a constraint for more ambitious interpretations of gravity anomalies.

A convenient surface to consider for our Gauss' Law integration is a plane at the surface of the Earth and a hemisphere below it:

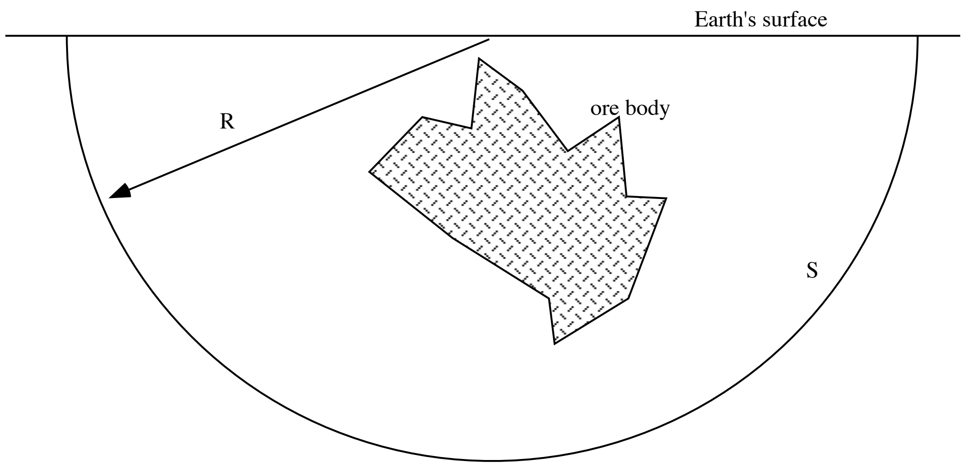

Figure 22: Geometry for excess mass calculation.

We have that

$$\int_S \mathbf{g} \cdot ds = \int_{\text{plane}} \mathbf{g} \cdot ds + \int_{\text{hemisphere}} \mathbf{g} \cdot ds = 4\pi GM$$

where now M denotes excess mass. Now, R, the radius of the hemisphere, may be made as large as we like, so large in fact that M appears as a point mass at the surface of the Earth. The integral over the hemisphere may then be evaluated by integrating the relation for a point mass:

$$\mathbf{g} = -G\frac{m}{R^2}\mathbf{r}$$

but it is much easier to apply Gauss's Law again and state that from symmetry, the integral over the hemisphere is $2\pi GM$; half the integral over the whole sphere which from Gauss' Law is simply $4\pi GM$. So now we have

$$\int_{\text{plane}} \mathbf{g}.ds = -2\pi GM$$

Since g_z is in the opposite direction to ds, we can drop the minus sign and write total excess mass M as

$$M = \frac{1}{2\pi G}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\ g_z$$

where g_z is the vertical component of gravity; both what we measure and the component perpendicular to our chosen surface. This integration may be applied to real data by using numerical integration procedures or simply by gridding and summing:

$$M = \frac{1}{2\pi G}\sum_{grid\ elements} g_i.A_i$$

where A_i is the area of each grid element and g_i is the gravity anomaly in that element. If all the element areas are the same (say, A), the A_i can be taken outside the sum:

$$M = A\frac{1}{2\pi G}\sum_{grid\ elements} g_i$$

Obviously the integration cannot be taken to infinity, so the integration or summation is terminated when the tails of the anomaly get very small. Herein lies the weakness of the excess mass calculation: for the result to be accurate:

a) The tails must be small, and

b) The residual must be well estimated (because the removal of the residual will very much determine the size of the anomaly tails.

The excess mass of a 2D body is given by:

$$M = \frac{1}{2\pi G}\int_{-\infty}^{\infty} dx g_z$$

where now M is the total excess mass per unit length of the body.

10. Depth rules in gravity.

We know that a point mass will produce the sharpest possible gravity anomaly from a given depth to the top of a body (see Figure 27). This is the basis for a simple maximum depth rule in gravity.

