

## Global mapping of the electrically conductive lower mantle

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**Abstract.** It is known that the electrical conductivity of the Earth's mantle increases to about 1 S/m at depths between 500 and 1000 km. This increase could equally well be ascribed to thermally activated conduction or to enhanced conductivity in high pressure mineral phases. Thermal activation would produce a smooth conductivity profile while conductivity associated with abrupt phase changes would also vary abruptly. Unfortunately, geomagnetic data alone cannot distinguish between these two models. However, under the assumption that the variation is indeed abrupt, we seek the best estimate for the depth to a conductivity jump. A peak in the geomagnetic spectrum around a period of 27 days produces an electromagnetic response at this period with least uncertainty and least bias associated with breakdown of an assumed  $P_1^0$  source-field geometry. Fitting such data using a simple two-parameter model of a buried conducting sphere, we estimate a conductivity of  $1.18 \pm 0.10$  S/m at a depth of  $650 \pm 20$  km. The coincidence of this result with estimated depths to the 660 km seismic discontinuity provides independent support for the hypothesis that the observed abrupt change in the elasticity of the mantle is also accompanied by an equally abrupt change in the electrical conductivity. Both physical properties are presumably associated with a mineral transition from an olivine-dominated upper mantle composition to perovskite/wüstite assemblage.

### Introduction

Temporal variations in the Earth's external magnetic field at periods of several hours to several months have long been used to estimate the electrical conductivity of the deep mantle. One of the earliest works [Lahiri and Price, 1939] indicated that conductivity increases rapidly with depth, and subsequent studies [e.g., Banks, 1969; Parker, 1970; Achache *et al.*, 1981; Hobbs, 1983; Jady and Paterson, 1983; Constable, 1993], while refining this result, have not changed the form of the conductivity curve in a substantial way. It is now generally agreed that conductivity rises from about 0.01 S/m to 1 S/m between the upper mantle and a depth of about 1000 km.

There are two possible explanations for this rise in conductivity. One lies in pressure and temperature driven activation of electrical conduction in silicate semiconductors. For example, measurements on dry olivine demonstrate a reproducible thermally activated conduction [e.g., Constable *et al.*, 1992], with indications of increased activation energy at the highest temperatures attainable in the laboratory (1300°C or so). It

is entirely possible that a high activation energy conduction mechanism in olivine or another phase is responsible for increased conductivity at depth.

The second explanation is associated with phase changes that occur within the mantle. At around 400 km depth, olivine begins to transform to the beta and gamma spinel phases, while at similar depths majorite garnet develops. At 660 km these minerals are thought to convert abruptly to silicate perovskite and magnesiowüstite. The higher pressure phases have all been proposed as being more conductive than untransformed olivine [Akimoto and Fujisawa, 1965; Kavner *et al.*, 1996; Shankland *et al.*, 1993; Li and Jeanloz, 1990, 1991], although measurement of electrical conductivity in the laboratory under high pressure represents a significant challenge.

The two different mechanisms described above would generate very different conductivity profiles. One would expect a thermally activated conductivity to increase smoothly with depth, albeit steeply for high activation energies. On the other hand, electrical conductivity changes associated with phase transitions would be abrupt, particularly for the perovskite transformation thought to be associated with the 660 km seismic discontinuity. Unfortunately, the diffusive nature of geomagnetic fields propagating in conductors means that one cannot distinguish between smooth and abrupt transitions in conductivity.

Although they do not require abrupt jumps in conductivity, the geomagnetic data are easily compatible with such jumps as local site analyses reveal [Egbert and Booker, 1992; Schultz *et al.*, 1993]. Schultz [1990] found adequate fits to various single site data sets with both jumps and sharp increases in conductivity between 500 and 1000 km, while in a global analysis Constable [1993] chose to assume a priori that a jump was associated with the 660 km seismic discontinuity. The question we ask in this paper is this: If we assume that an abrupt global conductivity jump between 500 and 1000 km exists, what is the best estimate we can make of its location and what is the uncertainty in this estimate? A coincidence of location with a phase boundary would support the hypothesis that the higher pressure phase is indeed more conductive than the overlying material.