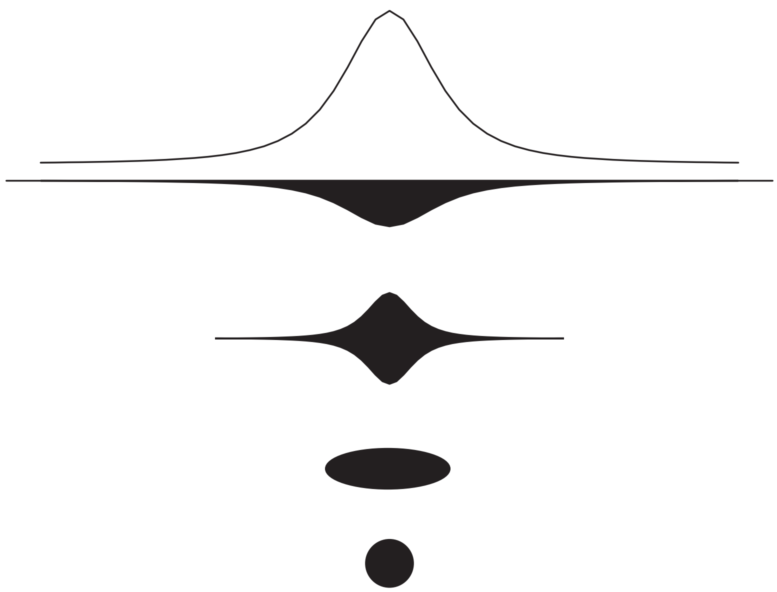

Figure 27. All these bodies will produce the same gravity anomaly (in a schematic sense), but the deepest one must be a point mass.

Consider a point mass at depth z. We know that g due to a point mass is given by

$$\mathbf{g} = \frac{GM}{r^2}\mathbf{r}$$

so the vertical component at P is given by

$$g_z = g\cos\phi = \frac{GM}{r^2}\cdot\frac{z}{r} = GM\frac{z}{(x^2+z^2)^{\frac{3}{2}}}$$

It should be clear that the maximum g_z occurs when $x = 0$ (i.e. directly over the anomaly) and so is

$$g_{max}(x=0) = \frac{GM}{z^2}$$

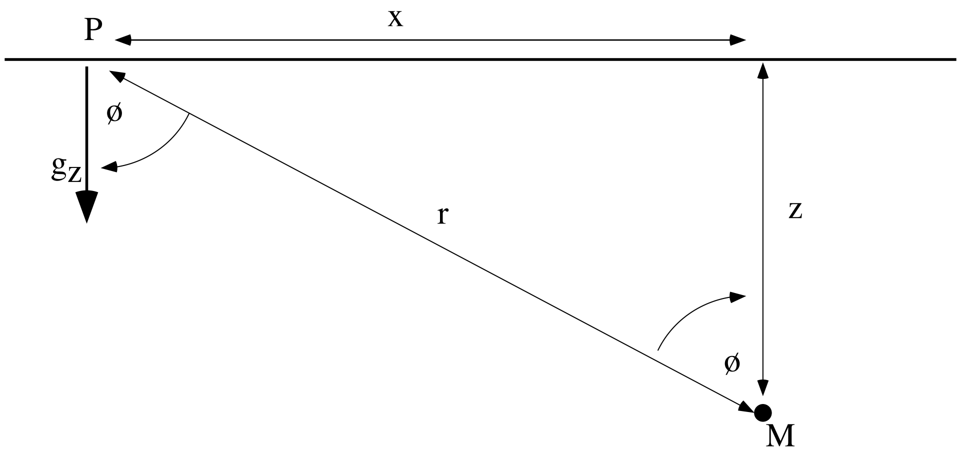

Figure 25: Geometry for point mass depth calculation.

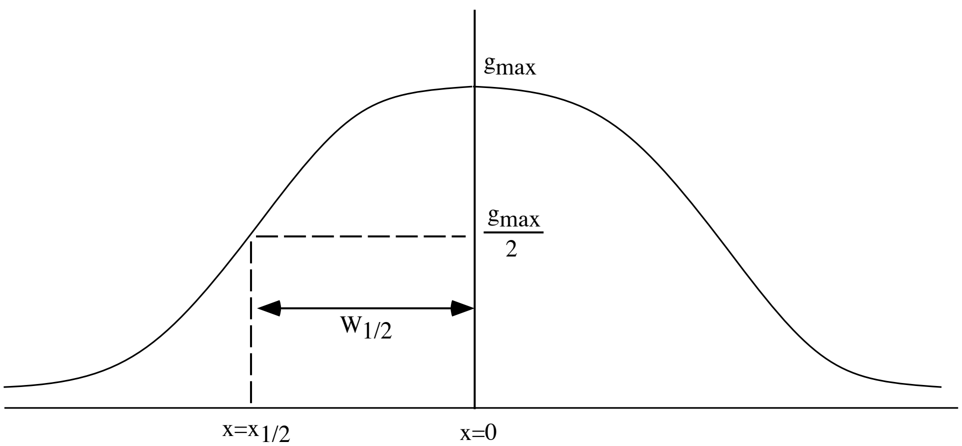

Figure 26: Definition of half width at half maximum.

Now, we know that the anomaly will be sharper as z gets smaller, but we need to quantify this, so we will introduce the half-width at half-maximum. (Be careful not to confuse this measure with the full width at half maximum.) The x coordinate of this half-width is found by solving

$$\frac{g_{max}}{2} = \frac{GM}{2z^2} = GM\frac{z}{(x_{1/2}^2 + z^2)^{\frac{3}{2}}}$$

so

$$2z^3 = (x_{1/2}^2 + z^2)^{\frac{3}{2}}$$

$$(2^{\frac{2}{3}} - 1)z^2 = x_{1/2}^2$$

$$z_{max} = \frac{x_{1/2}}{\sqrt{2^{\frac{2}{3}} - 1}}$$

or

$$z_{max} = 1.3x_{1/2}$$

Another maximum depth rule may be obtained from differentiation. This is a little complicated but yields:

$$z_{max} = \frac{0.86|g_{max}|}{|(\partial g/\partial x)_{max}|}$$

If the anomalies are known to be two-dimensional, then the analogue of the point mass is the line mass (a cylinder of zero diameter). The anomaly for such an object is broader than that of a point mass, so the above rules yield maximum depths which are uneccessarily conservative. However, a similar approach may be taken to yield

$$z = 1.0x_{1/2}$$

and

$$z_{max} = \frac{0.65|g_{max}|}{|(\partial g/\partial x)_{max}|}$$

for 2D bodies.

11. Forward modelling of chosen shapes.

If we have reason to believe that the structure causing a gravity, magnetic or electromagnetic anomaly is of a certain shape, we can compute the geophysical effects of the general form of such a shape, and then by trial and error or manipulation of the expressions for the shape we may use the data to determine the dimensions of our structure. Thus, if we have the gravity profile across a structure which we believe to be a normal fault of geometry we may estimate the gravity at the station from first principles.

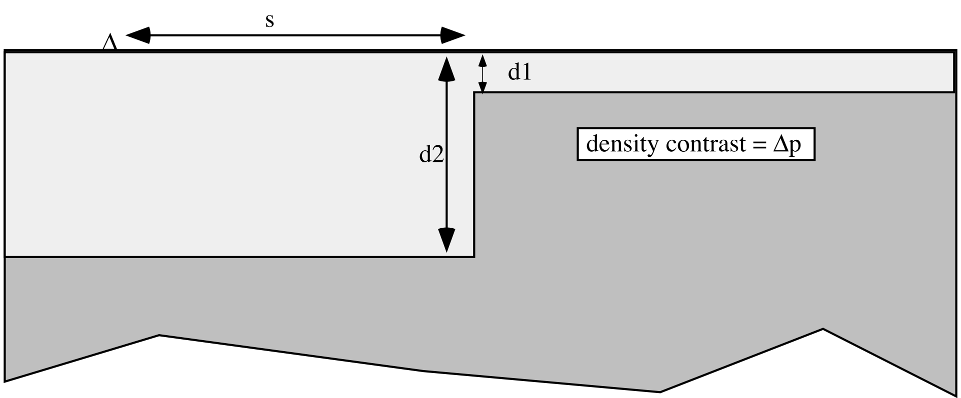

Figure 23: Geometry for calculation of gravitational effect of a fault.