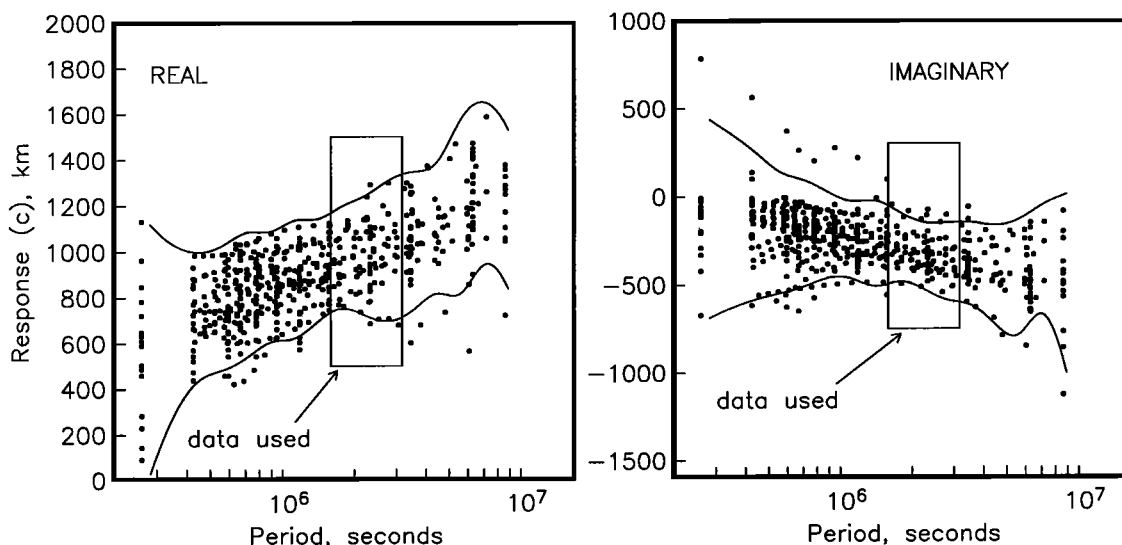
### Method and Results

In order to provide the best estimate for a conductivity jump between 500 and 1000 km depth, we seek the best data that are sensitive to these depths. To avoid sensitivity and attenuation in the upper mantle, we need periods of at least  $10^5$  s, based on simple skin depth considerations. Similarly, measurable attenuation in the deep conductive material requires periods shorter than  $10^7$  s. This is the band obtained by analysis of magnetic observatory data using the assumption of a  $P_1^0$  source-field ge-

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**Figure 1.** The global data set compiled by *Constable* [1993], replotted here without individual error bars, but with 2 standard deviation envelopes plotted around the average response. It can be seen that the variance is minimum around one month period, particularly for the imaginary component. The boxes indicate which data contribute to the response data used in this study.

ometry. Both *Roberts* [1984] and *Schultz and Larsen* [1987] published such data from a set of observatories, and *Constable* [1993] took band averages of their data on the grounds that averages provided a better estimate of radial conductivity. Figure 1 presents the compilation of *Constable* [1993], expressed as complex scale length  $c$  of *Schmucker* [1970]. These data are extensively described in *Constable's* study and the original references. They represent 39 independent estimates of the geomagnetic response from 29 different observatories. *Constable's* error bars were determined directly from the scatter of data in each band. Inspection of this scatter shows that it is a minimum at a period of around 27 days ( $2.3 \times 10^6$  s). To illustrate this, 2 standard deviation envelopes have been plotted in Figure 1 around the average response. Variations in upper mantle conductivity are presumably responsible for increased variance at shorter periods [*Roberts*, 1984; *Schultz and Larsen*, 1990], while at longer periods it is likely that the  $P_1^0$  source-field assumption begins to break down. It is also known that there is a strong peak in the geomagnetic power spectrum at 27 days, and so one would expect best signal to noise ratio and least bias from non- $P_1^0$  geometry at this period [*Banks*, 1969].

We selected the response from *Constable's* [1993] compilation at a period of 27 days, as well as the two adjacent periods (specifically data at periods of 1,778 ks, 2,238 ks, and 2,818 ks). At these periods the upper mantle is transparent, and so the simple conductive sphere model of *Chapman and Price* [1930] is adequate to represent a discrete jump in conductivity. In the more modern notation of *Chapman and Bartels* [1940] this model is given by:

$$Q_1^0 = \frac{q^3}{2} \left[ 1 - \frac{F_1}{F_0} \right]$$

where

$$F_0 = \frac{\sinh(x)}{x}$$

$$F_1 = \frac{3}{x^2} \left[ \cosh(x) - \frac{\sinh(x)}{x} \right]$$

and

$$x = (1 + i)/\beta$$

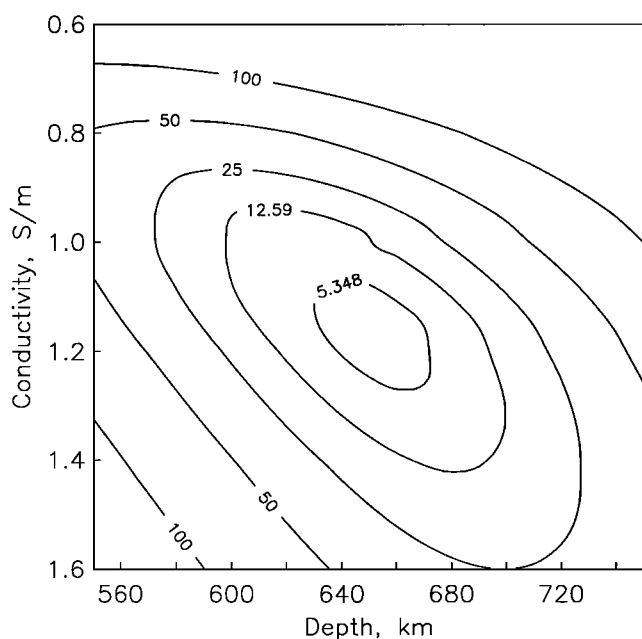
$$\beta^2 = \omega \mu \sigma q^2 a^2 / 2$$

Here  $a$  is the radius of observation (i.e. Earth's radius),  $qa$  is the conductive sphere radius, and  $\omega$ ,  $\mu$  and  $i$  are the usual representations of angular frequency, magnetic permeability, and  $\sqrt{-1}$ . The response  $Q$  is the ratio of internal to external magnetic fields, which for  $P_1^0$  source-field geometry is related to the inductive scale length  $c$  by

$$Q_1(\omega) = \frac{1/2 - c/a}{1 + c/a}$$

The elegance of the Chapman-Price model is that it is determined by only two parameters (the depth and conductivity of the sphere), and so we are able to contour the sum of squared misfits  $X^2$  between the data and the model predictions (Figure 2). If the data errors are independent, zero mean, and normally distributed then  $X^2$  is  $\chi_6^2$  distributed (the 6 degrees of freedom come from having real and imaginary components for the 3 data). It is probably safe to assume normal errors; *Constable* [1993] examined the distribution of the data in support of this claim. The 50% and 95% confidence levels for  $\chi_6^2$  of 5.348 and 12.59 respectively are contoured in Figure 1, along with  $X^2$  levels of 25, 50, and 100 for clarity. Taking the 50% contour to represent one standard error, we estimate parameters for our model of  $1.18 \pm 0.10$  S/m at a depth of  $650 \pm 20$  km. These are nonlinear forward calculations that do not rely on iteration or inversion, and because we are able to explore all parts of the relevant parameter space we can be confident that other minima do not exist.

The uncertainty in our parameter estimates is surprisingly small. Inclusion of additional data, at periods away from 27 days, markedly degrades the quality of the fit and increases the diameter of the 50% confidence level, lending support to our assertion that signal to noise is best at the 27-day period. Two-layer models were computed to demonstrate that until the upper mantle attains a conductivity of about 0.01 S/m, the



**Figure 2.** Contours of summed-squared misfit as a function of the two parameters in the Chapman-Price model; depth and electrical conductivity of a geocentric sphere. The 5.348 contour represents the 50% confidence level of the  $\chi^2_6$  statistic, while 12.59 represents the 95% level.

approximation of zero conductivity outside the Chapman-Price sphere is acceptable.

Shearer and Masters [1992] used long period precursors to the SS seismic phase to map the 660 km seismic discontinuity, and a depth variation of up to 30 km was observed. An estimated average depth of 653 km for the discontinuity was obtained by Shearer [1993], again with variations, up to 40 km. Thus our global estimate of the depth to a jump in conductivity agrees very well with the global estimates of the depth to the seismic discontinuity.

Our model conductivity of  $1.18 \pm 0.10$  S/m represents an average conductivity between 650 km and about 1000 km depth, based on simple skin depth considerations and also on a more sophisticated resolution analysis by Schultz [1990], who used a set of Backus-Gilbert resolving kernels [Backus and Gilbert, 1968, 1970] and a resolution technique of Smith and Booker [1988]. Our model assumes a constant conductivity below the jump, which is likely to be a good approximation given the lack of structure at these depths in previous models [e.g., Constable, 1993; Schultz *et al.*, 1993]. The magnitude of diamond-anvil measurements for perovskite and perovskite/wüstite conductivity [Shankland *et al.*, 1993] agree well with our lower mantle conductivity estimate. Variations in lower mantle conductivity with depth are probably not great, based on observations of a low activation energy for samples analysed by Shankland *et al.* [1993], which also supports our constant conductivity approximation.

## Conclusions

We stress that the presence of an abrupt jump in conductivity in the mantle is an assumption associated with our model, not a demonstrated requirement of the geomagnetic data. No finite data set can distinguish between a smooth and abrupt conductivity profile. However, the close association between our

global estimate of the depth to an inferred conductivity jump (650 km) and the depth to the seismic discontinuity (653 km) supports the hypothesis that the phase transformation from olivine+majorite to perovskite+wüstite is associated with an abrupt change in both seismic velocity and electrical conductivity.

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