Two dimensional anomalies. The above structure is 2D, and this will occur so often that it is worth deriving an expression for the general 2D case. We recall that the expression for the potential due to a general (3D) body is

$$U_p = G\Delta\rho \int\int\int \frac{dx\ dy\ dz}{R}$$

so for a 2D body we will have

$$U_p = G\Delta\rho \int dx \int dz \int_{-\infty}^{\infty} \frac{dy}{R}$$

The last integral is tricky to evaluate, but Telford et al. (page 8) show it to be equal to $-2\ln(R)$ so we have that

$$U_p = -2G\Delta\rho \int dx \int dz\ \ln(R)$$

and therfore

$$g_z = -\frac{\partial U}{\partial z} = 2G\Delta\rho \int\int \frac{z}{R^2}\ dxdz$$

remembering that $R = (x^2 + z^2)^{1/2}$.

Back to our fault: We know that the gravity method is only sensitive to lateral variations in density, so all rocks above the depth d_1 and below the depth d_2 will not contribute to the anomaly. That is, we need only integrate d_1 to d_2, which is equivalent to considering a semi-infinite slab. If the origin is considered to be at the station then

$$g_z = 2G\Delta\rho \int_{d_1}^{d_2} dz \int_{s}^{\infty} dx \frac{z}{x^2 + z^2}$$

$$= 2G\Delta\rho \int_{d_1}^{d_2} dz \left[z \frac{1}{z} \tan^{-1}(x/z) \right]_{x=s}^{\infty}$$

$$= 2G\Delta\rho \int_{d_1}^{d_2} dz \left[\frac{\pi}{2} - \tan^{-1}(s/z) \right]$$

We will have to integrate w.r.t. z, so we note that $\tan^{-1}(x) = \pi/2 - \tan^{-1}(1/x)$, which removes the $\pi/2$:

$$g_z = 2G\Delta\rho \int_{d_1}^{d_2} dz \tan^{-1}(z/s)$$

and allows us to look up $\tan^{-1}(x/a)$ in a table of integrals to get

$$g_z = 2G\Delta\rho \left[z \tan^{-1}(z/s) - \frac{s}{2} \ln(s^2 + z^2) \right]_{z=d_1}^{d_2}$$

So

$$g_z = 2G\Delta\rho \left[d_2 \tan^{-1}(d_2/s) - \frac{s}{2} \ln(s^2 + d_2^2) - d_1 \tan^{-1}(d_1/s) + \frac{s}{2} \ln(s^2 + d_1^2) \right]$$

$$= 2G\Delta\rho \left[d_2 \tan^{-1}(d_2/s) - d_1 \tan^{-1}\left(\frac{d_1}{s}\right) + \frac{s}{2} \ln\left(\frac{s^2 + d_1^2}{s^2 + d_2^2}\right) \right]$$

If the fault outcrops, *i.e.* $d_1 = 0$, then

$$g_z = 2G\Delta\rho \left[d_2 \tan^{-1}\left(\frac{d_2}{s}\right) + \frac{s}{2} \ln\left(\frac{s^2}{s^2 + d_2^2}\right) \right]$$

These are analogous to expressions 2.70 and 2.71 of Telford *et al.*, but arrived at by a more direct route.

How will these expressions help us? Well, we obviously can plug in some numbers for d_1, d_2 and $\Delta\rho$ to get a profile across the fault, but there are some more subtle ways. Firstly, we can obtain an expression for $g_z(0)$ by using $\tan^{-1}(d/s) = \pi/2 - \tan^{-1}(s/d)$ again, which equals $\pi/2$ if s is zero, so for the non-outcropping fault $g_z(0) = \pi G\Delta\rho(d_1 - d_2)$. Observe also that if $-s$ is substitued for s the expression just reverses sign, showing the anomaly to be symmetrical about the fault. If the field data are not symmetrical across then the fault model may be discarded immediately. Unfortunately, the above expressions are not in a nice form for getting the behaviour of $g_z(\pm\infty)$, but we know that an infinite distance away from the fault gravity must be zero, and so the symmetry about $g_z(0) = \pi G\Delta\rho(d_1 - d_2)$ gives $g_z(\infty) = 2\pi G\Delta\rho(d_1 - d_2)$. Compare this with the formula for a Bouguer slab! So we

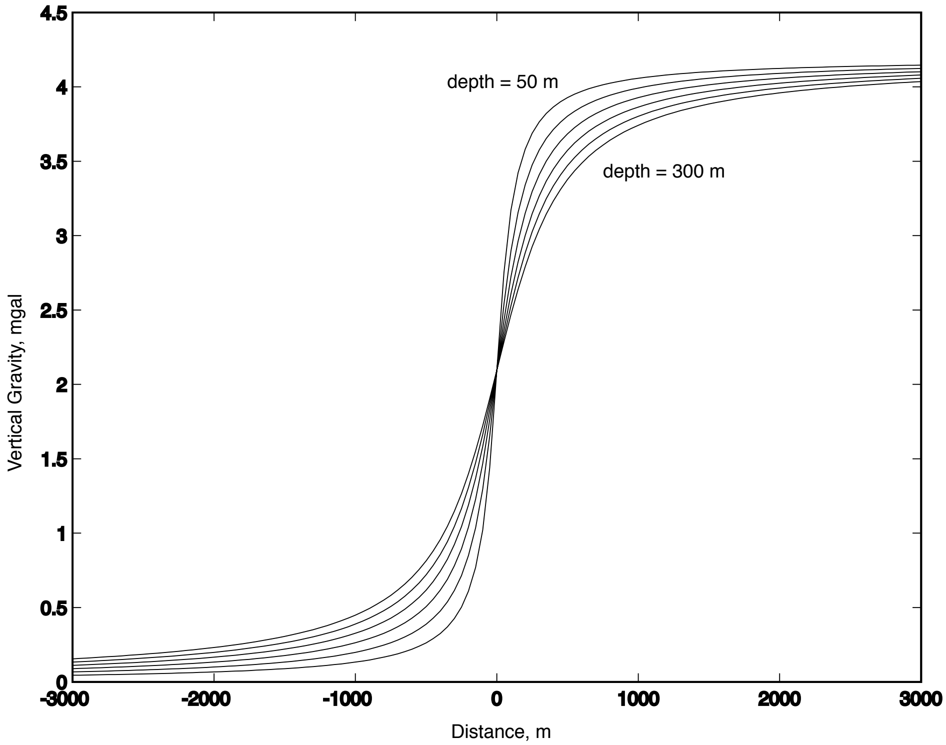

Figure 24: Gravity profiles over faults of throw 100 m and density contrast 1.0 g/cm^3 at depths of 50, 100, 150 m *etc*.

see that the total difference in gravity across the fault gives us an idea of $\Delta\rho$ times the fault's throw. How deep is the top of the fault? The shallower the fault, the sharper the anomaly, which we would expect.

We can quantify this by finding an expression for the slope at $s = 0$. We find that

$$\left(\frac{\partial g_z}{\partial s}\right)_{s=0} = G\Delta\rho \ln(d_2/d_1)$$

which we can invert to get

$$\frac{d_2}{d_1} = \exp\left(\frac{1}{2G\Delta\rho}\left(\frac{\partial s}{\partial g_z}\right)_{s=0}\right)$$

So, if we know $\Delta\rho$, we can solve for the rest of the fault geometry.

The above computations illustrate how forward modelling helps us to interpret geophysical data. If the reasoning behind this example is understood, there is no advantage to going through the calculations for a number of different bodies; one would only be learning more about integration and differentiation, not geophysics. Whenever a new problem is encountered, the required expressions may be either derived from one's understanding of the physics, or looked up in a text like Telford *et al.*